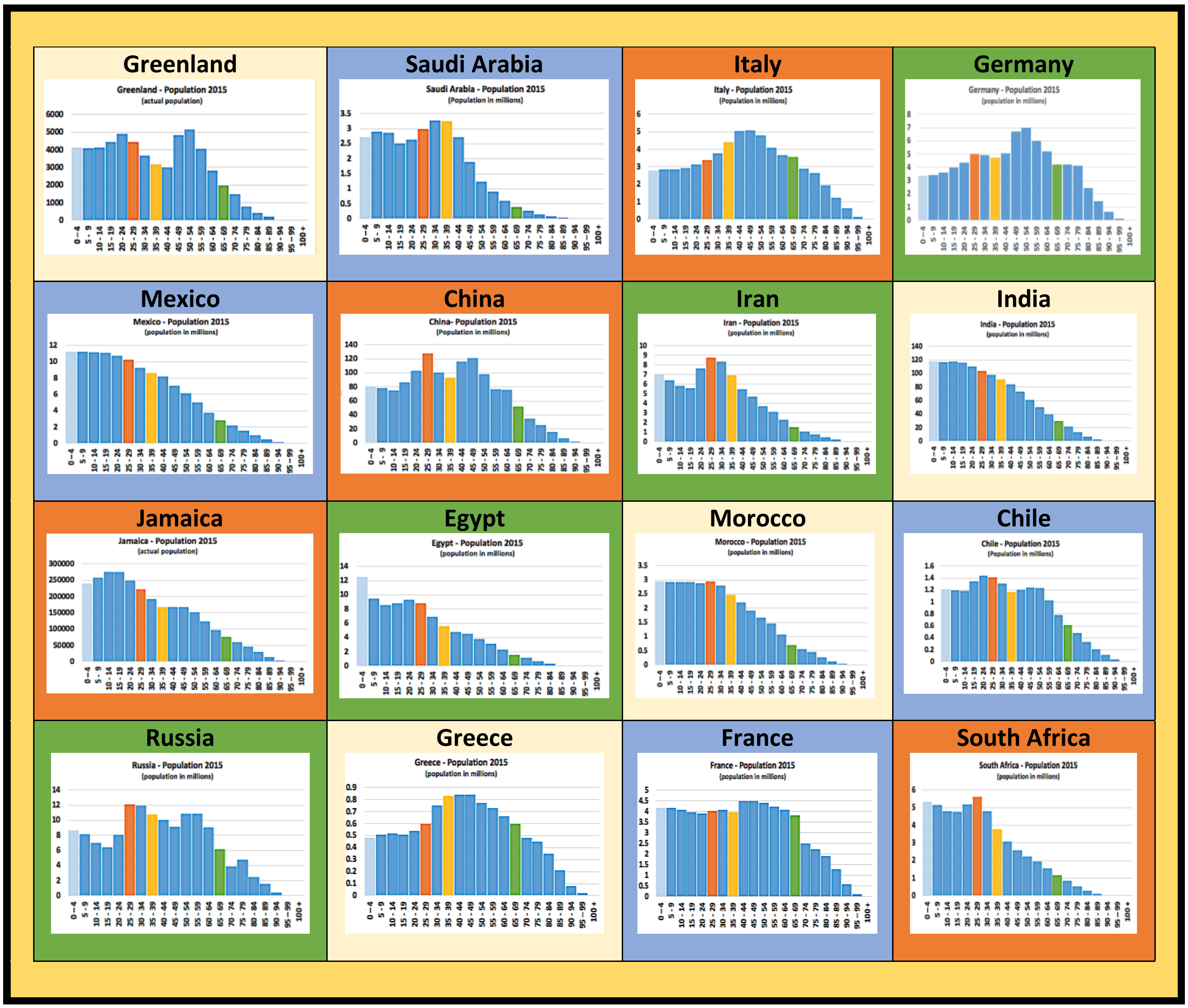
People Count!

(and their data stories)

Henry Kranendonk



Henry's Quilt of Countries

Data Stories

Modeling with Data

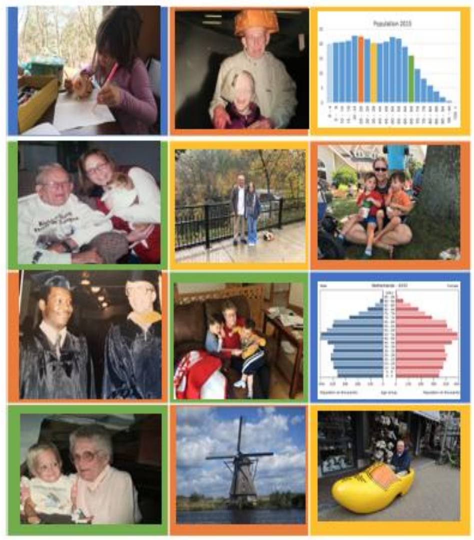
Student Edition

People Count!
and the stories of …

Kristin
Generation X

Abbey
The Millennial

Adeline
Generation ?



Parents
Baby Boomers

And generations yet to be named

* Henry's Quilt

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Marquette University, Retired

The population data used in this module were extracted from the **International Data Base** (**IDB**) website https://www.census.gov/programs-surveys/international-programs/about/idb.html. This website is housed and maintained by the United States Census Bureau. Data used in this module were obtained from the **IDB** in 2018 - 2020. Population estimates used in the module may be different than what is posted on the website due to periodic updates and revisions. The author is indebted to the United States Census Bureau for providing this valuable data and allowing it to be used in this module.

Acknowledgements

Let me start by thanking my former students at Rufus King High School in Milwaukee, Wisconsin who piloted the first several lessons of this module. Students in my International Baccalaureate (IB) Math Studies courses as well as students in my computer science courses were involved in working through problems in the early stages of developing several lessons.

I thank Gail Burrill for introducing me to population pyramid graphs and for her encouragement and support through the years. I thank David Ebert, a friend and fellow mathematics teacher, for his feedback and writing the **Foreword**. I also thank the early reviewers, namely Donald Cramer, Kevin McLeod, Mary Mooney, Michael Steele, Melissa Manley, and Steven Gorski. I especially thank Johnny Lott for reviewing the first draft and encouraging me to continue writing this module at a time I was not sure if it would have any value to students and teachers.

I thank my family for putting up with me while trying to weave this material together. In particular, I thank my wife Jean Kranendonk. I also thank my daughters Laura DeCicco, Kristin Moala, and Abbey Kranendonk, and my grandchildren Adeline, Paul, and Dominic DeCicco for allowing me to use their names as part of the data stories.

I would especially like to thank my daughter Kristin Moala for creating the illustrations that are included in several of the lessons and the data stories that include her name, and to my son-in-law Peter Moala for editing an early edition.

Finally, I would like to thank the Marquette University Upward Bound program. In particular, I thank Steven Robertson, Director of the Marquette University Upward Bound program, for providing the opportunity to pilot this module with high school students enrolled in the 2018 summer program. I also thank my Marquette University students for completing several lessons as part of the course work and for their feedback they provided as part of their evaluations.

I am especially grateful to Rebecca Nichols, Director of Education for the American Statistical Association, for her direction and encouragement during the process of getting this published in ***Statistics Teacher***. I am indebted to the NCTM/ASA Joint Committee on Curriculum in Statistics and Probability for organizing the review process and encouraging me to complete the module. I especially thank the Joint Committee for arranging an invaluable review by James Molyneux and Suyen Moncada Machado. James and Suyen extensively worked through the problems and provided critical suggestions and edits to an early draft. Their final words of encouragement after all of their work was a major inspiration to finish this module.

Finally, allow me to dedicate this module to the memory of my infant son, Jeffrey Scott Kranendonk. “There was too little time.”

Thanks again to everyone involved in this project.
Henry Kranendonk
2020

Foreword

I first met Henry Kranendonk at a workshop nearly twenty years ago. I was a young early career teacher, and Henry was a veteran mathematics teacher. That workshop was my first introduction to the population pyramid graphs that are featured in ***People Count***. I was, and continue to be, astounded by the beautiful simplicity of the graphs, and how such a simple graph can tell such a complex story. Henry introduced me to a new way of thinking about the visualization and analysis of data.

The National Council of Teachers of Mathematics' book, ***Catalyzing Change in High School Mathematics***, states that high school mathematics empowers students to expand professional opportunity, understand and critique the world, and experience wonder, joy, and beauty. Most high school mathematics and statistics textbooks and curricula do a good job expanding our students' professional opportunities, but fall short in bringing our students to understand and critique the world. Very few materials lead our students to experience wonder, joy, and beauty. ***People Count*** fills that void by guiding our students to an understanding of world population issues through the use of beautiful population pyramid graphs.

Too often, unfortunately, what is presented as "mathematical modeling" in many textbooks and curricula is either low on actual mathematics, or low on real-world connections. ***People Count*** is a great example of real, important data analysis that involves a high amount of mathematical rigor. Throughout the book, the knowledge and skills required by students builds on their experience in each successive unit, and each unit is another piece in the story of real people identified through their stories. The narratives present the mathematics as a student-centered story to be explored rather than a teacher-centered set of problems to be completed.

The book you have in front of you is the culmination of a career dedicated to sharing the simple complexity of population pyramid graphs with hundreds of students and teachers. It is a valuable addition to the field, and offers a compelling example of the investigative process of statistical problem solving called for in the American Statistical Association's ***Guidelines for Assessment and Instruction in Statistics Education*** (2007) or **GAISE**. Thank you, Henry, for bringing this material to hundreds of your students; and thank you, through this book, for sharing this material with thousands of students across the world.

David Ebert
High School Mathematics Teacher
Member, NCTM Board of Directors, 2016-2020

People Count!
(and their data stories)

Table of Contents
Student Edition 1

Introduction to Module

"One … two … three …" Our earliest introduction to mathematics was counting. Maybe we counted our fingers or toes as a child, or we counted money or points scored in a basketball game, or maybe we just counted people? Counting, however, is not necessarily simple (at least accurately counting). What we count, how we count, and why we count are challenging and important questions to answer.

Unless there is a purpose to counting, the investment in time and money needed to accurately count all the people in a country is not always feasible. The United States attempts, however, to count everyone in the country every 10 years as part of a ***census***. The United States census is mandated by its Constitution. The primary purpose of this census is to provide a fair representation of all the people in the House of Representatives, one of the two legislative bodies of the United States government. The goal of fair representation, however, is not without controversy. Nonetheless, the importance of the census has its origins in establishing representation in Congress.

Estimates of the count of people living in a country are important for several other reasons. Estimates of the population inform the government of the goods and services necessary to support the population. These estimates are also critically linked to the economy, health care, education, and the infrastructure needed to connect people by roads, bridges, and airports. Counting people is a major business. As current counts are analyzed, future counts of people are estimated by making assumptions involving birth rates, death rates, and immigration and emigration rates.

This module is about counting people. It examines counts by sex at birth and age in the United States, Kenya, and Japan that is provided by the International Data Base of the United States Census Bureau (or IDB). Each country's population is made up of young, old people, and middle-aged people. Why was the United States, Kenya, and Japan selected? The reasons will become more apparent as you complete the lessons in this module.

Do all young people or old people think the same way or spend their money the same way or seek out educational options the same way? Of course not. The diversity of the people living in a country is embedded in the make-up of its population by many factors other than age. The theme of this module, however, is to understand one aspect of a country's diversity based primarily on the distributions of ages and the implications connected to age. This module also looks back at age distributions in which major events (wars, economic booms or busts, diseases, cultural values) impacted the count of people living in the country. This module also considers challenges a country may face in the future. The past, present and future are combined through the stories of people who make up the countries represented in this module.

Henry's Quilt and Its Connection to this Module

The United States Census Bureau produced an inspiring set of classroom posters prior to the 2000 United States census. One of these posters has a picture of an elderly woman and a young girl. They are wrapped in a quilt as representatives of two important generations in this country (the young and the "old"). The picture also shows them working on *expanding* the quilt. The picture is a painting by a famous Hawaiian artist, Herb Kawaine Kane. The caption that accompanies the picture reads "Generations Are Counting on You". This poster was a major inspiration for this collection of lessons entitled ***People Count! (and their data stories)***.

What do you need to know about making a quilt? First you need some tools – needles, pins, measurement tools, and scissors. Then, of course, you need cloth and maybe yarn or thread – the "stuff." The end product (which is never done) is not just something to keep you warm, or to hang on a wall, or to store away for another generation to admire. It is also a story about the people who made the quilt.

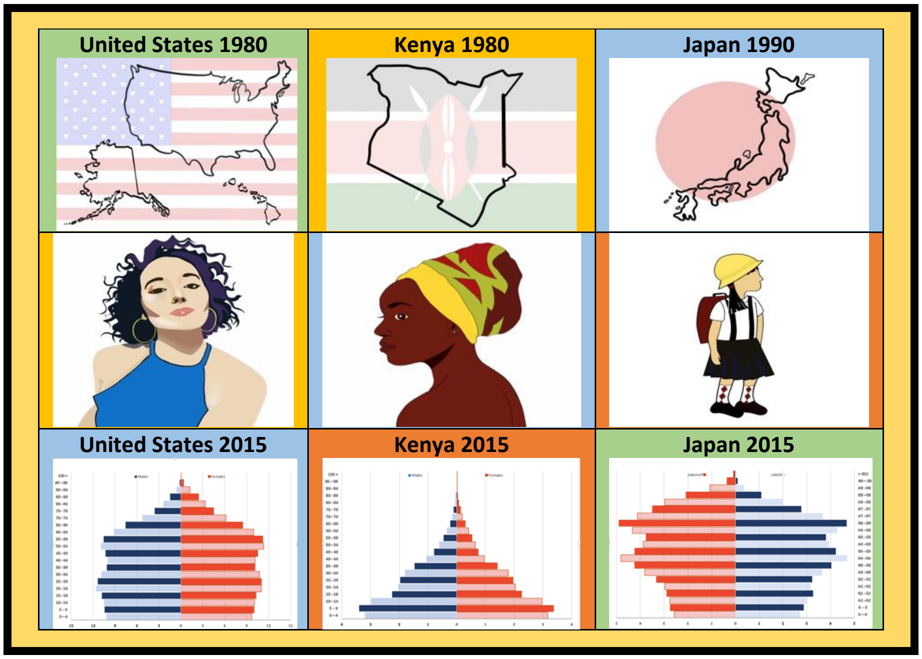
What do you need to construct a mathematical model? First you need tools. For this module, the tools will be described as proportions, ratios, population factors, foundation factors, and spreadsheets. Then you will need the stuff – in the lessons that follow, the stuff will be the population data of countries provided by the US Census Bureau. Finally, you build a model that is never complete – and the stories embedded in the model that link people to generations past, present, and the future. Constructing the model is a story of counting and people.

Quilts are more than just a blanket or a piece of cloth – they are also an art form. They are an art form that brings out sharing stories and history. The mathematical models in this module are also art forms – the symmetry (or lack of) found in a population pyramid graph and the shape of a population histogram are just like the art found in a quilt. And more importantly, this art is also about people.

Counting people may initially strike you as relatively "easy". ***People Count!*** connects real people and their stories to this process. It also explains why this process is important and a major challenge (and definitely not easy). Some people have actually stated that the survival of the United States is dependent on making sure that counting people is done correctly.

Henry Kranendonk
Author
2020

Unit 1
A Country's Shape



Introduction

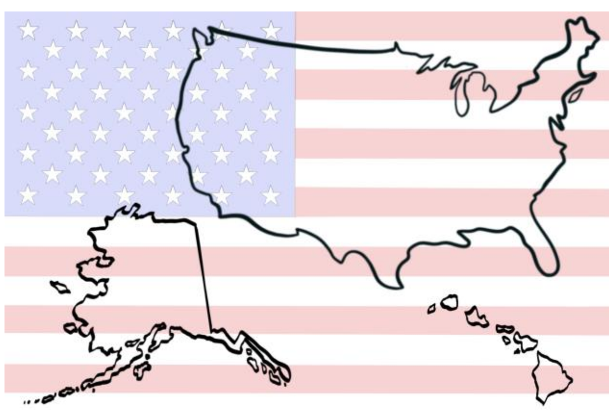
The United States, Kenya, and Japan, like all countries, are a collection of people often described by age, sex, race, education, and a number of other factors. Are these descriptions important? In particular, does the description of a country by age groups help us better understand a country? Unit 1 defines the shape of a country based on the counts of people living in the country by age. Age groups are displayed by way of population pyramid graphs and histograms. The graphs, and the summaries derived from them, begin to unpack stories about people living in these countries. Kristin, Raphine, Hana, their sisters and brothers and parents and nieces and nephews are people counted in these graphs. Their stories are not just about them. They are stories about special people who help us understand the United States, Kenya, and Japan.

