

STICKY NOTE STATISTICS

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Overview of Lesson

This set of activities allows students to analyze data from graphs created with sticky notes. Students are asked to estimate a string's length. Their estimates are put on sticky notes, forming dot and box plots. Following initial estimates and graphs, more estimation exercises are given to students (with strings of different lengths). Once again, teachers ask students to estimate, and sticky notes reveal students' estimations and graphs. Sticky notes provide ease in collecting and reviewing the data and a visualization of data. The statistical question is "Do students improve their estimates with repeated estimation experiences?" Students will justify their answers by applying their knowledge of graphs and summary statistics to describe center and spread.

Type of Data

- One quantitative variable
- Data generated or collected as a class

Learning Objectives

Students will:

- Estimate the length of an object using appropriate standard units with reasonable accuracy.
- Create and interpret a dot plot and box plot to visualize a distribution of quantitative values.
- Justify conjectures using measures of center and variability (mean, median, range, and interquartile range)
- CCSSM 6.SP.5: Summarize numerical data sets in relation to their context.

Audience

- The audience for this lesson is a middle school mathematics classroom.
- Before this lesson, students should have experience with length (inches) and creating dot and box plots as well as finding the mean, median, range, and interquartile range.

Time Required

90 minutes: Two 45 minute class periods or one 90 minute block scheduled class.

Technology and Other Materials

- A computer with internet access. The lesson involves using GeoGebra to create graphs using statistical features (see attached materials).
- Sticky Notes of the same color and a labeled axis (on poster board or white board)
- Strings of different lengths

Lesson Plan

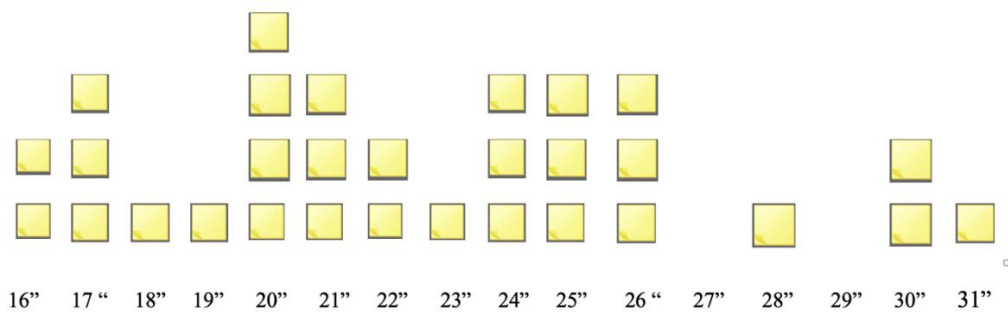
This lesson combines two essential skills in mathematics: estimation and interpretation of graphical and quantitative information. Specifically, students estimate the length of a string then construct graphs and calculate numerical summaries based on the student estimates written on sticky notes. This process is repeated with strings of different lengths. Students then discuss the statistical question “Do estimates improve after repeated experiences of estimation?”

Introducing the Investigative Question and Gathering Data

A string (preferably of a bright color) measuring 28 inches is shown to the students. The teachers can walk around the room so students can see the string up close. Students then write their estimates on sticky notes, and after the notes are collected, a dot plot is constructed based on their estimates as shown in Figure 1. Using the information from the dot plot, a box plot is then constructed. After creating both graphs, students discuss the center (mean, median) and spread (range, interquartile range) of the data.

Figure 1

Dot and Box Plots of Students' Estimates with Sticky Notes



Using the Five Number Summary, students construct a box plot with the given data in Figure 1.

