Exploring Fundamental Definitions with a Study of Fingerprint Types in the US

Investigative Cycle: When conducting any research, you will need to follow an investigative cycle. In general, an investigative cycle includes *asking a question*, *collecting data* to answer that question, *analyzing the data* and then *making conclusions*. This activity will follow this format.

Directions: With your partner(s), discuss and record answers to the questions that follow. Assign someone the responsibility of sharing your answers with the class. Do not reference any notes or technology. You will have 5 minutes to discuss.

a. What does "randomness" mean?

b. How would you take a sample from a population?

Question

Fingerprints are often used by law enforcement to fight crime and apprehend wrong doers. Each person has a unique fingerprint. If we look at the fingerprint of the thumb, there are three common characteristics: loops (lines that bend back on themselves), whorls (such as circles or ellipses) and arches (wave like patterns). In the United States, 60% of the population has loops, 35% whorls, and 5% arches.

Loop	Whorl	Arch
		

Here are some actual prints. Can you correctly identify the shape of each?

_______________ _______________ _______________

In this activity, we will address the following questions:

- *If you were to take several random samples of American adults, would the sample proportion of individuals with arches change from sample to sample? Why is this?*
- *What would be the most typical value of the sample proportions? Why does this make sense?*
- *How do the shape and center of a sampling distribution for a proportion change as the sample size is increased?*

We will use a bin filled with beads to represent the American population and explore these questions.

Data

Collecting a sample of beads:

1. Why might it be reasonable to take a random sample of beads rather than look at all the beads?

In the bin of beads, each bead represents an individual with a certain type of pattern on their thumb: White for loops, Green for whorls, and Black for arches. We are going to collect a sample of 20 beads and 100 beads using the given paddles.

Draw a sample of size n=20 and n=100. Complete the table provided below.

	n = 20	**n = 100**
x = Number of "Arches"		
n = Sample Size	**20**	**100**
Sample Proportion of "Arches" from your sample ($\hat{p} = x/n$)		

Now construct a bar graph to display the results of each sample.

Figure 1. Results for n=20

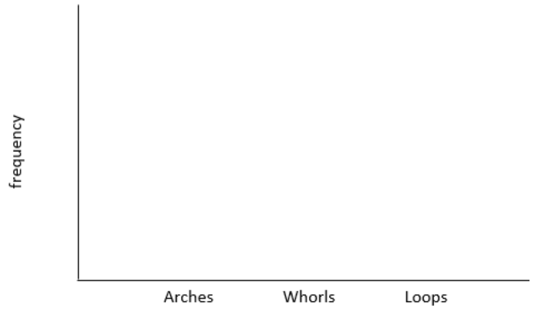
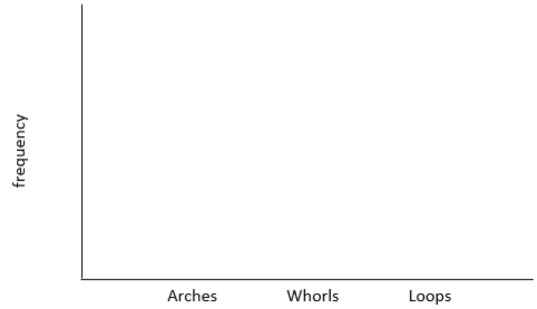


Figure 2. Results for n=100



Let’s continue to summarize our findings with some descriptive statistics.

2. Draw three more samples for n=20 and calculate their associated p-hats. Do you get the same value every time? Why is this?

3. If you were to take several random samples of American adults, would the sample proportion of individuals with <u>arches</u> change from sample to sample? Why is this?

<u>Discuss questions 1-3 as a class.</u>

Analysis

Let's look at the results of everyone in the class. Your instructor will put two axes on the board. Make a dot for each proportion of arches from your samples of size n = 20 and n = 100. Recreate the plots below.