The graphs are more than just visual summaries of each country. They serve another purpose that is also connected to the stories in this module. Each graph is a point on a country's timeline that tells us something about its past and possible future. From this point, we unpack a part of each country's history as well as speculate about its future. This module uses the count of people by ages to build a quilt that is understood by applying mathematical tools that emerge from a country's past and present shapes. Building this quilt, called a recursive model, starts by counting people. Let us begin the process, after all, "People Count!"

Lesson 1
The United States
A Lower Middle-Layered Country

Rectangle…square…trapezoid. Each of these words describes a shape that answers an important question in geometry. There are other examples in which shape is important in answering questions. For example, how would you describe the ***shape*** of a country? Why might studying the shape of a country be important? What questions are answered by understanding the shape of a country?

Let us start with United States. What is the shape of the United States? Most people would answer that question by sketching an outline of the United States as represented on a map. For example, the following sketch is an outline of the shape of United States that Kristin (a character in a data story you are about to read) found on a map. It is a unique shape. Although this shape is not described by words such as square or rectangle, the shape of the United States is important in answering questions about the country.



The geographical shape and placement of the United States in the world are important in understanding the United States' infrastructure, distances between important locations within the country, length of borders with other countries, coastal distances, and many other features.

There are other representations of the shape of a country that are also important to study. Kristin's story begins to reveal a different shape of the United States.

Kristin's Story – Chapter 1

It was January 1, 2015. Kristin, a 36-year old female, lived in Milwaukee, Wisconsin. She worked 40 hours a week as a health care researcher for a community clinic. Most people involved in her research were 60 years old or older. She was responsible for obtaining basic data that included weight, height, blood pressure, heart rate, previous health concerns, vaccinations, diet and sleeping issues. She felt that the start of a new year was a good time to think about her own future.

Kristin's mother was 66 years old at the start of 2015 and in good health. She came into the clinic at least once a year and generally did not require any follow-up visits. During her most recent visit, she stated something that confused Kristin. "Based on the shape of our country, I will be entering a new layer of our country's population in the next decade, along with lots of other people." Kristin was puzzled. What did she mean?

Kristin felt she could sketch the shape of the United States . Her sketch would be the shape she visualized during her study of geography in high school or college. She realized that her mother was reflecting on the fact that she was growing older, but what does age have anything to do with our country's shape? What did she mean that she was entering a new "layer"?

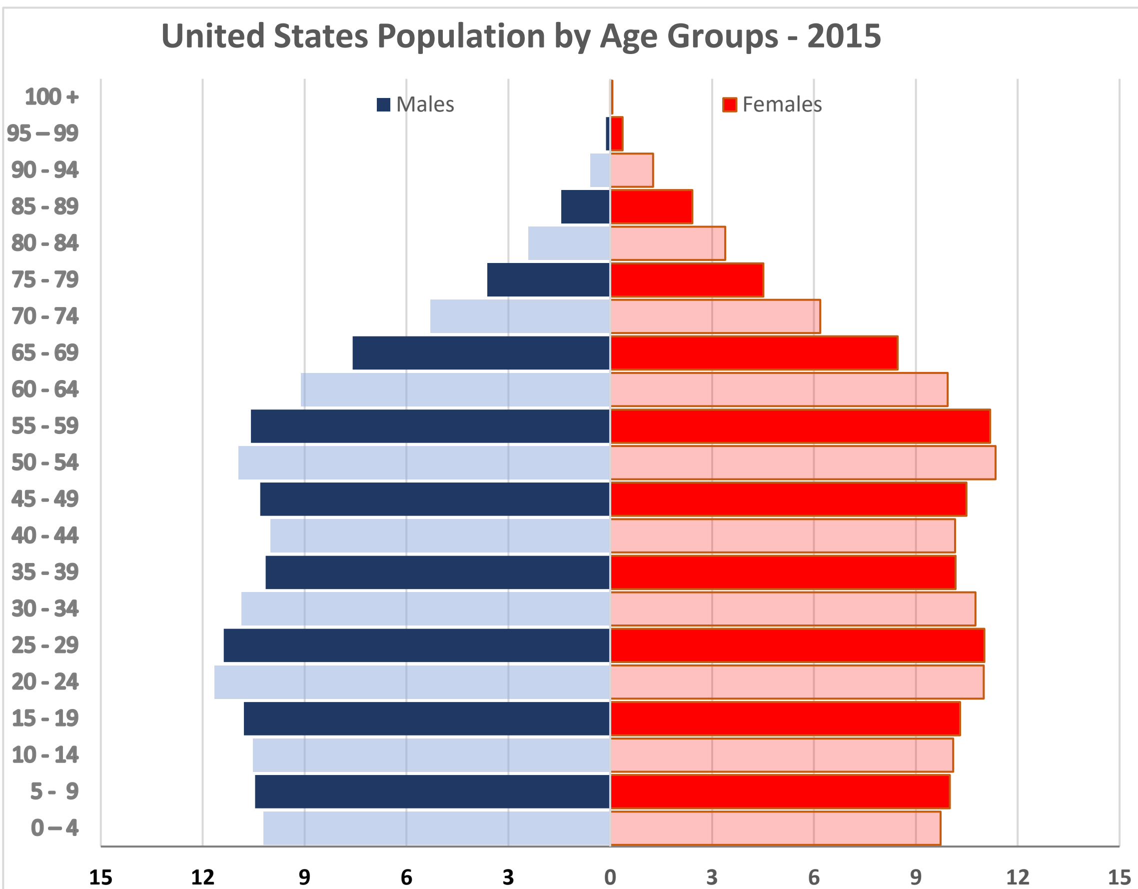
Shape suggested to Kristin something visual, like a square, a circle, or a triangle – something you studied in geometry. She thought again about her mother's comment. Kristin initially chalked up her mother's comment as something people say as they grow older. She found in her research, however, a ***population pyramid graph*** prepared by the United States Census Bureau. This graph had an interesting shape that might be used to explain her mother's comment. She did not, however, have an easy word (like square, rectangle, or circle) to describe this shape.

Kristin realized the graph is a summary of the ages of people in the United States. She wondered if the population pyramid graph's shape would explain why people her age were not as often highlighted by the media. In addition, Kristin and her friends did not particularly enjoy the latest shows offered on television or streaming services. She felt many of the popular movies seemed to either be something her parents (who were in their 60's and identified as Baby Boomers) or her 26-year old sister (who was identified as a Millennial) would enjoy. In her mid-thirties, Kristin and several of her friends were beginning to pursue leadership positions in government. But why did it seem the main issues addressed by the government were primarily issues involving her sister or her parents? Kristin looked closely at the population graph. The population pyramid graph prepared by the United States Census Bureau began sorting out some of her questions. Maybe the shape of this graph is another description of the shape of the United States?

Lesson 1 – Problems

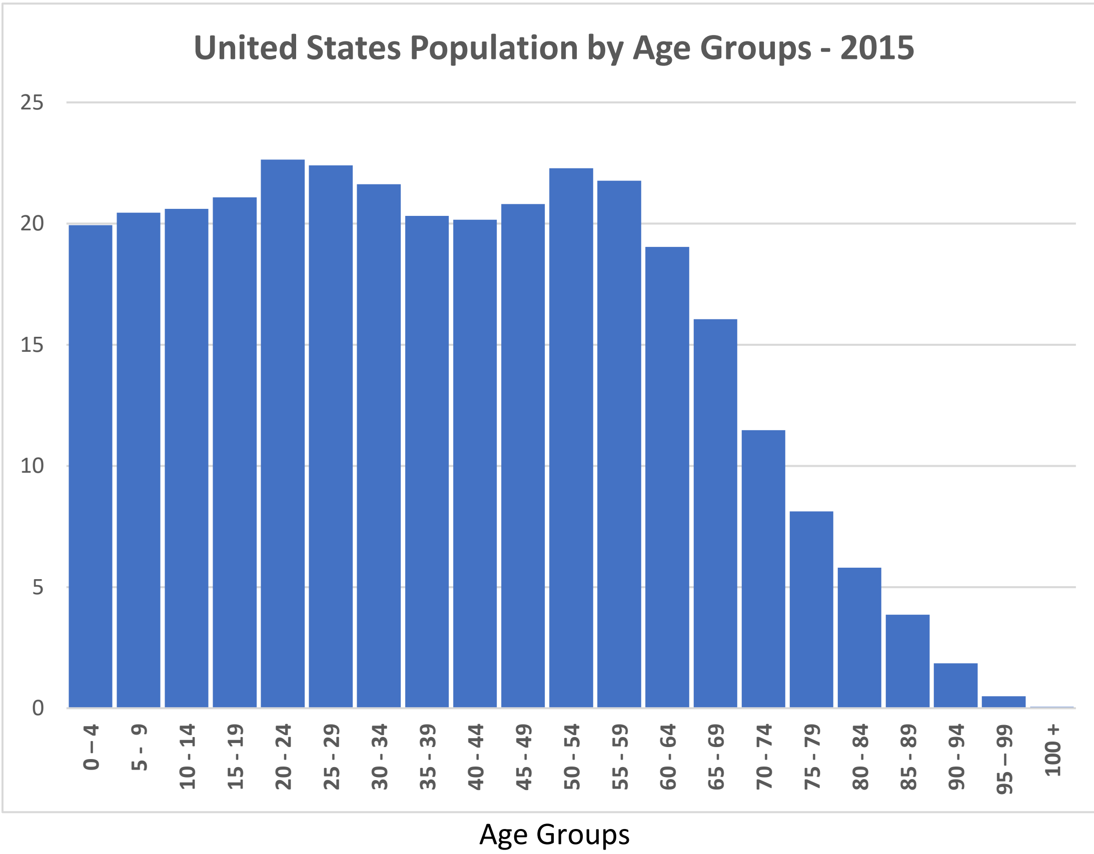
Handout needed to complete the following problems: Handout 1: *United States – 2015*

The United States Census Bureau provides graphs of the age distributions of countries around the world. The following graph summarizes the population of the United States at the start of 2015 and is called a ***population pyramid graph***. It is available at the United States Census Bureau's International Data Base (IDB) website (*https://www.census.gov/programs-surveys/international-programs/about/idb.html)*.



Population in millions of people
Data obtained from the US Census Bureau – International Data Base (IDB)

Another type of graph, called a ***histogram***, combines the counts of the females and males in each of the age groups. The histogram for the 2015 United States population is also a representation of the shape of a country.



Population in millions of people
Data obtained from the US Census Bureau – International Data Base (IDB)

Each of these graphs conveys important information about the United States. They provide visual representations of the *estimates* for the count of people in the designated age groups. The general shape of each graph summarizes important descriptions of the population of a country mentioned in Kristin's story.

Use **Handout 1: The United States – 2015** to answer the following problems. Derive your answers using the population pyramid graph, the histogram, or the table that is included with the handout.

1. What 5-year age group has more people (males and females) than any other age group?

2. In what age group was Kristin counted in the 2015 population pyramid graph or histogram? In what age group was Kristin's mother counted? In what age group was Kristin's younger sister counted?

3. What 5-year age group of just males has more counts of males than any other age group of males?

4. What 5-year age group of just females has more counts of females than any other age group of females?

A population distribution is defined by the following *layers*:

- The ***bottom-layer*** refers to the counts of people in the 0 to 24 years old age groups.
- The ***lower middle-layer*** refers to the counts of people in the 25 to 49 years old age groups.
- The ***upper middle-layer*** refers to the count of people in the 50 to 74 years old age groups.
- The ***top layer*** refers to the count of people in the 75 to 100+ years old age groups.

Based on the summary of layers, a country's shape is defined by the following terms:

- A country that has most of its people in the bottom-layer is identified as **Bottom-Layered Country**.
- A country that has most of its people in the lower middle-layer is identified as a **Lower Middle-Layered Country**.
- A country that has most of its people in the upper middle-layer is identified as an **Upper Middle-Layered Country**.
- A country that has most of its people counted in the top layer is identified as a **Top-Layered Country**.

5. Estimate what age group layer (bottom, lower middle, upper middle, top) you think will have the least number of people in the United States? Did you use the population pyramid graph, the histogram, or the table to make your estimate? Explain how you made your estimate using the graphs or table.

6. Estimate what age group layer you think will have the greatest number of people? Did you use the population pyramid graph, the histogram, or the table to make your estimate? Explain how you made your estimate using the graphs or table.

Identify on the ***population pyramid graph*** and the ***histogram*** where the above layers begin and end by age groups.

7. Kristin used the table included with Handout 1. She added up the count of people who were 0 – 4 years old, 5 – 9 years old, 10 – 14 years old, 15 – 19 years old, and 20 – 24 years old. The total count she obtained was 104,776,994 people. What is the percent of people 0 to 24 years old based on the estimates summarized by the table? Summarize your answer to the nearest tenth of a percent.

8. In a similar way, what is the percent of people 25 to 49 years old?

9. What is the percent of people 50 to 74 years old?

10. What is the percent of people 75 to 100+ years old?

11. Identify two different age groups in which the count of people in the first age group is approximately double the count of people in the second age group. (There are several examples to answer this problem.)

12. Kristin's data story indicates that she felt disconnected from the more popular choices of movies and political views. Look at the age group that includes Kristin at the start of 2015. Why might people older or younger than Kristin have different interests than Kristin in movies or political views? Using the population estimates provided in **Handout 1**, why might the entertainment choices or political views of people older or younger than Kristin be reported in the news or social media more often?

13. For this problem, adjacent age groups are age groups next to each other. For example, 0 – 4 years old is adjacent to 5 – 9 years old. In a similar way, the 45 – 49 years old age group is adjacent to the 50 – 54 years old age group. The age group 45 – 49 years old is also adjacent to the 40 – 44 years old age group. Identify two adjacent age groups that have approximately the same count of people in each age group. (There is more than one answer to this problem.)

14. Estimate the count of teenagers (13 to 19 years old). Explain how you derived your estimate.

15. What is the estimated count and percent of people who are under 10 years old?

16. What is the estimated count and percent of people who are 65 years old or older in the United States?

17. Why is it important that the count and percent of people under 10 years old and the count and percent of people 65 years old or older are given special attention when analyzing a country's population?

18. "Old" and "young" are subjective descriptions that in many cases are defined by several factors other than age (for example, health status, or income status). For this unit, however, consider the definition of "young" as people less than 10 years old, and the definition of "old" as people who are 65 years old or older. What is the ratio of "old" to "young" using the above definitions of young and old? Derive a decimal from this ratio and interpret it by describing the approximate count of "old people" to the count of one "young person." Express your answer to the nearest whole number.

19. Kristin's data story indicated that she worked in the health care field. Do you think that working in the health care field was a major area of employment in 2015? Explain your answer by referring to the data.

20. The voting age in the United States is 18 years old or older. Derive an estimate of the number of potential voters in the United States at the start of 2015.

21. Identify the age groups in which the count of males is estimated to be more than the count of females.

22. Identify the age groups in which the count of females is estimated to be more than the count of males.

23. Identify an age group that has approximately the same count of males and females.

24. Similar to the way you estimated the ratio of old to young people in problem 18, estimate the ratio of females to males for the following age groups. Derive a decimal from the ratio and estimate the number of females to one male in that age group (round your answers to the nearest whole number):

    a. In the age group of 85 to 89 years, there are approximately _____ females to one male.

    b. In the age group of 90 to 94 years, there are approximately _____ females to one male.

    c. In the age group of 95 to 99 years, there are approximately _____ females to one male.

    d. In the age group of 100+ years, there are approximately _____ females to one male.

25. Notice the changes in the ratio of females to males that you derived in problem 24 as the age groups grew older. Write a sentence or two that describes what is happening. Why might these changes be important for people interested in the health care of the population?

26. What questions would you like summarized for specific age groups? Answers to your questions would not necessarily be derived by the population graphs or table. Discuss with your class at least one of your questions and why you think the answer to your question is important.

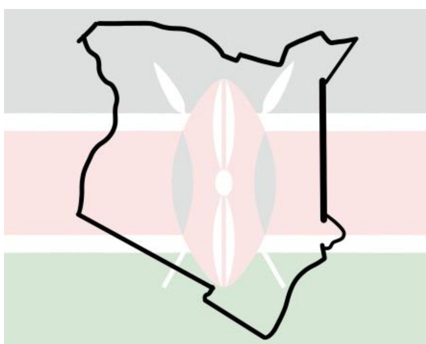
Age group	**Questions you would like summarized for this age group**
Example: 0 – 4 years old	How many children 0 – 4 years old are in a pre-school program? (The answer to this question is important in determining whether or not our country has enough trained pre-school teachers.) How many children 0 – 4 years old can count to 20? (The answer to this question would help determine what skills need to be addressed or what skills can be used to extend learning opportunities.)
15 – 19 years old	
35 – 39 years old	
An age group of your choice:	

Lesson 2

Kenya

A Bottom-Layered Country

Kenya … home to the well-known cities of Nairobi and Mombasa. A beautiful country in eastern Africa with unique connections to some of the most exotic animals in the world. A country whose history is marked by several events that altered the count of its people. The geographical shape of Kenya and its flag are represented below:



But, Kenya, just like the previous lesson about the United States, has another shape that is equally important in understanding this country. Kenya's shape is different than the shape of the United States and conveys its own stories about the people of Kenya. Faith, the twin sister of Raphine whose story is included in this lesson, is thinking about the geographical shape of her country Kenya and the population shape.