Minimum Value: 16

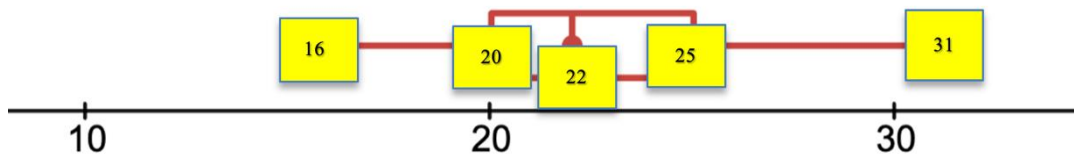
Maximum Value: 31

Median = 22

Quartile 1 = 20

Quartile 3 = 25

Analyzing the Data



The actual length is revealed (28"). Asking students how they estimate is an integral part of teaching estimation. Such as “What was your referent? , “Did you use any objects to compare the

length of the string?” “Does it help to move the string vertically or horizontally?” These questions emphasize the importance of having a referent—a familiar object like a pencil or a shoe—to compare the length of the string.

Analysis of Measures of Center and Spread

In this stage of the lesson, the teacher asks students to estimate again, this time with strings of different sizes. The goal of this part of the lesson is to encourage students to compare the dot and box plots of multiple estimations and to see if the estimates are closer to the actual measures and there is less variation as they get more experiences in estimation. The teacher asks “How can each graph show us if we are getting better at our estimates?” Instead of using the sticky notes to create a graph, this time, the sticky notes will be used to simply collect estimates to be used in GeoGebra and to provide the 5 number summaries. This particular program enables an efficient way to sort any numeric data from highest to lowest values, calculate the mean and median, and produce graphs and tables to show learner’s progress over time in estimation. Step-by-Step directions on using the Geogebra Statistics program are found in Appendix A.

Table 1.

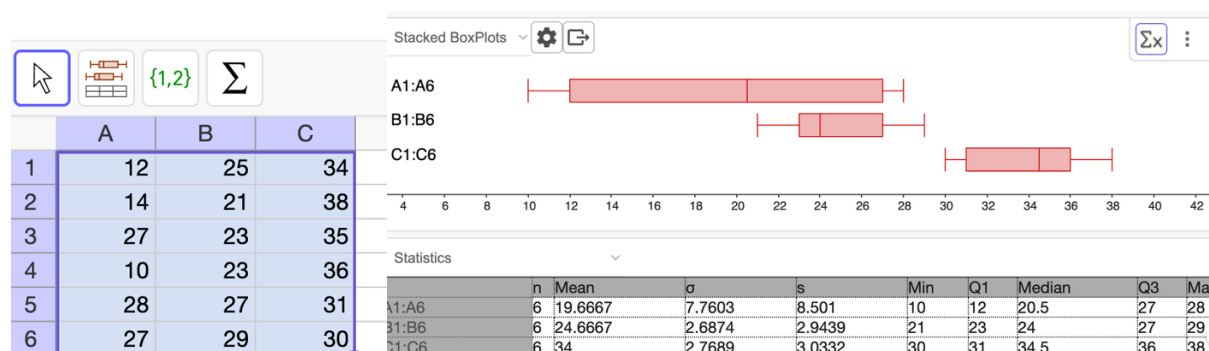
Using Geogebra to Compare Data From Multiple Estimations

What the Teacher Will Do	What the Students Will Do
Again strings of different lengths are introduced to students.	Students write their estimates on sticky notes and turn them in.
1. Teachers enter data within the GeoGebra Statistics Program (see Appendix A) https://www.geogebra.org/classic#spreadsheet 2. Enter students’ estimates of the three string lengths in three separate columns. 3. The teacher then shows the three box plots to the class (selecting the  button. Then select Box Plots making sure that all three columns are selected) and asks,. What do you notice? What do you wonder about the graphs? 4. After students provide their observations, press the  button (upper right of the box plot screen). This will provide summary statistics.	1. Students take note of the data being entered and make a prediction about whether the data points are getting closer and explain how they know. 2. Students examine the graphs and identify the minimum, maximum, Q1, median, and Q3 from the box plots. 3, Examine the box plots and determine if they show better estimates each time. Students may mention changes of center (related to accuracy) and/or changes of spread (related to precision). 4 Students make observations and conclusions based upon the box plots and the summary statistics.