Figure 3. **n = 20**

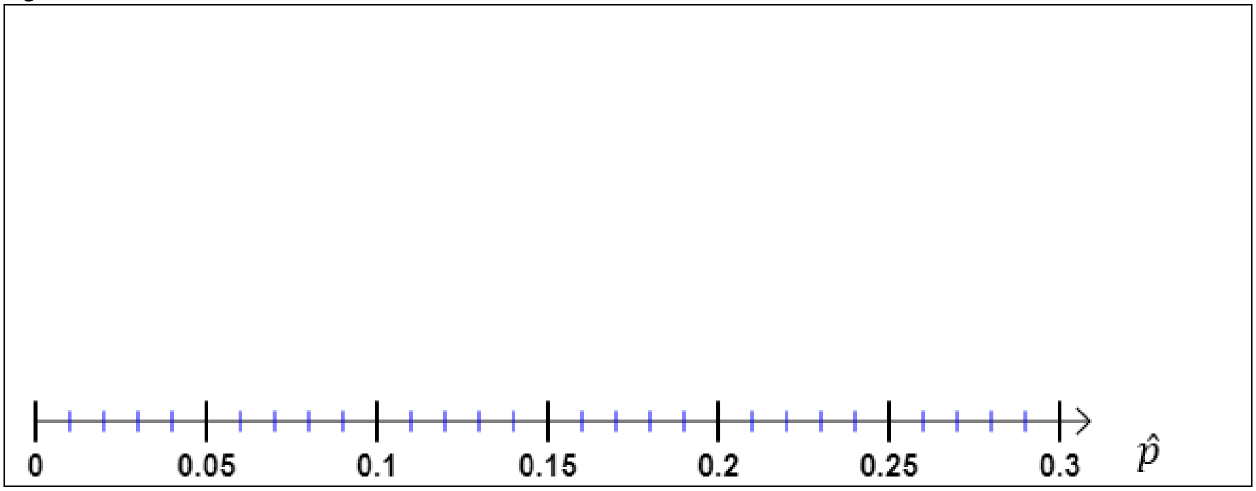
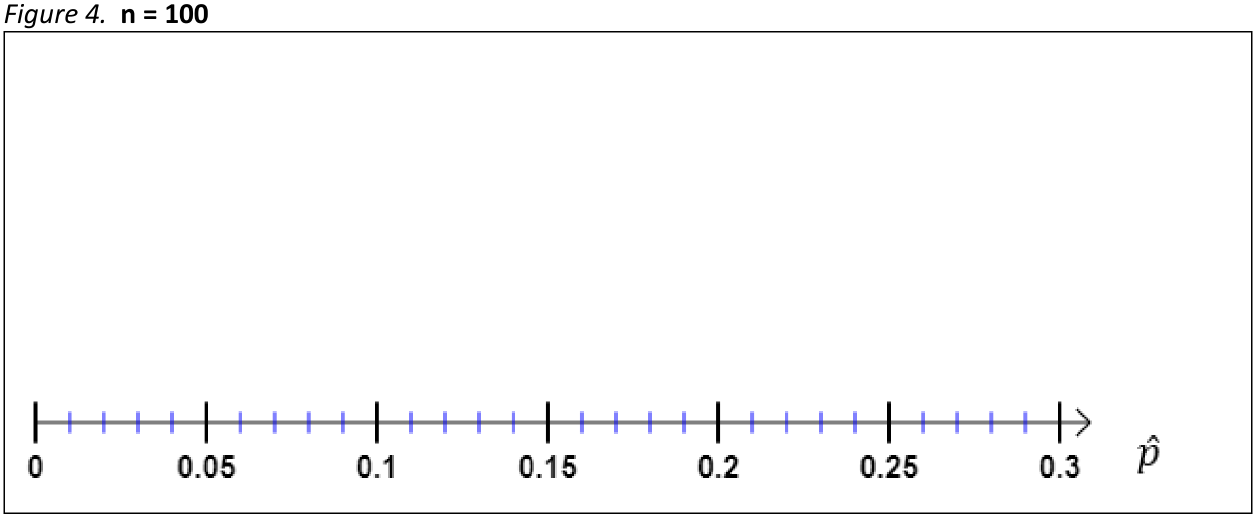


Figure 4. **n = 100**



Let's now compare the features of the graphs in Figures 1 & 3.