Kristin and Raphine's Story – Chapter 2

Kristin and Raphine's Story – Chapter 2
Raphine was born in a small village north of Nairobi, Kenya. In 2015, he celebrated his 36th birthday and felt a bit uneasy. He currently is caring for his elderly mother and grandmother, along with several younger brothers and sisters. He is the headmaster of an elementary school in his village, a position he held for the last 12 years. He teaches math to nearly 40 young children who are 5 to 14 years old. The children attending the school live near his village. He loves his job. There are many families hoping their children will be able to attend his school. Unfortunately, the school is just unable to accept everyone who wants to attend.

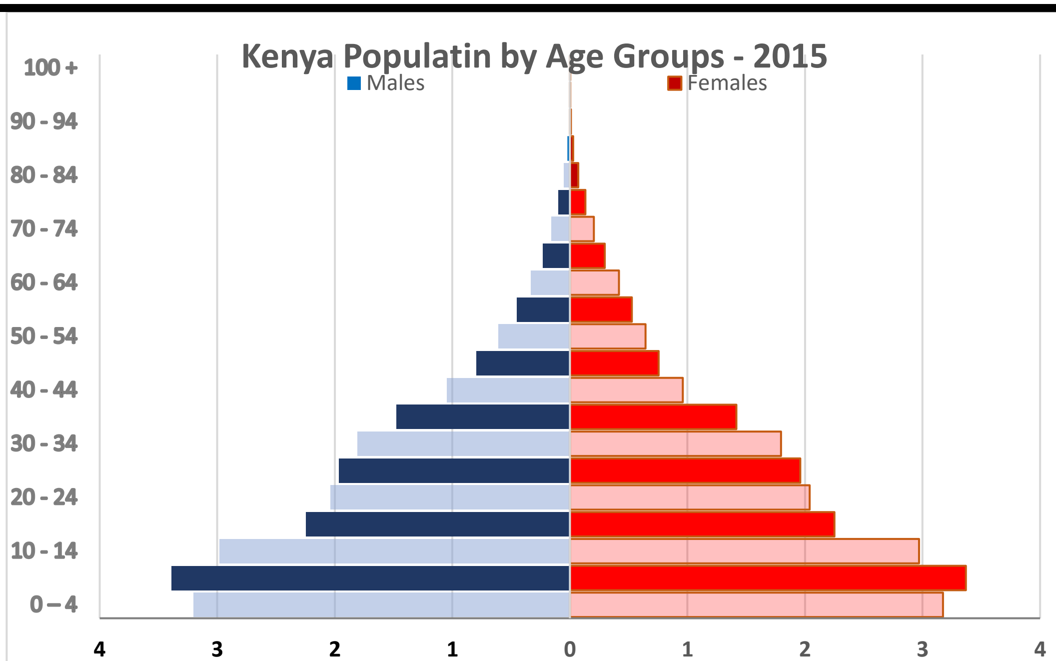
Raphine also has an interest in becoming a doctor. There is limited health care for his mother and twin sister Faith. He is quite sure Faith suffers from asthma that often makes life difficult for her to care for his younger brothers and sisters and nieces and nephews. The kids in his school also have many health problems. For the last couple of years, Raphine was looking into a program at a university in the United States to pursue a personal dream of becoming a doctor or a nurse. He recently applied to participate in the program and received a letter that his application was accepted. The university has offered Raphine a full scholarship to live and study in the United States. The university is also encouraging Raphine to attend a conference in Milwaukee, Wisconsin to learn more about the paperwork he will be required to complete to become a fulltime student. Kristin is responsible for identifying United States families who will assist Raphine with his transition if he accepts this offer. He is also encouraged to communicate through e-mail with Kristin to become more familiar with the obvious challenges of a possible move to the United States.

Raphine is nervous. Leaving Kenya for 3 or 4 years is not an easy decision. Who will run his school? Who will take care of his family members? He is working with other teachers in his school who might address some of his concerns.

Lesson 2 – Problems

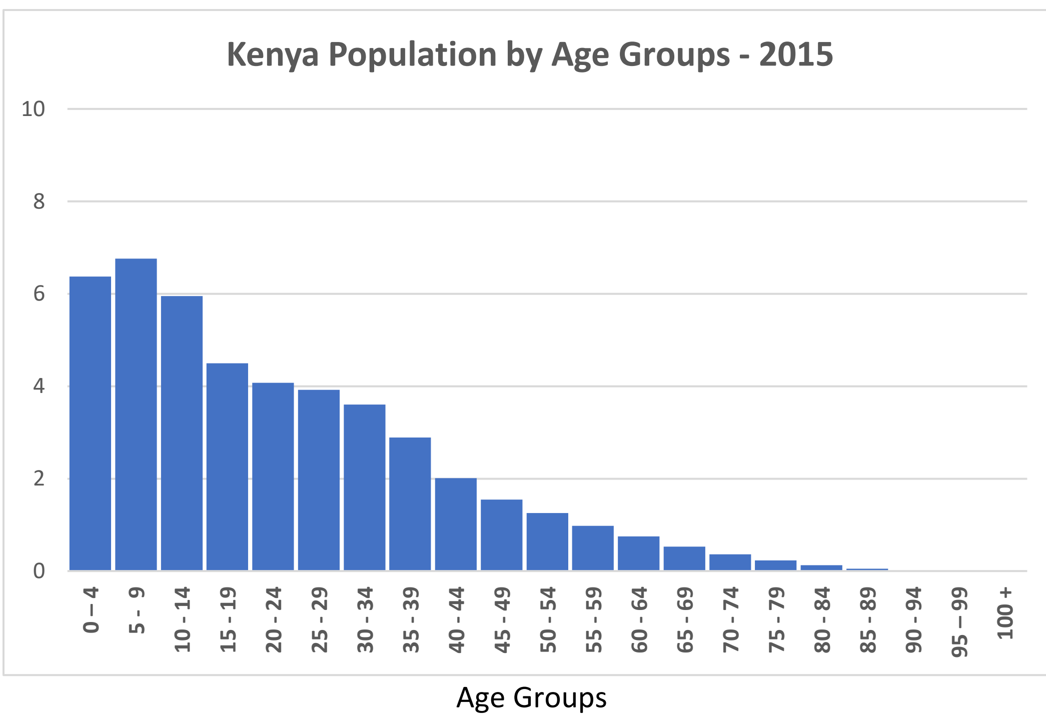
Handout needed to complete the following problems: Handout 2: *Kenya – 2015*

The US Census Bureau provides special graphs of the age distribution of countries around the world. The following graph and data were obtained from the United States Census Bureau's International Data Base (IDB) at *https://www.census.gov/programs-surveys/international-programs/about/idb.html.* The population pyramid graph of Kenya represents the 2015 age distribution of Kenya.



Population in millions of people
Data obtained from the US Census Bureau – International Data Base (IDB)

The following ***histogram*** combines the female and male counts of people in Kenya:



Population in millions of people
Data obtained from the US Census Bureau – International Data Base (IDB)

The population pyramid graph and histogram convey information about the count of people in Kenya in the same way the graphs conveyed information about the United States in Lesson 1. They provide visual representations of the estimates for the count of people in special age groups. The different shapes, however, indicate different challenges for Kenya and the United States.

Use **Handout 2: *Kenya – 2015*** to answer the following problems. You can use the population pyramid graph, the population histogram, or the table provided on the handout to answer these problems.

1. What 5-year age group has more people counted in it than any other age group?

2. In what age group was Raphine counted in the 2015 population pyramid graph or histogram? In what age groups were the students in his school counted at the start of 2015?

Review again the definitions from Lesson 1:
A population distribution is defined by the following *layers*:

- The ***bottom-layer*** refers to the counts of people in the 0 to 24 years old age groups.
- The ***lower middle-layer*** refers to the counts of people in the 25 to 49 years old age groups.
- The ***upper middle-layer*** refers to the count of people in the 50 to 74 years old age groups.
- The ***top layer*** refers to the count of people in the 75 to 100+ years old age groups.

Based on the summary of layers, a country's shape is defined by the following terms:

- A country that has most of its people in the bottom-layer is identified as **Bottom-Layered Country**.
- A country that has most of its people in the lower middle-layer is identified as a **Lower Middle-Layered Country**.
- A country that has most of its people in the upper middle-layer is identified as an **Upper Middle-Layered Country**.
- A country that has most of its people counted in the top layer is identified as a **Top-Layered Country**.

3. Estimate what layer (bottom, lower middle, upper middle, top) you think will have the least number of people in Kenya? Explain how you made your estimate.

4. Estimate what layer you think will have the greatest number of people? Explain how you made your estimate.

Identify on the ***population pyramid graph*** and the ***histogram*** where the layers begin and end.

5. Using the table of population counts on **Handout 2**, what is the percent of people 0 to 24 years old? (Round your answer to the nearest 10^{th} of a percent.)

6. In a similar way, what is the percent of the count of people 25 to 49 years old?

7. What is the percent of the count of people 50 to 74 years old?

8. What is the percent of the count of people 75 to 100+ years old?

9. Identify two age groups in which the count of people in one age group is approximately double the count in the other age group. Is the larger age group younger or older than the other age group?

10. Raphine's data story indicates that he was unable to accept all of the kids who wanted to attend his school. In what way do the graphs indicate that finding a school for all of the young people might be one of Kenya's challenges?

11. Identify one of the age groups from the table that has more than 10% of the total population of Kenya.

12. Estimate the count and percent of teenagers (13 to 19 years old). Explain how you derived your estimate. (Estimates will vary.)

13. What is the count of people who are under 10 years old?

14. What is the percent of the count of people who are under 10 years old?

15. What is the count of people who are 65 years old or older?

16. What is the percent of the count of people who are 65 years old or older?

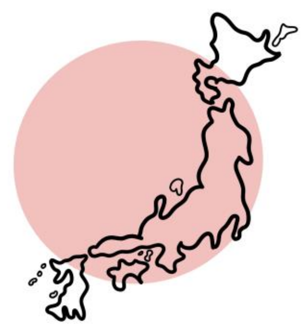
17. Why is it important that the count and percent of people under 10 years old and 65 years old or older are given special attention?

18. "Old" and "young" are subjective descriptions that in many cases are defined by several factors other than age (for example, health status, or income status). For this unit, however, consider the definition of "young" as people less than 10 years old, and the definition of "old" as people who are 65 years old or older. What is the ratio of "old" to "young" using the above definitions of young and old? Derive a decimal from this ratio and interpret it by describing the approximate count of "old people" to the count of "young people." Express your answer to the nearest whole number.

19. If there are approximately 500 students in a typical school for students who are 5 to 14 years old, estimate the number of schools needed to educate the students who are 5 to 14 years old.

Lesson 3
Japan
An Upper Middle-Layered Country

At the beginning of 2015, Japan was the 11th largest country in population. Just like the previous lessons that highlighted the unique geographical shapes of the United States and Kenya, Japan has an unique geographical shape outlined by its formation of several islands. Japan's population pyramid graph and population histogram also have a distinctive shape. The population graphs tell us a lot about the people who live in Japan. (if you research more about Japan, you will discover that the people of Japan have a special affection of cats. Several of Japan's past and present stories involve their admiration of cats.)



Scientists are particularly interested in Japan's population because some of the oldest people in the world live in Japan. People over 110 years old are part of its population. A research study entitled ***Blue Zones*** (https://www.bluezonesproject.com) highlights areas of the world where a large count of people live to be 100 years old or older. Japan is a country included in this research.

Kristin and Hana's Story – Chapter 3

Hana lives in Tokyo, Japan. She celebrated her 36th birthday in 2015 with her parents and grandmother at a popular restaurant in Tokyo. She especially enjoys these celebrations with her grandmother who is in her late 90's. Sara, Hana's best friend, was also there. What was particularly memorable for Hana was that her grandmother gave her a picture album that had several photos of her mother and her grandmother when they were young girls. Hana has no aunts, uncles, or cousins,. She also has no brothers or sisters, so this birthday celebration was small, but memorable.

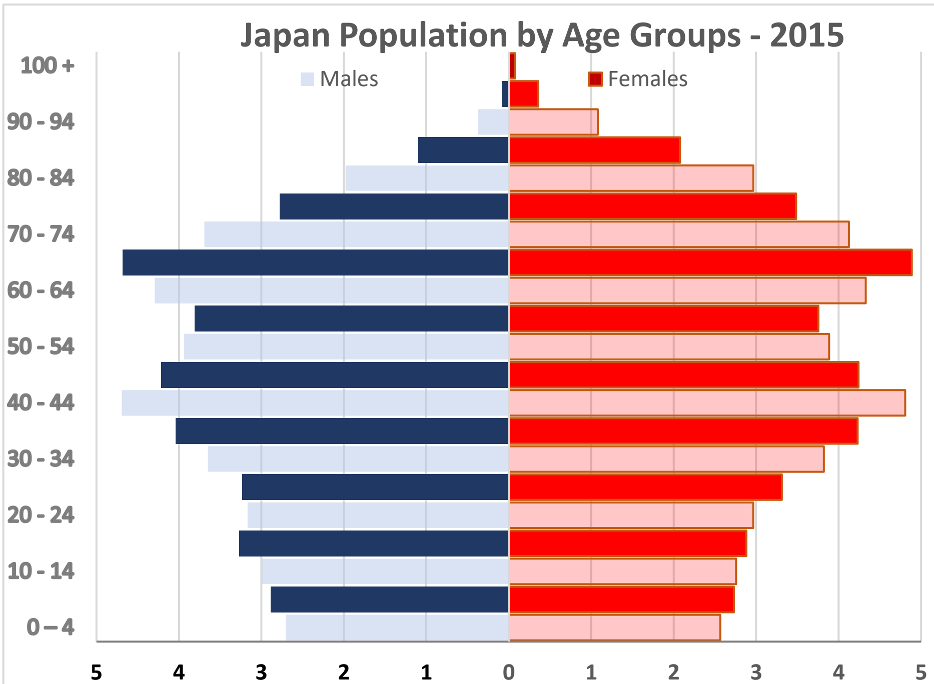
Hana is a graphic artist and works for a company that develops online advertising for companies that sell motorcycles. Her job involves traveling to other countries. She especially enjoyed her trips to Norway, Australia, and the United States. Her trip to the United States included a tour of a manufacturing plant of motorcycles in Milwaukee, Wisconsin. Sara was able to go along on this trip. Sara is a nurse, and persuaded Hana to attend a conference in Milwaukee that was particularly interesting to Sara. It was at this conference that Hana met Kristin. Kristin was distributing information about setting up health care facilities that would provide care for older people. Although not involved in the health care profession, Hana was interested in some of the information presented at this meeting as she is concerned about her parents and grandmother as they get older. Hana is especially concerned that there may be a time that they will not be able to live independently.

Hana and Kristin continue to exchange e-mails. Kristin particularly enjoyed Hana's pictures of her cat. Both Kristin and Hana identify themselves as "cat people". Hana is hoping that she can arrange for Kristin to visit Japan in the near future. Kristin is also hoping that this will be possible.

Lesson 3 – Problems

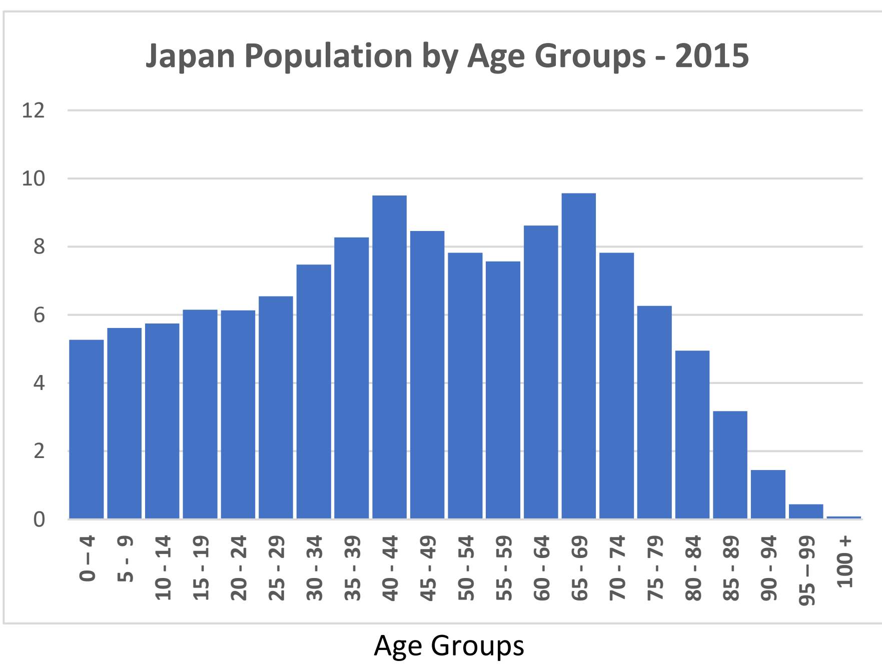
Handout needed to complete the following problems: Handout 3: *Japan – 2015*

The US Census Bureau provides special graphs of the age distribution of other countries. The following graph and data were obtained from the United States Census Bureau's International Data Base (IDB) at *https://www.census.gov/programs-surveys/international-programs/about/idb.html.* The population pyramid of Japan represents the population distribution of Japan at the beginning of 2015.