A display of the Geogebra screen is found in Figure 2 showing students' estimates when the strings are each 17", 24", and 34" respectively.

Figure 2

Geogebra Statistics Display of Sample Data (Step-by-Step explanation found in Appendix A)



Using Center and Spread Statistics to Analyze the Statistical Question

The above displays are helpful in examining the statistical question in this lesson plan, yet for a more detailed justification, we use the Geogebra Statistical program to calculate (step 4 above) the mean, median, range, and interquartile range.

Table 2: Measures of Center for Sample Data

Mean and median for estimates in Column A	Mean and median for estimates in Column B	Mean and median for estimates in Column C
Mean: 19.7 inches Median: 20.5 Actual Length: 17 inches	Mean: 24.7 inches Median: 24 Actual Length: 24 inches	Mean: 34 inches Median: 34.5 Actual Length: 34 inches

The Range shows the spread between the smallest and largest values.

The Inter Quartile Range measures the spread of the middle 50% of the data.

Table 3: Measures of Spread (range and interquartile range) for Sample Data

Range and Interquartile Range for Estimates in Column A	Range and Interquartile Range for Estimates in Column B	Range and Interquartile Range for Estimates in Column C
Range: 18 Interquartile Range: 15 Actual Length: 17 inches	Range: 8 Interquartile Range: 4 Actual Length: 24 inches	Range: 8 Interquartile Range: 5 Actual Length: 34 inches

Wrap-up Questions for Discussion:

- How has the mean or median changed and what does that tell us? (*Sample answer:* The mean and median have changed by getting closer to the actual length. Students' estimates are becoming closer to the actual.)
- Are the mean and median values getting closer together? (*Sample answer:* The mean and median are getting closer together. This closeness indicates that the data sets are more symmetrical).
- Are the range and interquartile range getting smaller? If so, what does that tell us about our estimation skills? (*Sample answer:* When the range is getting smaller the difference between the maximum and minimum values are less extreme or outliers. When the interquartile range gets smaller, the difference between the Q1 and Q3 are closer together meaning the data points are getting closer to the median. Both of these show students are getting more consistent with their estimates.)

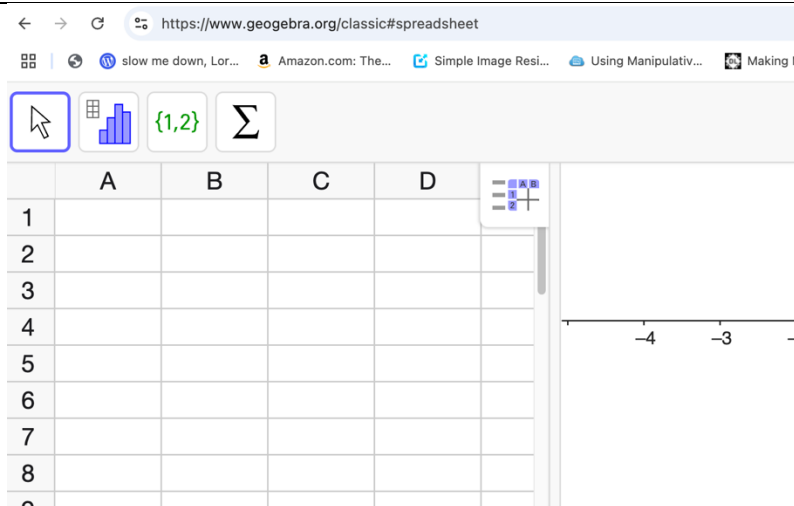
Teachers may use Appendix B which introduces students to a variety of estimation exercises by Steve Wyborney for future estimation exercises.