For Figure 1:

4. What does the x-axis represent?

5. What does the y-axis represent?

6. Does this graph show the result of one sample or results from many samples?

For Figure 3:

7. What does the x-axis represent?

8. Does this graph show the result of one sample or results from many samples?

9. What does each dot on the graph represent?

<u>Discuss questions 4-9 as a class.</u>

10. What values of $\hat{p}$ should you reasonably expect when sampling? [Consider Figures 3 & 4.]

    For n=20 at a time:

    For n=100 at a time:

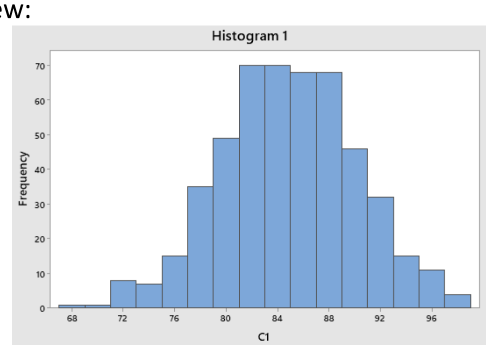
11. Why do you think the sample proportions we calculated for each iteration (draw) were rarely (if ever) above 0.20?

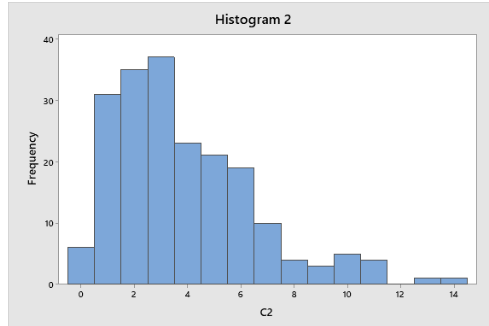
Discuss questions 10 - 11 as a class.

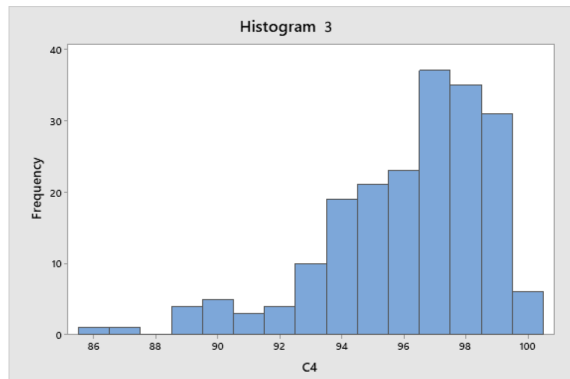
Let's establish some definitions:

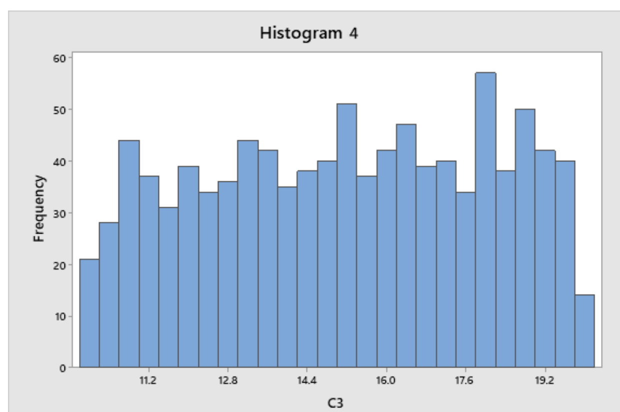
Distribution –

Certain distributions tend to show up often. Here are a few:




Histogram 2
Frequency
C2
0
10
20
30
40
2
4
6
8
12
14


Histogram 3
Frequency
C4
0
10
20
30
40
86
88
90
92
94
96
98
100


Histogram 4
Frequency
C3
0
10
20
30
40
50
60
11.2
12.8
14.4
16.0
17.6
19.2

Let's define the following properties and identify them on each of the above graphs:
Center (graphically) –