Population in millions of people
Data obtained from the US Census Bureau – International Data Base (IDB)

The following ***histogram*** combines the female and male counts of people in Japan:



Population in millions of people
Data obtained from the US Census Bureau – International Data Base (IDB)

The population pyramid graph and histogram convey information about the count of people in Japan in the same way the graphs conveyed information about the United States in Lesson 1 and Kenya in Lesson 2. They provide visual representations of the estimates for the count of people in the designated age groups. The different shapes mapped out by these graphs indicate different challenges for each country. Japan has a distinctive shape when compared to most of the countries of the world. The following problems provide a summary of the counts of people living in Japan.

Use **Handout 3: *Japan – 2015*** to answer the following problems. Your answers can be obtained from the population pyramid graph, the population histogram, or the table that are provided on **Handout 3**.

1. What 5-year age group records the greatest count of people?

2. Identify the age group that has the second greatest count of people.

3. Identify the age group of people younger than 80 years old with the least count of people.

4. The story indicated that Hana celebrated her 36th birthday in 2015. Identify on the population graph or the population histogram Hana's age group in 2015.
   a. What is the age group adjacent to Hana's age group that is younger? Is that age group "less than" or "greater than" that the count in Hana's age group?

   b. What is the age group adjacent to Hana's age group that is older? Is that age group "less than" or "greater than" that the count in Hana's age group?

   c. Is the above summary of the age groups younger and older than Hana different than the summary of these same age groups in the United States? If yes, describe the difference. (See **Handout 1** to review the count of people in these age groups in the United States.)

Review again the definitions from Lesson 1:
A population distribution is defined by the following *layers*:

- The ***bottom-layer*** refers to the counts of people in the 0 to 24 years old age groups.
- The ***lower middle-layer*** refers to the counts of people in the 25 to 49 years old age groups.

- The ***upper middle-layer*** refers to the count of people in the 50 to 74 years old age groups.
- The ***top layer*** refers to the count of people in the 75 to 100+ years old age groups.

Based on the summary of layers, a country's shape is defined by the following terms:

- A country that has most of its people in the bottom-layer is identified as **Bottom-Layered Country**.
- A country that has most of its people in the lower middle-layer is identified as a **Lower Middle-Layered Country**.
- A country that has most of its people in the upper middle-layer is identified as an **Upper Middle-Layered Country**.
- A country that has most of its people counted in the top layer is identified as a **Top-Layered Country**.

5. Estimate what layer (bottom, lower middle, upper middle, top) you think will have the least count of people in Japan? Explain how you made your estimate.

6. Estimate what layer you think will have the greatest count of people? Explain how you made your estimate.

Identify on the ***population pyramid graph*** and the ***histogram*** where the layers begin and end.

7. Use **Handout 3** to derive an estimate of the percent of the count of people who are 0 to 24 years old in Japan. (Round your answer to the nearest 10^{th} of a percent.)

8. In a similar way, what is the percent of the count of people 25 to 49 years old?

9. What is the percent of the count of people 50 to 74 years old?

10. What is the percent of the count of people 75 to 100+ years old?

11. Based on the above definitions of a country's shape, what is the description of the shape of Japan's population graphs?

12. Identify two age groups in which the count of people in one age group is approximately double the count in the other age group. Is the larger age group younger or older than the other age group?

13. Hana's data story indicates that she was concerned about her parent's future. In what way do the graphs indicate why she might be concerned about their future?

14. Identify at least one age group that has approximately 1 million more females than males.

15. Estimate the count of teenagers (13 to19 years old). Explain how you derived your estimate. (Estimates will vary.)

16. What is the count of people who are under 10 years old?

17. What is the percent of the count of people who are under 10 years old?

18. What is the count of people who are 65 years old or older?

19. What is the percent of the count of people who are 65 years old or older?

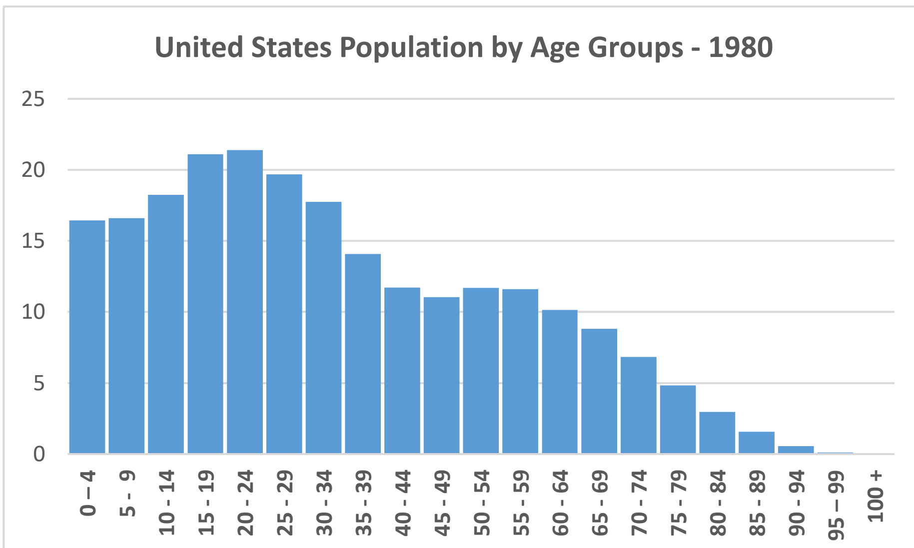
20. Why is it important that the count and percent of people under 10 years old and 65 years old or older are given special attention?

21. “Old” and “young” are subjective descriptions that in many cases are defined by several factors other than age (for example, health status, or income status). For this unit, however, consider the definition of “young” as people less than 10 years old, and the definition of “old” as people who are 65 years old or older. What is the ratio of “old” to “young” using the above definitions of young and old? Derive a decimal from this ratio and interpret it by describing the approximate count of “old people” to the count of one “young person.” Express your answer to the nearest whole number.

22. If there are approximately 500 people in a typical special care facility designed for people 90 years old or older, how many facilities were possibly needed in 2015?

Lesson 4
The Center and Spread of a Country's Shape

Kristin was 36 years old at the start of 2015. Her birthday was in February of 1979. The first histogram that counted Kristin was the 1980 population histogram. She was counted in the 0 – 4 years old age group of that histogram.



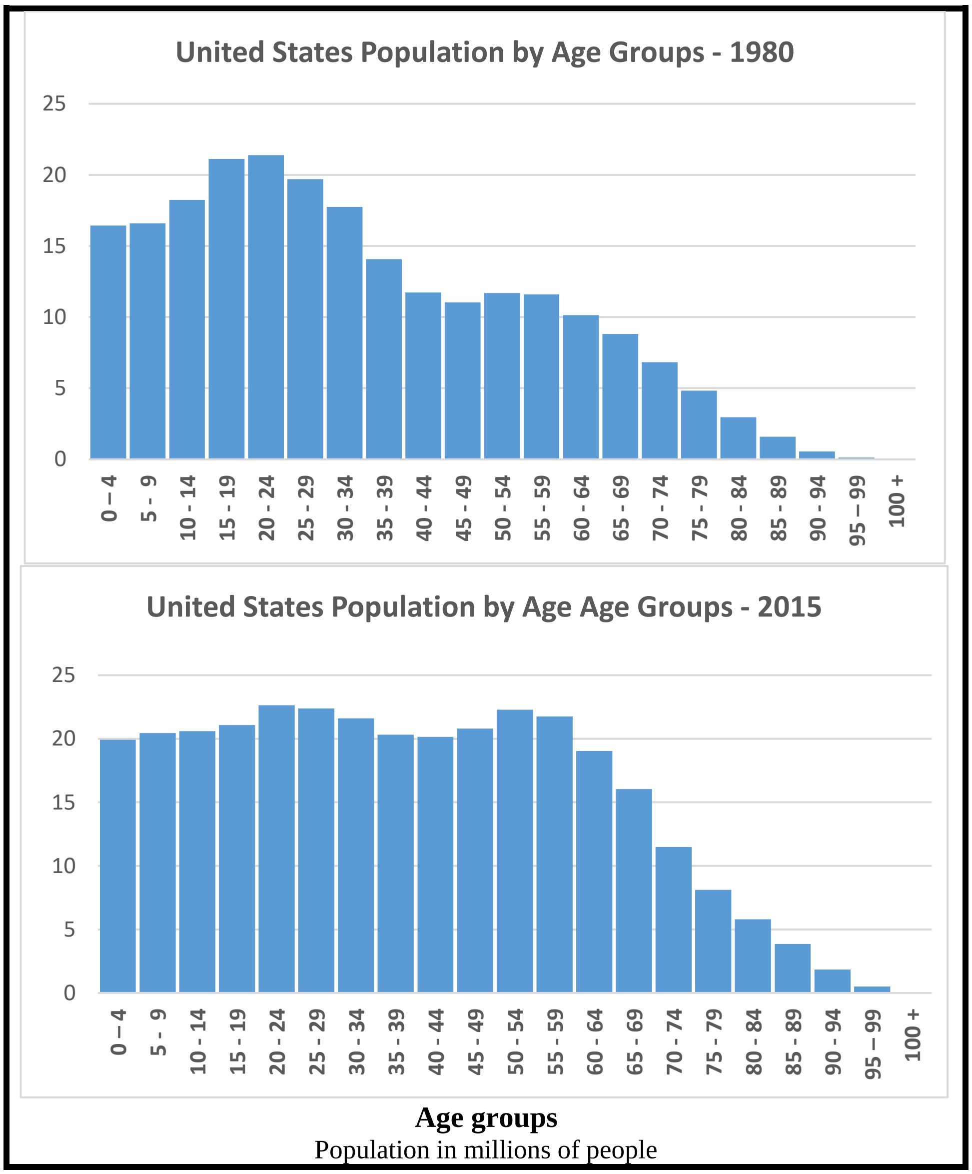
Age Groups
Population in millions of people
Total population is 227.24 million people

A 1980 report prepared by the United States Census Bureau indicated a typical person in the United States was 34 years old, with a little more than 50% of the country's population between the ages of 15 and 54 years old. Describing the age of a typical person does not mean that most of the people are 34 years old. Identify in the 1980 population histogram the age group in which a person 34 years old would be counted. Note that the count of people who were 15 to 19 years old, 20 to 24 years old, and 25 – 29 years old in 1980 had more people than the 30 – 34 years old age group. Why was 34 years old used to identify a typical person in the United States?

The problems in this lesson derive two summaries of the distribution of ages, namely the median age and the mean age. Although other summaries could be included in this list, the mean age and the median age are often used to describe a “typical” age of a person in a country. This lesson will also explain that there is more to the story of a typical age than just the mean or median age. Equally important in understanding a country’s population distribution is the ***spread*** of the ages. The problems in this lesson will also derive the spread of the ages based on an estimate of the median age.

Lesson 4 – Problems

To begin this lesson, compare the 1980 population histogram to the 2015 population histogram:



1. Answer the following based on the 1980 and the 2015 population histograms.
    a. Do you think the population increased from 1980 to 2015? Explain your answer.

    b. What two age groups recorded the greatest count of people in 1980? What two age groups recorded the greatest count of people in 2015?

    c. Identify the age group where a person 34 years old would be counted on the 1980 population histogram. Also identify the age group where a person 34 years old would be counted on the 2015 population histogram. Do you think the typical age of a person in 2015 is also 34 years old? If not, do you think the typical age will be older or younger? Explain your answer. Several of the problems in this lesson will develop an answer to these questions more precisely.

"Typical" generally starts by describing a ***center*** of a data set. The first center derived in this lesson is the *median age*. The second center derived is the *mean age*. What these centers summarize about the population is explored in the following problems.

Finding the Median Age of the United States Population in 2015

An important summary of the population is the median age. The exact median age requires access to everyone's age in the country. Is that possible? Of course not. Imagine the number of ages this would represent (over 300 million). Also, at any moment in time, there is a person who leaves the country, a person who dies, a person who enters the country, and a person who is born. The process outlined in this lesson is an estimate of the median age based on the count of people in each of the age groups represented in the 2015 histogram.

What is the median age? If it were possible to collect the ages of all the people in the country, the first step would be to put those ages in order from 0 years old to 100+ years old (or from the youngest to the oldest person). **The median age is the age that marks where approximately one-half of the population, or 50%, is below this age and approximately one-half, or 50% of the population, is above this age.** A more precise definition of the median age is based on data compiled from every person's age in a population of over 300 million people. For this lesson, an estimate of the median age is derived using the counts of people in each of the age groups. The result will be an estimate of the median age and not the exact median age.

2. Are the ages of the people summarized in the histograms from 1980 and 2015 in order from youngest to oldest? Explain your answer.

3. Is it possible to estimate how many people in the United States were exactly 34 years old? Explain your answer.

4. Answer the following questions based on the histogram of the 2015 population provided in this lesson.
   a. Do you think a person who is 20 years old would be an estimate of the median age? Why or why not?

   b. Do you think 40 years would be an estimate of the median age? Why or why not?

   c. Do you think 80 years would be an estimate of the median age? Why or why not?

The table that follows is used to derive an estimate of the median age, or identify the age where approximately 50% of the population is greater than this age and 50% of the population is less than this age. For the purposes of this lesson, the 2015 population is provided in millions of people to the nearest hundredth. Given the size of the population (over 300 million people), approximations listed in the table are sufficient for deriving an estimate of the median age.

Carefully examine the column headings of this table. In particular, note the column labeled **Cumulative count** of people. The first value of the **Cumulative count** of people is the count of the 0 – 4 years old age group. The next value of the cumulative count represents the count of people 0 – 9 years old. This cumulative count is the sum of the 0 – 4 years old and the 5 – 9 years old age groups. In the same way, the next value of the cumulative count represents the count of people 0 – 14 years old. It is the sum of the previous cumulative count (or the 0 – 9 years old age groups) and the 10 – 14 years old age group. This process continues until the entire population of the country is summarized in the cumulative count.

Study the following partially completed table designed to estimate the population median age:

United States - 2015

Age group	**Count** of people in each age group (in millions of people to the nearest hundredth)	**Cumulative count** of people (in millions of people to the nearest hundredth)	**Proportion of cumulative count** of people to the total population of the country (to the near thousandth)	**Proportion as a percent** (to the nearest tenth of a percent)
0 – 4	19.91	19.91	$\frac{19.91}{320.91} = 0.062$	6.2%
5 - 9	20.48	40.39	$\frac{40.39}{320.91} = 0.126$	12.6%
10 - 14	20.61	61.00	$\frac{61.00}{320.91} = 0.190$	19.0%
15 - 19	21.09	82.09	$\frac{82.09}{320.91} = 0.256$	25.6%
20 - 24	22.69	104.78	$\frac{104.78}{320.91} = 0.327$	32.7%
25 - 29	22.40		$\frac{127.18}{320.91} = 0.396$	
30 - 34	21.62			46.4%
35 - 39	20.31	169.11	$\frac{169.11}{320.91} = 0.527$	52.7%
40 - 44	20.16		$\frac{189.27}{320.91} = 0.590$	
45 - 49	20.80	210.07	$\frac{210.07}{320.91} = 0.655$	65.5%
50 - 54	22.29		$\frac{232.36}{320.91} = 0.724$	72.4%
55 - 59	21.77	254.13		79.2%
60 - 64	19.04	273.17	$\frac{273.17}{320.91} = 0.851$	85.1%

65 - 69	16.05		$\frac{289.22}{320.91} = 0.901$	
70 - 74	11.48	300.70		93.7%
75 - 79	8.12			
80 - 84	5.80	314.62		98.0%
85 - 89	3.86		$\frac{318.48}{320.91} = 0.992$	
90 - 94	1.85	320.33		99.8%
95 – 99	0.50		$\frac{320.83}{320.91} = 0.999$	99.9%
100+	0.08	**320.91**	$\frac{320.91}{320.91} = 1.000$	100.0%
Total	**320.91**			

5. What does each of the columns of the above table summarize? What does the cumulative count summarize? Answer each of the following questions based on the table to indicate your understanding of each of the columns.
   a. How many people are younger than 5 years old?
   b. What is the percent of people younger than 5 years old?
   c. How many people are younger than 10 years old?
   d. What is the percent of people younger than 10 years old?
   e. What is the percent of people younger than 20 years old?
   f. How many people are younger than 50 years old?
   g. What is the percent of people younger than 50 years old?