Appendix A

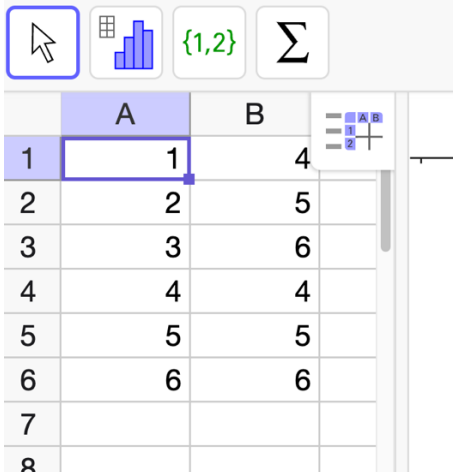
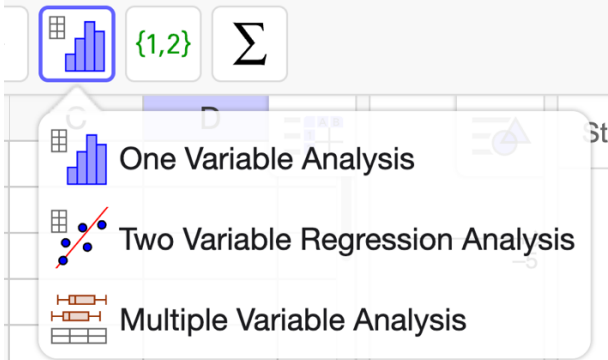
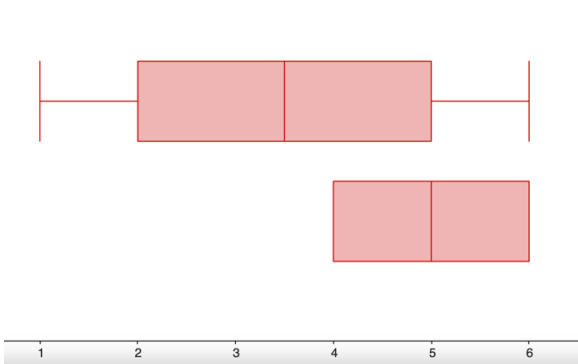
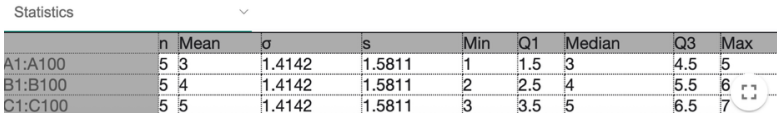
Instructions for Creating Parallel Boxplots with GeoGebra Classic

Visit:
<https://www.geogebra.org/classic#spreadsheet>

You will notice a spreadsheet on the left side and on the right side is where we get our box plots.



The screenshot shows the GeoGebra Classic web interface. On the left is a spreadsheet with columns A, B, C, and D, and rows 1 through 9. On the right is a box plot view with a horizontal axis labeled -4 and -3. The interface includes a toolbar with icons for selection, data entry, and calculation.

To enter data, use the columns on the left side The goal is to compute statistics of both of the columns at the same time.																																																			
Highlight the columns of data. Go to the histogram purple icon just above the A Column and select multiple variable analysis.																																																			
After you click on that icon you will see the box plots as one of the graph options.																																																			
To see summary data, press the Σx button (upper right of the box plots). This will provide summary statistics.	 <table><tr><th colspan="10">Statistics</th></tr><tr><th></th><th>n</th><th>Mean</th><th>σ</th><th>s</th><th>Min</th><th>Q1</th><th>Median</th><th>Q3</th><th>Max</th></tr><tr><td>A1:A100</td><td>5</td><td>3</td><td>1.4142</td><td>1.5811</td><td>1</td><td>1.5</td><td>3</td><td>4.5</td><td>5</td></tr><tr><td>B1:B100</td><td>5</td><td>4</td><td>1.4142</td><td>1.5811</td><td>2</td><td>2.5</td><td>4</td><td>5.5</td><td>6</td></tr><tr><td>C1:C100</td><td>5</td><td>5</td><td>1.4142</td><td>1.5811</td><td>3</td><td>3.5</td><td>5</td><td>6.5</td><td>7</td></tr></table>	Statistics											n	Mean	σ	s	Min	Q1	Median	Q3	Max	A1:A100	5	3	1.4142	1.5811	1	1.5	3	4.5	5	B1:B100	5	4	1.4142	1.5811	2	2.5	4	5.5	6	C1:C100	5	5	1.4142	1.5811	3	3.5	5	6.5	7
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Appendix B