Symmetry –

Tail(s) –

Sampling distribution of a statistic –

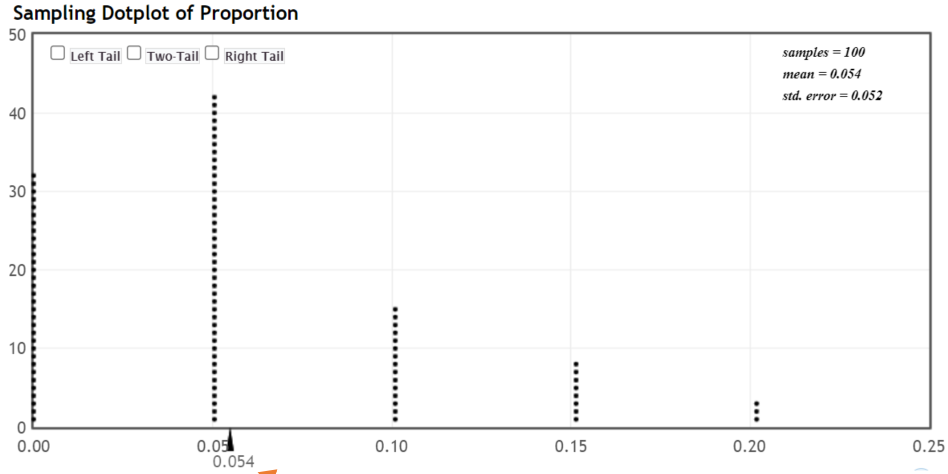
Important Note: The above properties are <u>not</u> used to describe graphs that display distributions of categorical or qualitative data (x-axis shows categories), as shown in the bar graphs we created in Figures 1 & 2.

These properties are only used to describe graphs that display distributions of quantitative data (x-axis is a number line). Examples of acceptable graphs include the four histograms provided above and the dotplots we created in Figures 3 through 4 to display our sampling distributions.

Below are the results from using *StatKey* to run 100 samples for the two sample sizes, n=20 and n=100. Comparing these graphs to the ones we constructed for our earlier samples (Figures 3 & 4), we see that as the number of samples is increased, the shape becomes more pronounced in each sampling distribution.

Figure 5.

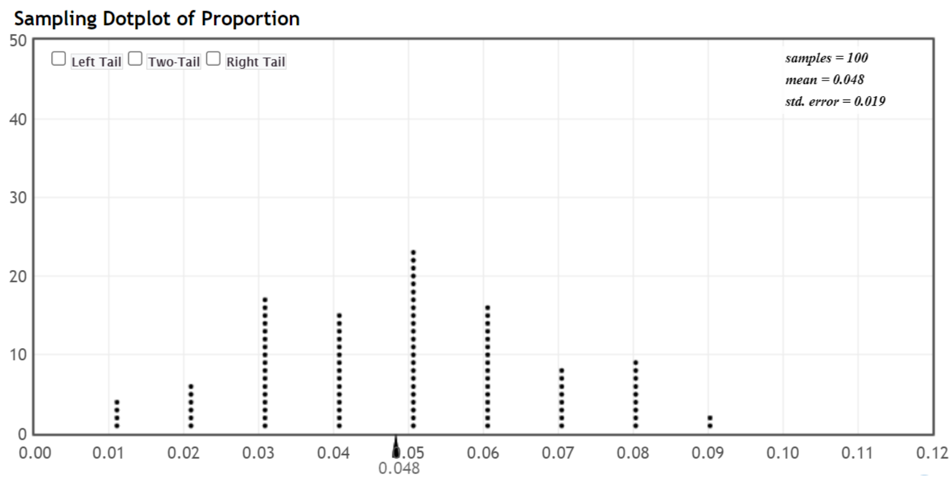
n = 20:



NOTE: The 0.054 shown below the x-axis is the mean of the 100 p-hats in this sampling distribution.

Figure 6.

n = 100:



NOTE: When comparing Figures 5 & 6 to our sampling distributions in Figures 3 & 4, we see that as the number of samples is increased, the shape becomes clearer in each sampling distribution.

12. Describe the shape, location of center, and symmetry (if any) of the sampling distribution for n=20 (Figure 5).

13. Describe the shape, location of center, and symmetry (if any) of the sampling distribution for n=100 (Figure 6).

Discuss questions 12-13 as a class.

Conclusion

Let’s return to the questions posed at the start of this activity.

14. If you were to take several random samples of American adults, would the sample proportion of individuals with arches change from sample to sample? Why is this?

15. What would be the most typical value of the sample proportions? Why does this make sense?

16. What property was the same in the n = 20 and n = 100 sampling distributions?

17. What properties changed in the n = 20 and n = 100 sampling distributions?

Discuss questions 14-17 as a class.

18. Describe each of these terms **in the context of this investigation**.

   f. *random* sampling

   g. sample

   h. *statistic* (give a written description and the associated symbol)

   i. population

   j. *parameter* (give a written description, the associated symbol, and the value)