6. There are several blank cells in the table. For each blank cell, complete the expected calculations.

7. What is the first age group that captures at least 50% of the cumulative population?

8. Estimate the **age group** in which the median age is located.

9. Is it possible to identify the exact age within the age group identified in problem 8 that would be the median age (or the age where at most 50% of the population would be less than this age)? Explain your answer.

In addition to the center or median age of the population, the ***spread*** of the population based on the above table also provides an important description of the ages in the United States. There are several summaries that are used to describe spread. For this lesson, spread is identified by two important age groups. The first age group captures 75% of the cumulative population. The second age group captures 25% of the cumulative population. Between these two age groups at least 50% of the ages of every person in the country are captured. Note that the age group capturing the median age is within these two age groups. **The *difference* of the age groups that captures 75% of the population and the age group that captures 25% of the population summarizes a spread.** This difference is derived by subtracting the youngest age of the age group capturing 25% of the population from the oldest age of the age group capturing 75% of the population.

10. Using the above description of spread and the completed table, answer the following:
    a. What is the first age group that captures 75% of the cumulative population?

    b. What is the first age group that captures 25% of the cumulative population?

    c. Calculate the spread of the population based on the difference in ages of the two age groups in 10(a) and 10(b) using the oldest age from the age group that captures 75% of the cumulative population and the youngest age from the age group that captures 25% of the cumulative population. Approximately what percent of the total population is captured between the two ages used to calculate the spread?

    d. Why is the last cell in the column representing the proportion as a percent equal to 100.0%?

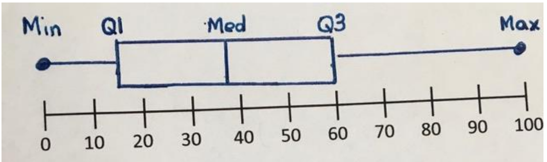
11. Describe the typical person in 2015 based on the median age and the spread. How does this person differ from the 1980 typical person?

A helpful visual of the median age and spread of a distribution is a **box plot**. For this lesson, the box plot is a modified box plot as the data are grouped in intervals of 5-years. A more precise box plot is based on using the ages of the entire population.

The box plot for the United States is defined by a ***5-number summary***.

- The first number of the summary is called the minimum (or **Min**) and represents the youngest age in the data set. For the United States population that age would be 0 years old.
- The second number of the summary is called **quartile 1** (or **Q1**) and is the youngest age of the age group capturing 25% of the population.
- The next number is the median age (or **Med**). For this lesson, this age is an age within the age group that captures 50% of the population.
- The next number is **quartile 3** (**Q3**). For this lesson, Q3 is the oldest age of the age group capturing 75% of the population.
- Finally, the last number is the maximum age (or **Max**). For this lesson, the value of 100 years will be used (although we know there actually are individuals older than 100 years old.

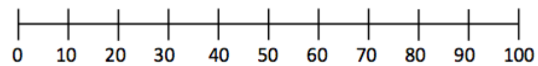
Putting these 5 numbers derived in this lesson forms the following box plot:



The United States

12. Sketch a histogram of a country with the same median age as the United States in 2015 but with a spread that is one-half of the value derived for the United States. Also sketch an approximate box plot of this country using the following grid:

Sketch a possible histogram:	Sketch a possible box plot:



Finding the Mean Age of the United States Population in 2015

The mean age of the United States population in 1980 was reported to be 33.6 years. Similar to a median age, the mean age is also considered a center. Recall that a mean of a set of data is the arithmetic average of the data. The mean age is derived by dividing the sum of the ages of all people in the United States by the total count of people. The mean is also identified as the "balance point" of a set of data (or ages) plotted on a number line. Is it possible to estimate the mean age using the counts in the age groups provided by the Census Bureau rather than all ages of the population? To answer this question, an estimate of the mean age of the United States in 2015 will be derived from the grouped data and then compared to the 1980 mean age. The following template will be used to derive an estimate of the mean age. Study the following column headings used to organize your calculation of this estimate.

United States - 2015

Age group	Mid-interval Age of Age group	Count of people (in millions of people to the nearest hundredth)	Sum of the ages in each age group: (Estimated in millions of years)
0 – 4	2	19.91	2 X 19.91 = 39.82
5 - 9	7	20.48	7 X 20.48 = 143.36
10 - 14	12	20.61	12 X 20.61 = 247.32
15 - 19	17	21.09	17 X 21.09 = 358.53
20 - 24		22.69	
25 - 29	27	22.40	27 X 22.40 = 604.8
30 - 34	32	21.62	32 X 21.62 = 691.84
35 - 39	37	20.31	
40 - 44	42	20.16	42 X 20.16 = 846.72
45 - 49		20.80	47 X 20.80 = 977.6
50 - 54	52	22.29	52 X 22.29 = 1159.08
55 - 59	57	21.77	57 X 21.77 = 1240.89
60 - 64	62	19.04	
65 - 69		16.05	
70 - 74		11.48	72 X 11.48 = 826.56
75 - 79	77	8.12	77 X 8.12 = 625.24
80 - 84	82	5.80	82 X 5.80 = 475.6
85 - 89	87	3.86	
90 – 94		1.85	
95 – 99	97	0.50	97 X 0.50 = 48.5
100+	102	0.08	102 X 0.08 = 8.16
	Total	**320.91**	**12306.52**

13. Explain how the mid-interval ages were determined for each age group.

14. Do you think all of the 19.91 million people who were 0 – 4 years old are 2 years old? Why might, however, the mid-interval age of 2 years be a reasonable estimate of the age of each of the children in the 0 – 4 years old age group?

15. To determine the mean age, the sum of the ages in each age group is needed. Consider the age group 0 – 4 years old. If 2 years is a good estimate of the age of all of the people in that age group, what does the product of 2 and 19.91 represent?

16. In the same way, what does the product of 7 and 20.48 represent?

17. Analyze what is missing in each of the blank cells of the above table. For each blank cell, complete the expected calculations.

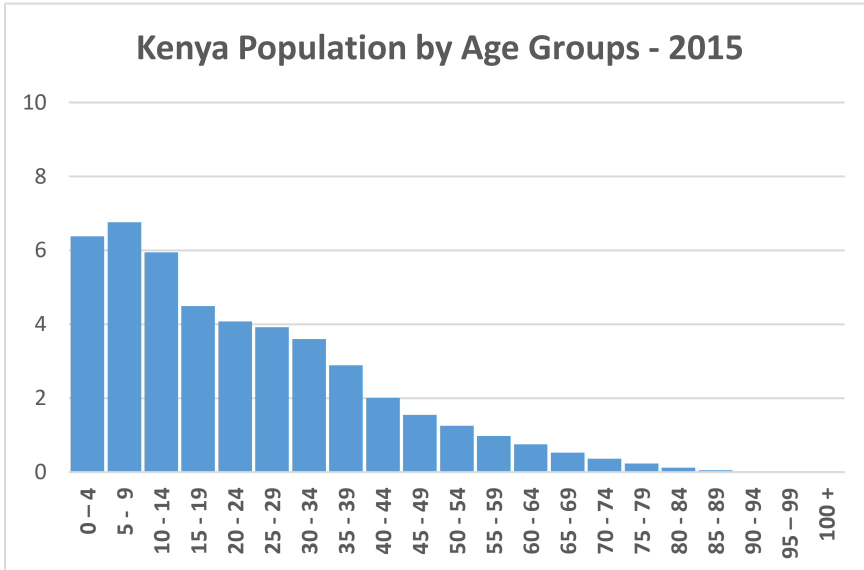
18. The last column of the above table represents an estimate of the sum of the ages for each age group. To determine the mean, the sum of all of the ages for 320.91 million people is needed. Based on the estimates recorded for each age group, 12306.52 million is the approximate sum of all of the age groups in that column. Describe the last step needed to calculate an estimate of the mean age of a person in the United States.

19. What is an estimate of the mean age of a person in the United States?

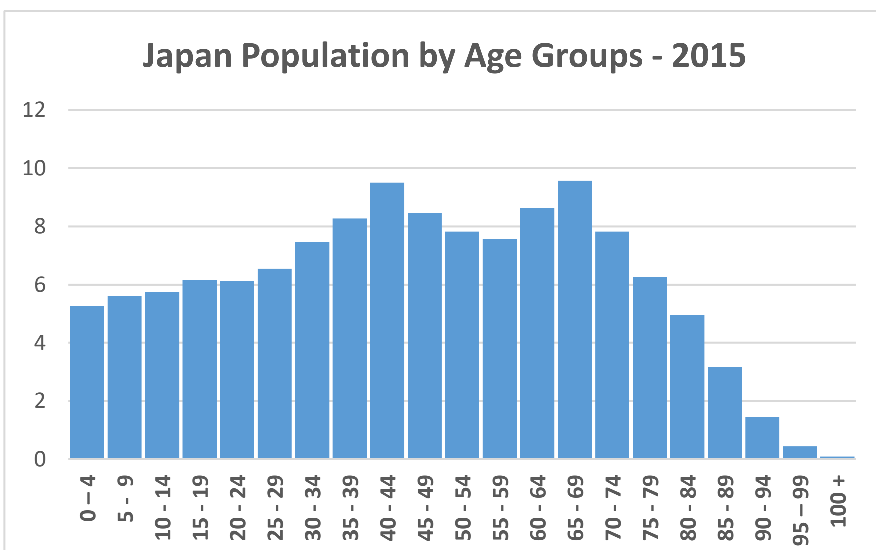
20. Do you think the estimated mean age is a good description of a typical person in 2015? Explain why or why not.

21. The estimate of the mean age in 2015 is greater than the estimate for 1980. What does this indicate about the change in the population during this time?

The estimates of the mean age and the median age are similar for the United States but not exactly the same. This similarity is not always the case, however, for other countries. Use the following templates prepared to derive an estimate of the median ages and spread and the mean ages for Kenya and Japan in 2015. One of these two countries will have a noticeable difference in the estimates of the median age and the mean age. What does it indicate about the population if the two centers are different?



Age Groups
Population in millions of people
Total Population is 45.93 millions of people



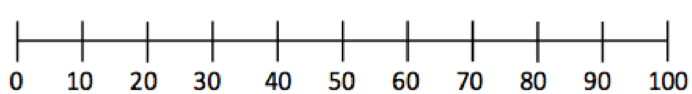
Age Groups
Population in millions of people
Total Population is 45.93 millions of people

Kenya 2015

Template for finding the estimate of the median age:

Age group	Count of people (in millions of people to the nearest hundredth)	**Cumulative count** of people (in millions of people to the nearest hundredth)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	6.38			
5 - 9	6.76			
10 - 14	5.95			
15 - 19	4.49			
20 - 24	4.08			
25 - 29	3.92			
30 - 34	3.60			
35 - 39	2.89			
40 - 44	2.01			
45 - 49	1.55			
50 - 54	1.25			
55 - 59	0.98			
60 - 64	0.75			
65 - 69	0.53			
70 - 74	0.36			
75 - 79	0.23			
80 - 84	0.12			
85 - 89	0.05			
90 - 94	0.01			
95 – 99	0.01			
100+	0.01			
Total	45.93			

22. Derive an estimate of the median age group for Kenya and a description of the spread based on the definition of spread in this lesson. After you have estimated the median age and the spread, sketch a modified box plot using the following grid:



Kenya – 2015
Template for finding the estimate of the mean age:

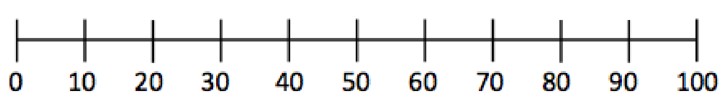
Age group	**Mid-interval Age of Age group**	**Count of people** (in millions of people to the nearest hundredth)	**Sum of all ages** in age group: (Estimated in millions of years)
0 – 4	2	6.38	
5 - 9	7	6.76	
10 - 14	12	5.95	
15 - 19	17	4.49	
20 - 24	22	4.08	
25 - 29	27	3.92	
30 - 34	32	3.6	
35 - 39	37	2.89	
40 - 44	42	2.01	
45 - 49	47	1.55	
50 - 54	52	1.25	
55 - 59	57	0.98	
60 - 64	62	0.75	
65 - 69	67	0.53	
70 - 74	72	0.36	
75 - 79	77	0.23	
80 - 84	82	0.12	
85 - 89	87	0.05	
90 – 94	92	0.01	
95 – 99	97	0.01	
100+	102	0.01	
	Total	45.93	

23. Derive an estimate of the mean age of Kenya. Compare the estimated mean age to the median age. Are they similar? Explain.

Japan 2015
Template for finding the estimate of the median age:

Age group	Count of people (in millions of people to the nearest hundredth)	**Cumulative count** of people (in millions to the nearest hundredth)	**Proportion of cumulative count of people to total population** (to the near thousandth)	Proportion as a percent
0 – 4	5.27			
5 - 9	5.61			
10 - 14	5.75			
15 - 19	6.15			
20 - 24	6.13			
25 - 29	6.54			
30 - 34	7.47			
35 - 39	8.27			
40 - 44	9.50			
45 - 49	8.46			
50 - 54	7.82			
55 - 59	7.57			
60 - 64	8.62			
65 - 69	9.57			
70 - 74	7.82			
75 - 79	6.26			
80 - 84	4.95			
85 - 89	3.17			
90 - 94	1.45			
95 – 99	0.44			
100+	0.09			
Total	**126.91**			

24. Derive an estimate of the median age group for Japan and a description of the spread based on the definition of spread in this lesson. After you have estimated the median age and the spread, sketch a modified box plot using the following grid:



Japan – 2015
Template for finding the estimate of the mean age:

Age group	**Mid-interval Age of Age group**	**Count of people** (in millions of people to the nearest hundredth)	**Sum of ages in age group:** (Estimated in millions of years)
0 – 4	2	5.27	
5 - 9	7	5.61	
10 - 14	12	5.75	
15 - 19	17	6.15	
20 - 24	22	6.13	
25 - 29	27	6.54	
30 - 34	32	7.47	
35 - 39	37	8.27	
40 - 44	42	9.50	
45 - 49	47	8.46	
50 - 54	52	7.82	
55 - 59	57	7.57	
60 - 64	62	8.62	
65 - 69	67	9.57	
70 - 74	72	7.82	
75 - 79	77	6.26	
80 - 84	82	4.95	
85 - 89	87	3.17	
90 – 94	92	1.45	
95 – 99	97	0.44	
100+	102	0.09	
	Total	**126.91**	

25. Derive an estimate of the mean age of Japan. Compare Japan's estimate of the mean age to the median age. Are they similar? Explain.

Lesson 5
My Country

The next several lessons of this module continue to examine the population graphs of the United States, Kenya, and Japan prior to 2015. Distributions for 2020 and in the future will also be examined. Several lessons involve estimating the future shape of these countries based on assumptions that affect the counts in key age groups. Each country has a distinctive population pyramid graph and histogram that suggests challenges and opportunities. Changes in the population distribution over time indicate possible new challenges and opportunities.

Now it is your turn. In this lesson, you will generate a population distribution of a made-up country. What shape do you want your country's population to represent (bottom-layered, lower middle-layered, upper middle-layered, top-layered) and what age groups might pose some challenges or special concerns for your country now or in the future? Your country will be referenced in several of the upcoming lessons. These lessons will allow you to address some of the possible changes in your country's future in the same way that they impact the future of the United States, Kenya, or Japan. The lessons in Unit 4 will encourage you to work with your country using a projection model that is initially organized for analyzing the United States, Kenya, Japan and other countries. (If interested, provide a name for your country. The lessons in the next unit of this module will reference the country you design as My Country.)

Lesson 5 – Problems

Consider the following scenarios before you decided to generate a population distribution:

Scenario 1: The Perfect Shape?

The United States, Kenya, and Japan each face unique challenges and opportunities that are connected to their current population distributions. Finding the resources to care for a growing segment of older people, or finding the resources to care for a significant count of children were part of what the United States or Kenya or Japan face or will face in the future. A student completing this module wondered if there was a "perfect shape" of a population distribution. Several opinions were shared regarding that question, but the most interesting opinion of a "perfect shape" was a suggestion that the population distribution resulted in a pyramid graph that looked like a house. Consider creating a population distribution that results in a pyramid graph fitting this description. Why do you think this shape was suggested as a "perfect shape"?

Scenario 2: A Top Layered Country

There are currently no population distributions that would define a country as a top-layered country. Consider creating a country that is top-layered. What challenges do you think will be faced by a country with this shape?

Scenario 3: A Lower and Upper Middle-Layered Country

It is possible that a country has an equal percent of people in more than one of the layers of the population distribution. Consider creating a country that has an equal or nearly equal percent of people in the lower middle-layer and the upper middle-layer in which the percent in these layers is greater than the percent in the bottom or top layers.