Additional Resources for Estimation and Comparison of Data

Studies show that students improve their estimation abilities when they are provided with repeated opportunities to compare quantities (Siegler, & Opfer, 2003). Through frequent exposure to comparisons between numbers or measurements, students become better at recognizing relative sizes and making more accurate judgments. The following resource by Steve Wyborney gives students many opportunities to show their estimation skills with the potential of using sticky notes to tally results. The *Estimation Clipboard* is a set of lessons that each include four highly similar images. Each image provides an invitation to estimate. Then as new images are introduced, the students' context and intrigue will grow – and so will their estimation skills. The following table is a step-by-step description of the process in Estimation Clipboard.

Estimation Clipboard: Step-by-Step (permission granted by Steve Wyborney)

Slides	Instructions
	1. The first image is revealed. If you hear answers from a small number of students, you are also hearing silence from nearly all of your class. Anticipate engaging all students in mathematical reasoning by the time you reach the third image.
	2. Reveal the answer.
	3. When the <u>second image</u> appears, invite the class to share some estimates aloud again. You will likely hear estimates from more students than the first time. You may want to spend a little more time on the second image, but the power of the Estimation Clipboard is yet to come!

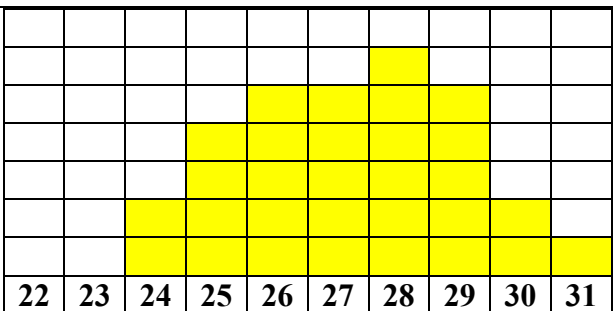
18 pencil sharpeners

A clear glass jar is filled with 18 colorful pencil sharpeners. The sharpeners are in various colors including pink, yellow, blue, and green. They are piled together in the jar, which sits on a light-colored wooden surface. The background is a dark, textured wall.

4. Reveal the answer.	



5. When the third image appears, change your approach. You may not have heard from several students at this point, but everyone's interest is growing. When you show the third image, instead of asking for answers aloud have all of the students write down their estimate on a sticky note. Then have them discuss these two questions with a partner: "What was your estimate? Why did you choose it?" Listen carefully to the reasoning.



6. After all of the students have placed their answers on sticky notes, place them on a dot/note graph. Results of Sticky Note Graph appear.

27 pencil sharpeners

A clear glass jar is filled with 27 colorful pencil sharpeners. The sharpeners are in various colors including yellow, green, blue, and pink. They are stacked on top of each other, with some visible at the top and others partially obscured. The jar is sitting on a light-colored wooden surface.

7. Eventually, perhaps after you have tried several sets, introduce the concept of using a range to estimate – rather than using a single number. Look for opportunities to encourage your students to self-select whether a range or a single number would be more useful.

References

- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, 14(3), 237–243. <https://doi.org/10.1111/1467-9280.02438>
- Steve Wyborney. (n.d.). *Estimation clipboard*. <https://www.stevewyborney.com/estimation-clipboard/> (Used with permission).