Consider one of the above scenarios to complete the following problems in this lesson.

1. Generate counts for your country for the beginning of 2015. Complete the following table (estimate counts in millions of people to the nearest hundredth – for example, 16,293,232 will be represented as 16.29 million):

Country: **My Country** in 2015

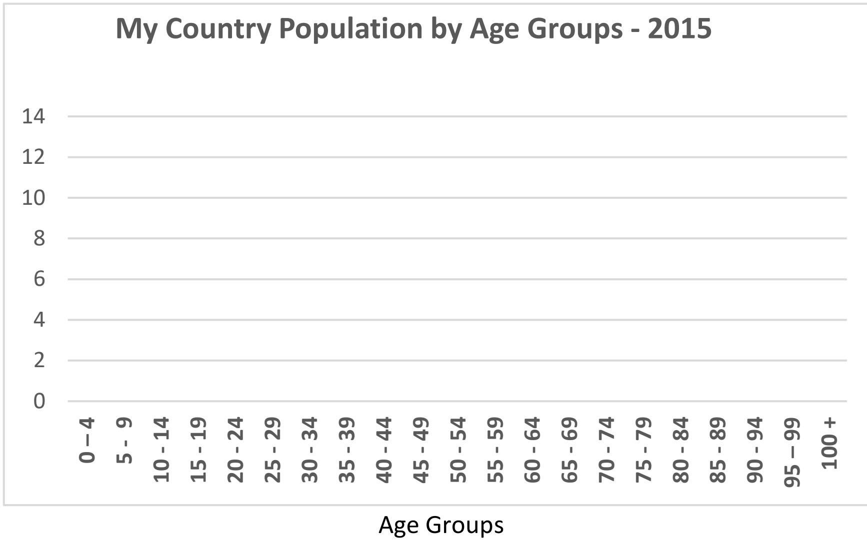
Age Group	Males	Females	Total
0 - 4			
5 - 9			
10 - 14			
15 - 19			
20 - 24			
25 - 29			
30 - 34			
35 - 39			
40 - 44			
45 - 49			
50 – 54			
55 - 59			
60 - 64			
65 - 69			
70 - 74			
75 - 79			
80 - 84			
85 - 89			
90 - 94			
95 – 99			
100+			

2. Based on the table of counts in each age group, use the following template to complete a pyramid graph of **My Country**:

Male **My Country** - 2015 Female

					100+					
					95 - 99					
					90 - 94					
					85 - 89					
					80 - 84					
					75 - 79					
					70 - 74					
					65 - 69					
					60 - 64					
					55 - 59					
					50 - 54					
					45 - 49					
					40 - 44					
					35 - 39					
					30 - 34					
					25 - 29					
					20 - 24					
					15 - 19					
					10 - 14					
					5 - 9					
					0 - 4					
					Age group					
				0		0				

3. Create a histogram of My Country.



Population in millions of people
(adjust the scale if necessary based on the counts of your country)

4. Determine if your country is bottom-layered, lower middle-layered, upper middle-layered, or top-layered or a combination of these descriptions.

5. Identify any features of your country you want to watch as you look into the future.

6. Use the templates provided to calculate estimates of the median age, the spread as defined in Lesson 4, and the mean age for My Country.
   Median age:
   Spread of ages from approximately 25% to 75% of the population:
   Mean age:

7. Describe a typical person in your country.

My Country – 2015
Template for finding the estimate of the mean age:

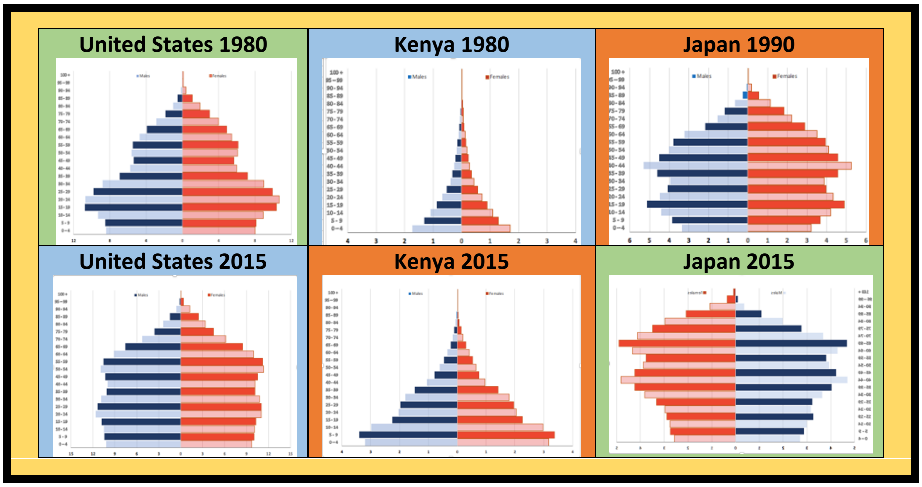
Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2		
5 - 9	7		
10 - 14	12		
15 - 19	17		
20 - 24	22		
25 - 29	27		
30 - 34	32		
35 - 39	37		
40 - 44	42		
45 - 49	47		
50 - 54	52		
55 - 59	57		
60 - 64	62		
65 - 69	67		
70 - 74	72		
75 - 79	77		
80 - 84	82		
85 - 89	87		
90 – 94	92		
95 – 99	97		
100+	102		
	Total		

My Country 2015

Template for finding the estimate of the median age:

Age group	Count of people (in millions of people)	**Cumulative coun**t of people (in millions)	Proportion of cumulative count of people to total population	Proportion as a percent
0 – 4				
5 - 9				
10 - 14				
15 - 19				
20 - 24				
25 - 29				
30 - 34				
35 - 39				
40 - 44				
45 - 49				
50 - 54				
55 - 59				
60 - 64				
65 - 69				
70 - 74				
75 - 79				
80 - 84				
85 - 89				
90 - 94				
95 – 99				
100+				
Total				

Unit 2
Looking Back



Introduction

Kristin, Raphine and Hana are part of the same age group. While Raphine finds most of the people in his country younger and Hana finds most of the people older, Kristin finds most of the people older or younger. Each of them belongs to an age group that plays a major role in telling the stories of their countries. Although each of them was 36 years old at the start of 2015, the population pyramid graphs of their countries tell different stories over time.

The United States, Kenya, and Japan experienced events that resulted in their unique shapes. Natural disasters such as hurricanes, earthquakes, volcanic eruptions, or unusual drought or flood conditions resulted in a loss of life that reshaped each population distribution for decades. Other events, such as major wars, also decreased each country's population and reshaped the population distribution reflected in the population pyramid graphs. In the same way, periods of relative peace and positive economic conditions increase a country's population that is also evident in a country's past and current shape.

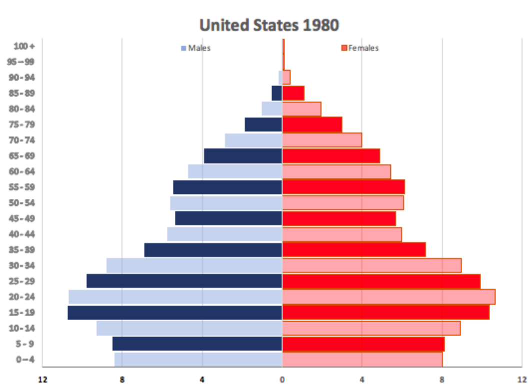
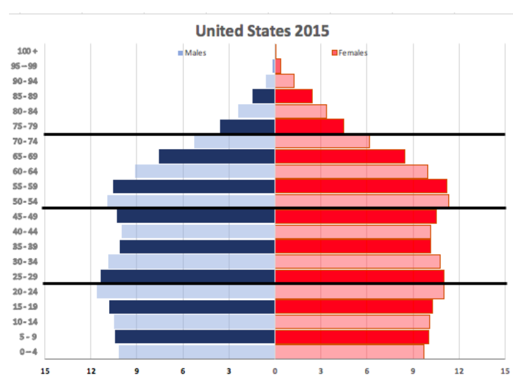
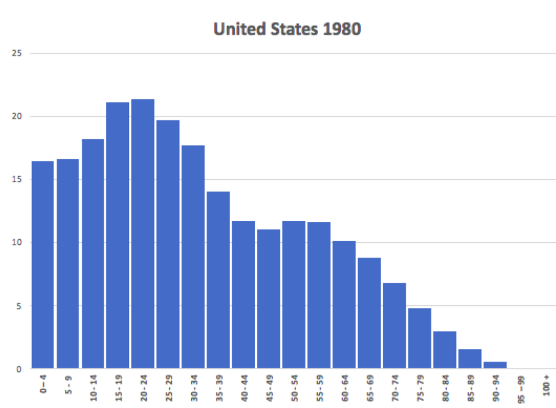
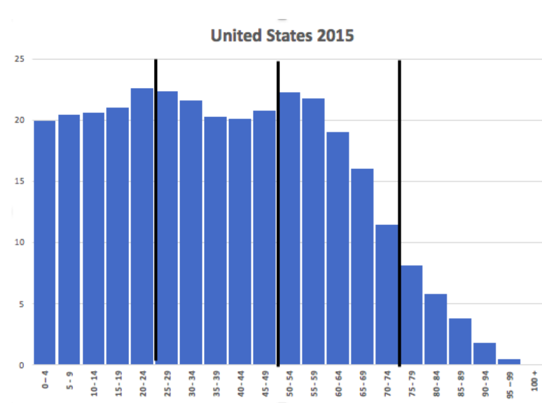
"Looking Back" is more than just a reflection of how things have changed. It is identifying those events that resulted in increasing or decreasing the population in ways that are evident in each country's past and current shape. This short unit looks back at each of our countries through the pyramid graphs and the histograms that tell us where the generation that includes Kristin, Raphine, and Hana began or made their impact.

Lesson 6
Looking Back at the Shapes of the United States

The shapes of the United States population pyramid graphs and histograms varied over time. The pyramid graphs and the population histograms displayed in this lesson provide a visual summary of the changes in the population of the United States from 1980 to 2015.

Lesson 6 – Problems

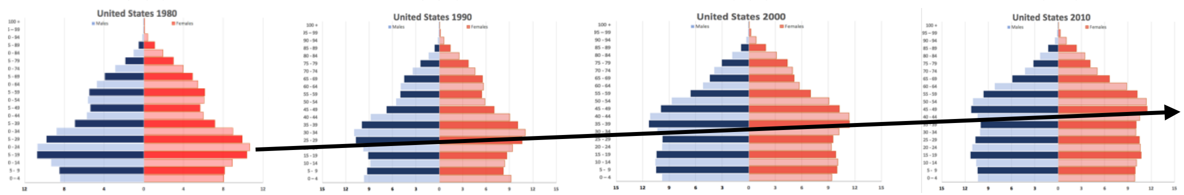
1. Lesson 1 defined the age group layers (bottom-layered, lower middle-layered, upper middle-layered, and top-layered) for the 2015 pyramid graph or histogram. Based on the percent of people in each layer, the United States was identified as a lower middle-layered country in 2015. Identify the layers in the 1980 pyramid graph and the 1980 histogram similar to the way they are identified in the 2015 graphs.

1980	2015
 Total population: 227.2 million people	 Total population: 320.9 million people
 Population in millions	 Population in millions

2. Based on the layered marks, what identification would summarize the shape of the country in the year 1980? Which graph, the pyramid graph or the histogram, did you use to make your identification? Explain why you selected the graph you identified.

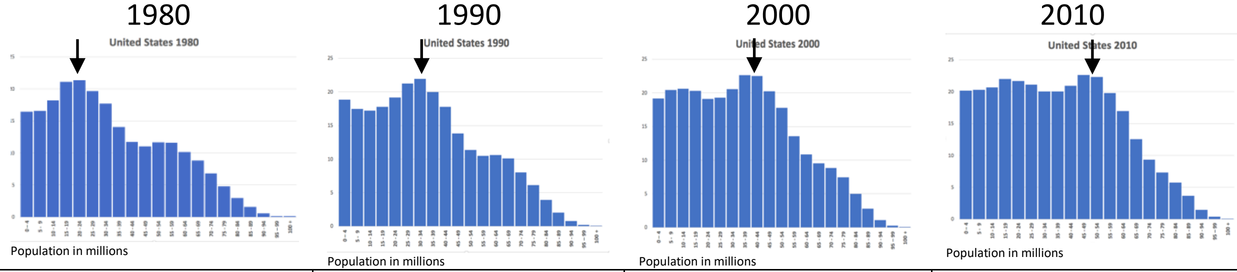
3. The change in the shape from 1980 to 2015 is highlighted by observing the aging of certain age groups over two to three decades. The arrows sketched on the following pyramid graphs and histograms trace the aging of people who were 20 – 24 years old at the start of 1980 through three decades. What is significant about this age group in 1980?

1980	1990	2000	2010



Total population: 227.2 million people	Total population: 249.6 million people	Total population: 282.2 million people	Total population: 309.3 million people

1980	1990	2000	2010



Arrow highlights the 20 – 24 years old age group or 21.4 million people	Arrow highlights the 30 – 34 years old age group or 21.9 million people	Arrow highlights the 40 – 44 years old age group or 22.4 million people	Arrow highlights the 50 – 54 years old age group or 22.4 million people

4. Based on the above timeline, the people 20 - 24 years old in 1980 would be counted in the 30 – 34 years old age group of the 1990 Census if they did not move out of the country or die. What is significant about the count of the 30 – 34 years old age group in 1990?

5. Observe that the people 20 – 24 years old in 1980 are counted in the 40 – 44 years old age group in the 2000 census if they did not move out of the country or they did not die. Did the count of people in the 40 – 44 years old age group in 2000 increase, decrease, or stay the same when compared to the count of people in the 20 – 24 years old age group in 1980? Explain what happened during the 20 years from 1980 to 2000 that might increase or decrease the count of people 20 – 24 years old in 1980 to the count of people 40 – 44 years old in 2000.

6. Calculate the following based on the above pyramid graphs and histograms highlighted in this lesson:
   a. What percent of the population in 1980 were 20 – 24 years old?

   b. What percent of the population in 1990 were 30 – 34 years old?

   c. What percent of the population in 2000 were 40 – 44 years old?

   d. What percent of the population in 2010 were 50 – 54 years old?

   e. The count of people in 2015 who were 55 – 59 years old was 21.8 million people. What percent of the population in 2015 were 55 – 59 years old?

The people who were 20 – 24 years old in 1980 and then 50 - 54 years old in 2010 and 55 – 59 years old in 2015 are part of what demographers (or people who study and research populations) identify as the ***Baby Boom Generation***. A generation refers to a range of birth years that identify a group of people with common historical experiences. Although a subjective topic, most research involving the Baby Boom Generation identify this group as people born in 1943 to 1960.

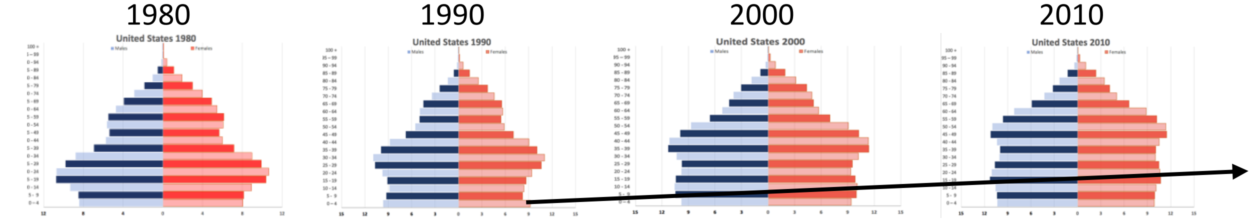
7. Recall that for this module, the count of people in an age group is based on ages ***at the start of the year***. Therefore, a person born in 1943 was counted as a one-year old at the start of the 1945 United States Census count, although this person turned two years old sometime in 1945. If this person continued to live in the United States, he or she was six years at the start of 1950 and turned seven sometime in 1950. Based on that interpretation of the age groups represented in the graphs, answer the following:
   a. What was the age of a person whose birth year is 1943 at the start of 2015?

   b. What was the age of a person whose birth year is 1960 at the start of 2015?

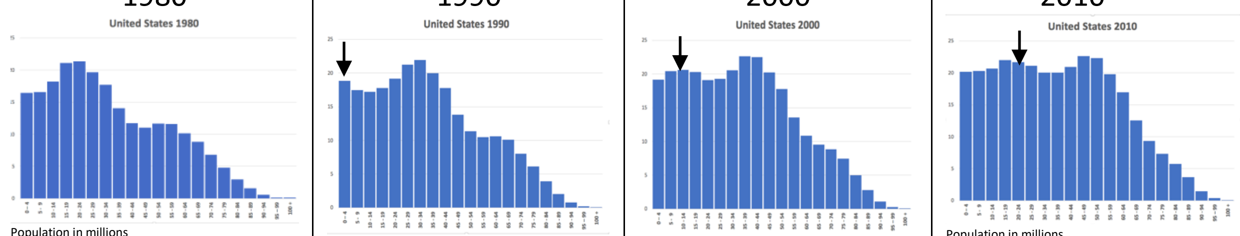
c. What age groups in the 2015 population graphs would include most of the people born in 1943 to 1960?

d. What do the population graphs from 1980 to 2010 indicate is happening to the Baby Boom Generation?

In 1990, people who were 0 – 4 years old are identified by the arrows in the following pyramid graphs and histograms. In a similar way to the previous problems, the people 0 – 4 years old at the start of 1990 are identified in the following graphs and the aging of that age group for two decades.



1980	1990	2000	2010
Total population: 227.2 million people	Total population: 249.6 million people	Total population: 282.2 million people	Total population: 309.3 million people



1990	2000	2010
Arrow highlights the 0 – 4 years old age group or 18.8 million people	Arrow highlights the 10 – 14 years old age group or 20.6 million people	Arrow highlights the 20 – 24 years old age group or 21.7 million people

8. What is significant about the 0 – 4 years old age group in 1990?

9. People 0 – 4 years old at the start of 1990 (or born in 1985 to 1989) would be 20 - 24 years old at the start of 2010 if they remained in the country or did not die. Did the count of people in the 0 – 4 years old age group in 1990 increase, decrease, or stay the same when compared to the count of people in the 20 – 24 years old age group in 2010? Explain what might increase or decrease the count of the people who were 0 – 4 years old in 1990 and then 20 – 24 years old in 2010.

Demographers identify the people born in 1982 to 2004 as the ***Millennial Generation***. (The birth years for people in this generation are frequently debated, but at the time of the writing of this module, most research involving the ***Millennial Generation*** identify this group as people born in the years 1982 to 2004.)

10. Calculate the following summaries of the population using the values included in the timeline:
    a. What is the percent of the population 0 – 4 years old in 1990?

    b. What is the percent of the population 10 – 14 years old in 2000?

    c. What is the percent of the population 20 – 24 years old in 2010?

11. Kristin was directed by the administrators at the health clinic to write a report highlighting the changes of the United States population by ages from 1980 to 2015. Which one of the following summaries would be appropriate for her report based on the descriptions of the Baby Boom Generation and the Millennial Generation in the previous problems? Explain why you think your selection is the most accurate.

    a. The Baby Boom Generation is growing in numbers.

    b. The Millennial Generation is catching up to the Baby Boom Generation.

    c. The Millennial Generation is decreasing in numbers.

    d. The percent of the United States population in 2010 who are considered the Baby Boom Generation and the Millennial Generation is less than most other age groups.

12. Why are these two generations (Baby Boom Generation and the Millennial Generation) important in understanding the United States in the following areas:
    a. entertainment watched on TV or cable or streaming services?

    b. musical preferences?

    c. movies?

    d. health care?

An Overview of Generations

A description of age groups in the United States is often done by references to a generational classification. Several labels have been created to identify people by the year they were born or their birth year There is debate among sociologists on the labels used to identify generations as well as the birth years used to classify the generations. Several sociologists, however, have defined generational labels when describing the United States population. The generational labels used by sociologists William Strauss and Neil Howe are often cited in research studies. Although the dates are periodically revised, the following generational labels were defined by William Strauss and Neil Howe:

Generation label (by Strauss-Howe)	Birth years
Silent Generation	1925 to 1942
Baby Boom Generation	1943 to 1960
Generation X or Generation 13	1961 to 1981
Millennial Generation	1982 to 2004

Source: https://en.wikipedia.org/wiki/Strauss–Howe_generational_theory

(The above dates are also cited in their book ***Generations: The History of America's Future*** by Strauss and Howe. Note the interesting title of this book as it indicates the impact of the past to the future.)

13. Let us return to the characters of the data stories in this module. Complete the following table (Kristin has been completed for you):

	Age at start of 2015	Birth year	Age at start of 1980	Generation
Kristin	36	1978	1	Generation X
Abbey (Kristin's sister)	26			
Kristin's mother	66			

14. Answer the following:
    a. In what age group did Kristin's mother belong in the 1980 population histogram?

    b. In what age group did Kristin belong in the 1980 population histogram?

15. Use the 2015 population histogram to answer the following:
    a. Identify the three age groups immediately younger than Kristin's age group in 2015. Are the counts in these age groups greater than or less than the count in Kristin's age group?

    b. Identify the three age groups immediately younger than Kristin's mother's age group in 2015. Are the counts in these age groups greater than or less than the count in her mother's age group?

16. In what age group was Abbey counted in 2015?. Identify the three age groups immediately younger than Abbey's age group in 2015. Are the counts in these age groups greater than or less than the count in Abbey's age group?

17. Kristin's mother was 30 years old when Kristin was born and 40 years old when Abbey was born. Members of the Baby Boom Generation generally had children at an older age than people in the generations older than the Baby Boom Generation. In what way did the decision of Baby Boomers to have children at an older age possibly impact the counts of people in age groups that included Kristin's age group and 3 age groups immediately younger than Kristin's age group?

The increased count of people in the age groups identified in the Baby Boom Generation is generally explained by the impact of World War II. This horrific war (1939 - 1943) directly involved millions of people and affected the counts of several age groups that is reflected in the 2015 population histogram. Several of the age groups identified as the **Silent Generation** were born just prior to this war and during the war. The counts of people born during this time were less than the counts before and after this generation. Wars have a major impact on the shape of a country.

What events happened to the Baby Boom Generation during their birth years and approximately 15 beyond their birth years? What events happened to Generation X during their birth years and approximately 15 beyond their birth years? What happened during the birth years and approximately 15 years beyond the birth years to the Millennial Generation? What events happened and are happening in the birth years of your generation that is still being defined? The events and changes in society due to economic conditions, changes in technology, changes in transportation and education are often researched to understand a generation.

Consider researching what happened in the United States during the birth years of the generations identified in the table. Also consider interviewing a person in a different generation, discussing with them some of the events they identify as important (for example, 9/11, the moon landing, the Vietnam War, the disaster of the Challenger mission)***. As this possible interview can result in some personal discussions that might be difficult for people to share, carefully write out what you plan to ask in your interview and share your plan with your teacher before you conduct the interview.*** And, respect the decision of a person to decline an interview.

18. Determine the percent change in the age groups from 1980 to 2015 by completing the following table (round off the percent increases to the nearest tenth of a percent):

Age Group	1980 Population	2015 Population	Percent change from 1980 to 2015
0-4	16,451,184	19,912,499	(19,912,499 – 16,451,184)/16,451,184 0.210 = 21.0%
5-9	16,602,353	20,481,130	(20,481,130 - 16,602,353)/16,602,353 0.234 = 23.4%
10-14	18,236,335	20,605,579	
15-19	21,110,940	21,084,710	
20-24	21,385,705	22,693,026	(22,693,026 - 21,385,705)/21,385,705 .061 = 6.1%
25-29	19,685,966	22,401,168	
30-34	17,742,706	21,617,533	
35-39	14,076,734	20,312,646	(20,312,646 - 14,076,734)/14,076,734 0.443 = 44.3%
40-44	11,728,497	20,156,736	
45-50	11,048,040	20,801,156	
50-54	11,694,715	22,289,734	
55-59	11,611,382	21,767,855	(21,767,855 - 11,611,382)/11,611,382 0.875 = 87.5%
60-64	10,142,668	19,038,554	(19,038,554 - 10,142,668)/ 10,142,668 0.877 = 87.7%
65-69	8,809,479	16,049,246	
70-74	6,841,235	11,477,776	(11,477,776 - 6,841,235)/6,841,235 0.678 = 67.8%
75-79	4,829,832	8,119,847	
80-84	2,955,279	5,798,910	
85-89	1,580,234	3,864,289	(3,864,289 - 1,580,234)/1,580,234 1.445 =144.5%
90 - 94	557,241	1,851,620	
95 – 99	119,057	495,362	(495,362 - 119,057)/119,057 = 316.07%
100+	15,099	77,242	
Totals	227,224,681	320,896,618	

It is important to look back when designing a model that is designed to look forward. The 2010 and 2015 counts recorded by the United States Census Bureau are considered accurate ***estimates*** of the actual population at the start of 2010 and 2015. Population estimates beyond 2015 will be identified in the following lessons as ***projections.*** It may be possible that the actual counts from the US Census Bureau for the year 2020 or later are available at the time you are working with this module. Consider modifying the projection models presented in the following lessons that result in a match of the projections for 2020 to the estimates of the actual counts for 2020.

Remember, these recorded counts are based on either an actual census (e.g., 2010) or an adjusted census (e.g., 2015). Also remember, although these counts are recorded "to the nearest person or nearest whole number", they are nonetheless estimates – the Census Bureau's best estimates! In several of the lessons that follow, these counts will be rounded off to other units to make the calculations more manageable.

The United States

Age Group	2010	2015
0-4	20,189,589	19,912,499
5-9	20,331,807	20,481,130
10-14	20,681,215	20,605,579
15-19	21,983,206	21,084,710
20-24	21,704,549	22,693,026
25-29	21,145,232	22,401,168
30-34	20,070,096	21,617,533
35-39	20,079,840	20,312,646
40-44	20,905,848	20,156,736
45-50	22,637,291	20,801,156
50-54	22,353,471	22,289,734
55-59	19,795,182	21,767,855
60-64	16,990,224	19,038,554
65-69	12,521,439	16,049,246
70-74	9,336,583	11,477,776
75-79	7,320,106	8,119,847
80-84	5,759,428	5,798,910
85-89	3,640,827	3,864,289
90 - 94	1,471,494	1,851,620
95 – 99	376,356	495,362
100+	54,410	77,242
Totals	309,348,193	320,896,618

19. Identify age groups that you would like to follow as projection estimates are derived in the following lessons. Why are you interested in these age groups?

*__Henry's Quilt Problem:__
Identify one country from the cover (Henry's quilt) that you think has a Baby Boom Generation. Explain why you selected this country. Also identify one country that you think has a Millennial Generation. Explain why you selected this country.

Lesson 7
Looking Back at the Shapes of Kenya and Japan and My Country

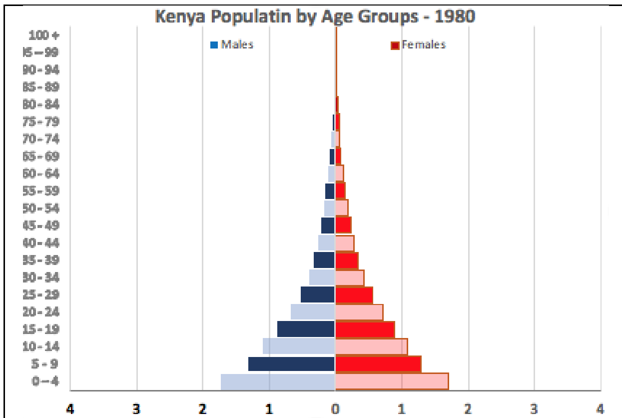
This lesson looks back at the population data for Kenya and Japan in the same way Lesson 6 looked back at the United States population. Kenya and Japan do not have a constitutional mandate to conduct a census every ten years. Each country periodically conducts a census, however, the time between each census varies. The data used in this lesson were obtained from the International Data Base (IDB) and are considered as accurate estimates of the population of the countries. Statistical procedures other than a census were used to estimate the population of Kenya and Japan.

Lesson 7 – Problems

Kenya

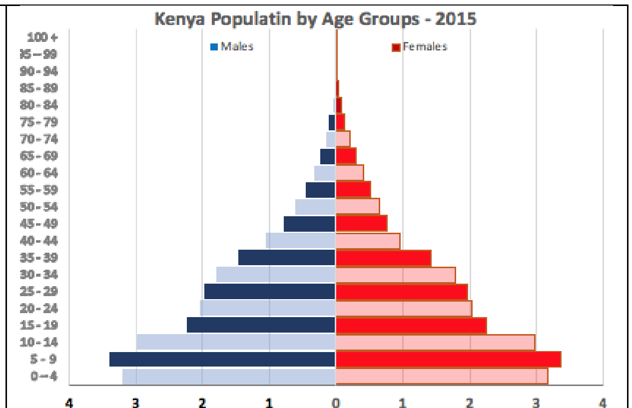
The 1980 population and the 2015 population are summarized by the following population pyramid graphs and histograms.

Kenya -1980

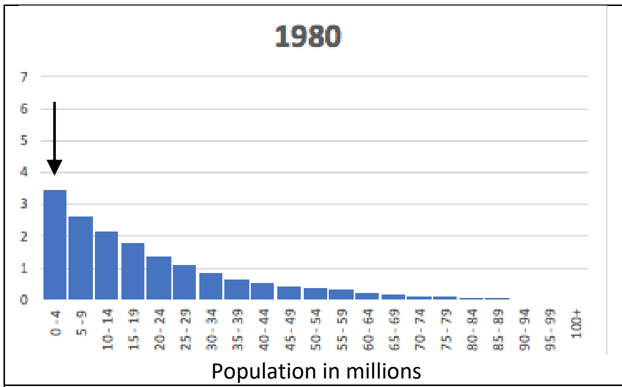


Total population: 16.33 million people

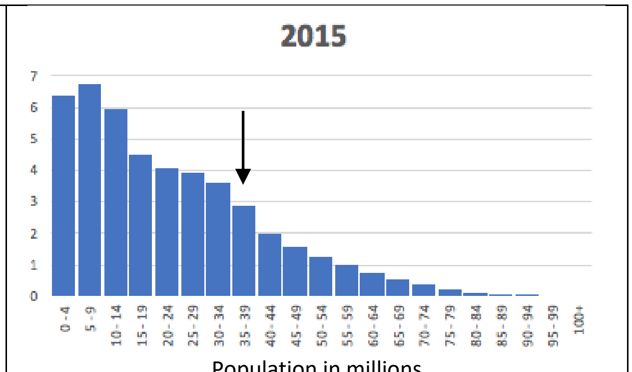
Kenya - 2015



Total population: 45.93 million people



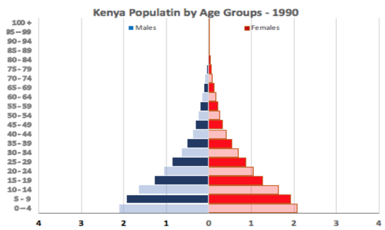
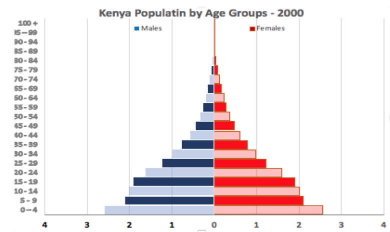
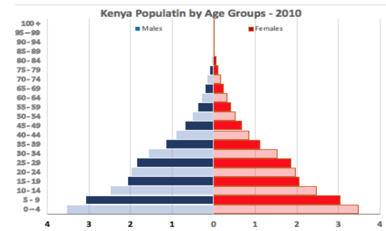
Arrow highlights the 0 – 4 years old age group or 3.48 million people

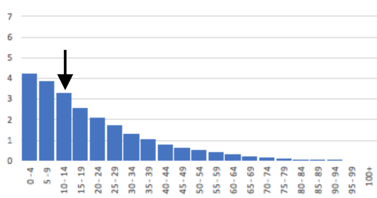
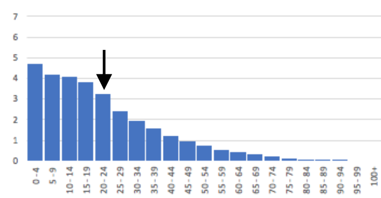
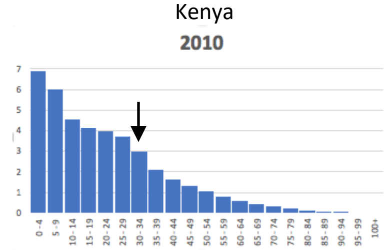


Arrow highlights the 35 – 39 years old age group or 2.89 million people

The population pyramid graphs and histograms of the intervening years of 1990, 2000, and 2010 are also provided. Note the changes in the shape of the population pyramid graphs and histograms.

Journey through the intervening years:

Kenya 1990	**Kenya 2000**	**Kenya 2010**
		
Total population: 23.36 million people	Total population: 30.61 million people	Total population: 40.84 million people

Kenya 1990	Kenya 2000	Kenya 2010
		
Population in millions	Population in millions	Population in millions
Arrow highlights the 10 – 14 years old age group or 3.28 million people	Arrow highlights the 20 – 24 years old age group or 3.22 million people	Arrow highlights the 30 – 34 years old age group or 3.00 million people

1. The information provided above indicates that the total population of Kenya increased from 16.33 million people in 1980 to 45.93 million people in 2015.
   a. Calculate the percent increase of the population from 1980 to 2015.

   b. Describe how the graphs (either the population pyramid graph or the population histogram) also indicate an increase of the population from 1980 to 2015.

2. The 0 – 4 years old in 1980 are highlighted by an arrow in the 1980 histogram. If the people in the 0 – 4 years old age group did not move to another country or did not die in the next 35 years, they were also counted in the 35 – 39 years old age group in 2015. An arrow is used to also identify the 35 -39 years old age group in 2015.
   a. What count and percent of the people in Kenya were 0 – 4 years old at the beginning of 1980?

   b. What count and percent of the people in Kenya were 35 – 39 years old at the beginning of 2015?

   c. Explain what factors contributed to the decrease in the population of the age group 0 – 4 years old in 1980 to the 35 – 39 years old in 2015 while the total population of Kenya increased.

3. Answer the following questions using the population graphs and the information provided about the total population and the population within highlighted age groups:
   a. In what age group in the 1990 population were people counted who were 0 – 4 years at the start of 1980?

   b. What percent of the people in 1990 belonged to the age group identified in 3(a)?

4. Continue to use the population graphs and the information provided to answer the following:
   a. In what age group in the 2000 population were people counted who were 0 – 4 years at the start of 1980?

   b. What percent of the people in Kenya belonged to the age group you identified in 4(a)?

c. In what age group in the 2010 population were people counted who were 0 – 4 years at the start of 1980?

d. What percent of the people in Kenya belonged to the age group you identified in (c)?

5. Summarize the change in the count and percent of the people who were 0 – 4 years old at the start of 1980 to the count and percent of people 35 – 39 years old in 2015.

6. Although a decrease in both the count and the percent of people were noted in the 0 – 4 years old age group to the 35 – 39 years old age group, there are other summaries that indicate changes in the population of Kenya from 1980 to 2015.
   a. Complete the following table by calculating the percent of the population in the given year who were 35 – 39 years old. (The calculation for 1980 has been completed as an example. Calculate your answer to the nearest tenth of a percent.)

Year	Count in age group 35 – 39 years old (millions of people)	Count in total population (millions of people)	Percent of 35 – 39 years old in the total population
1980	0.66	16.33	4.0%
1990	1.05	23.36	
2000	1.56	30.62	
2010	2.10	40.83	
2015	2.89	45.93	

   b. Based on the above changes in this age group, and similar changes in several older age groups, describe a change in the median age of people in Kenya from 1980 to 2015.

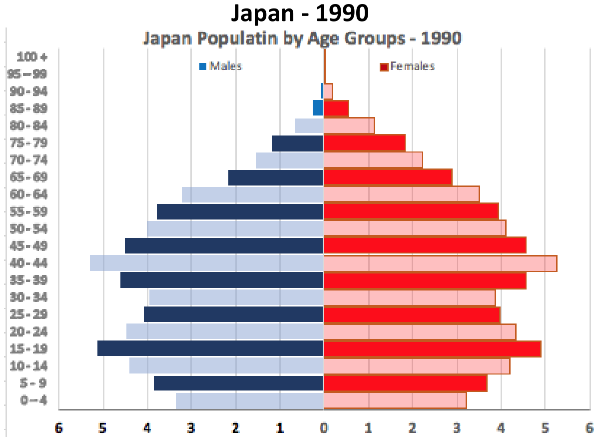
   c. Why was the change in the median age an encouraging summary for the population of Kenya?

7. In what way do you think the mean ages has also changed from 1980 to 2015 in Kenya? Explain your answer.

Japan

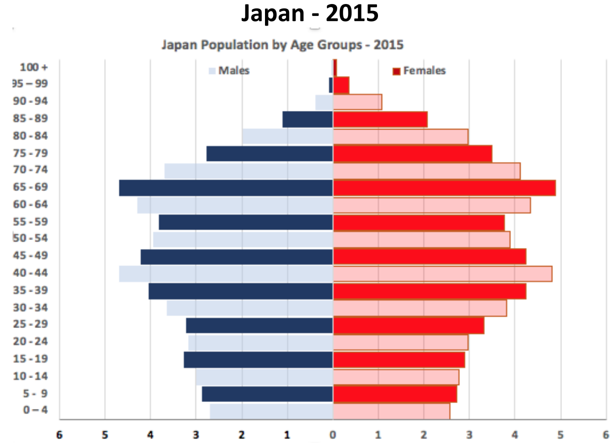
The 1980 population and the 2015 population are summarized by the following population pyramid graphs and histograms.

Japan - 1990

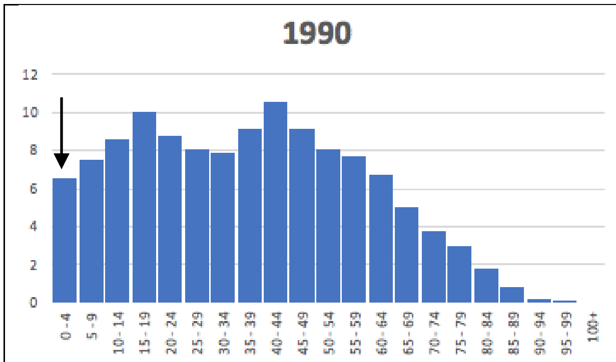


Total population: 123.54 millions of people

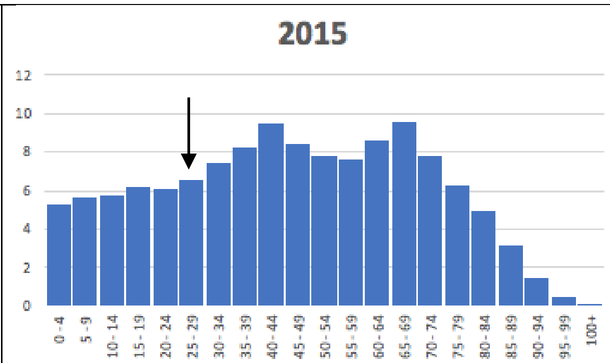
Japan - 2015



Total population: 126.91 millions of people



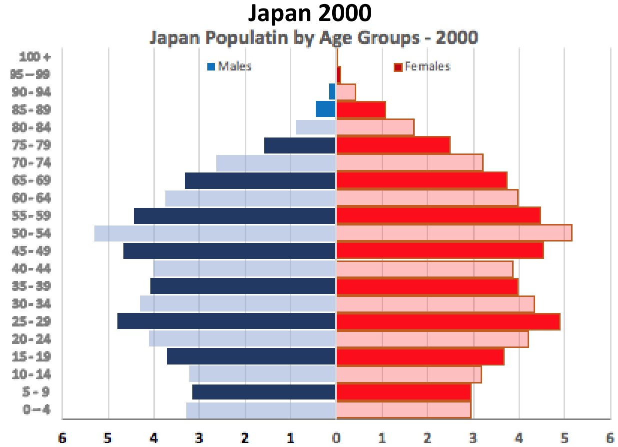
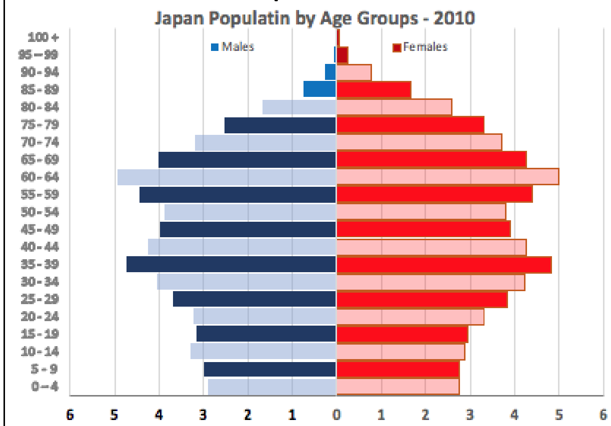
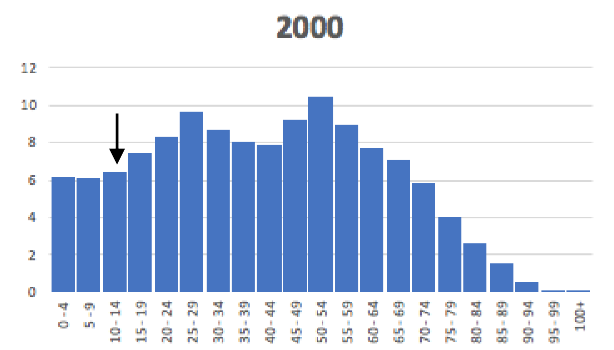
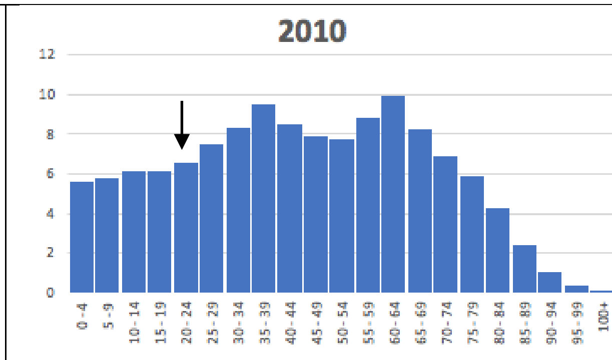
Arrow highlights the 0 – 4 years old age group or 6.57 million people



Arrow highlights the 25 – 29 years old age group or 6.54 million people

The population pyramid graphs and histograms of the intervening years of 2000 and 2010 are also provided. Note the changes in the shape of the population pyramid graphs and histograms.

Journey through the intervening years:

Japan 2000	Japan 2010
	
Total population: 126.78 million people	Total population: 127.56 million people
	
Population in millions	Population in millions
Arrow highlights the 10 – 14 years old age group or 6.42 million people	Arrow highlights the 20 – 24 years old age group or 6.55 million people

8. What do the graphs tell us about the changes in the population of Japan from 1990 to 2015? Identify at least 2 changes highlighted by the graphs.

9. Use the 1990 and 2015 population pyramid graphs or histograms to answer the following questions.
   a. What is the count of people 0 – 4 years old in 1990?
   b. What is the percent of people 0 – 4 years old in 1990?
   c. What is the count of people 25 – 29 years old in 2015?
   d. What is the percent of people 25 – 29 years old in 2015?

10. Use the 2000 and 2010 population pyramid graphs or histograms to answer the following questions?
    a. What is the count of people 10 – 14 years old in 2000?

    b. What is the percent of people 10 – 14 years old in 2000?

    c. What is the count of people 20 – 24 years old in 2010?

    d. What is the percent of people 20 – 24 years old in 2010?

11. Based on the counts and percent derived in questions (9.) and (10.), what happened to the people and the country of Japan who were 0 – 4 years old at the start of 1990 that changed the counts and the percent from 1990 to 2015?

12. In what way do you think the median ages changed from 1990 to 2015 in Japan? Explain your answer.

13. In what way do you think the mean ages changed from 1990 to 2015 in Japan? Explain your answer

14. If you knew a person from Japan who was well aware of the history of Japan, what questions would you ask this person based on the above pyramid graphs or histograms?

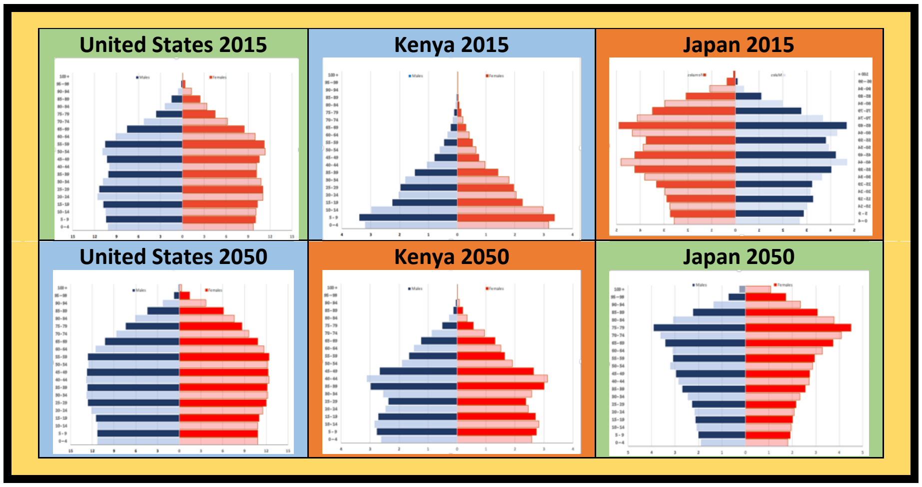
15. Complete the following summary table of the United States, Kenya, and Japan:

Country	Explain the changes of the counts of the age groups over time that were highlighted in this lesson.	Summarize the changes in the overall population count and shape of the countries during the years highlighted in the lesson.
United States		
Kenya		
Japan		

Use the population totals for 2010 and 2015 as ***anchor years*** in observing changes over time for the next unit. The totals represent estimates of the country's population in millions of people. The summaries from 2010 and 2015 provide a summary of the past and a basis for speculating on the future. If you are completing this module when census counts of the population are known for the years 2020 or beyond, consider adjusting the anchor years. Also create 2015 population estimates for the country you set-up in Lesson 5, or the My Country example. Create counts for the country set-up in 2010 that when linked to the 2015 counts can be used to estimate future counts.

	Kenya		Japan		My Country	
Age Group	**2010**	**2015**	**2010**	**2015**	**2010**	**2015**
0 – 4	6.87	6.38	5.63	5.27		
5 – 9	6.01	6.76	5.76	5.61		
10 - 14	4.55	5.95	6.16	5.75		
15 - 19	4.13	4.49	6.13	6.15		
20 - 24	3.99	4.08	6.55	6.13		
25 - 29	3.70	3.92	7.50	6.54		
30 - 34	3.00	3.6	8.30	7.47		
35 - 39	2.10	2.89	9.55	8.27		
40 - 44	1.63	2.01	8.52	9.50		
45 - 49	1.32	1.55	7.91	8.46		
50 - 54	1.04	1.25	7.69	7.82		
55 - 59	0.81	0.98	8.84	7.57		
60 - 64	0.59	0.75	9.92	8.62		
65 - 69	0.43	0.53	8.27	9.57		
70 - 74	0.31	0.36	6.89	7.82		
75 - 79	0.20	0.23	5.86	6.26		
80 - 84	0.10	0.12	4.27	4.95		
85 - 89	0.04	0.05	2.40	3.17		
90 - 94	0.01	0.01	1.03	1.45		
95 - 99	0.01	0.01	0.33	0.44		
100+	0.00	0.01	0.05	0.09		
Totals	40.84	45.93	127.56	126.91		

Unit 3
Looking Forward



Introduction

"Looking Forward" is a series of lessons to build models and rebuild models that estimate the future counts of people in a country. The models are put together by mathematical tools (common ratios, population factors, foundation factors) that are carefully developed in the lessons. In this unit, assumptions will be made, data will be selected and organized, and procedures or algorithms will be designed to generate outputs that give us a possible glimpse into a country's future. The models speculate about a country's shape for future decades based on well-defined assumptions. Ultimately these forecasts, however, will be evaluated on whether or not the predicted outcomes actually happen. In other words, we need to stick around and wait to see the results of the count of people in a country. The stories continue!

Lesson 8
Looking Forward with an Arithmetic Sequence and a Linear Model

Why would estimating the population of a country 10, 20, or 30 years from the present be important? What if the estimates, or ***population projections***, are not accurate? What if the actual count of people turns out to be considerably higher than the estimates projected by the model? What if the actual count of people turns out to be considerably lower than the model estimates?

Kristin's Story – Chapter 4

It was 2018 and a vicious strain of flu was affecting much of the country. Complications from the flu were particularly serious for infants and older people. There was a flu vaccine, but either people did not get the flu shot or it was not as effective as vaccines were in the past. Kristin's clinic was overwhelmed with sick patients. Many other people were also asking for the flu shot, although doctors were not sure it would be effective this late into the flu season.

Kristin's job involved reporting the number of flu cases the clinic treated and the number of flu shots administered during a year. A vaccine for future strains and the next flu season were already in development. Estimating how many people the clinic should be prepared to service for the next several years was her challenge. This was frankly a major responsibility as the vaccines were expensive and often were not adequately covered by the clinic's budget. It was an even more important responsibility, however, to provide adequate care for patients. If an estimate of the number of people needing the vaccine turned out to be too high, the clinic would suffer a major financial loss, and as a result, health care services in other areas would be affected. If an estimate was too low, people would be put at risk of getting the flu, or at least, a more serious case of the flu.

Kristin needed to carefully track the number of flu cases and the number of flu shots administered to make an estimate for the next several years. She also analyzed the ages of the patients serviced during the last few years and their health history. These numbers were then matched with future estimates of the count of people in the age groups targeted for the clinic's services as well as the total population of people in the region. A lot was riding on accurate recordings of the clinic's services in the past and on obtaining reliable estimates of the future counts of people needing services. She wished she had some magical ability to see into the future.

Population models are designed to generate estimates of the future count of people for situations like the one described in Kristin's story. This lesson will focus on a specific type of model, called a linear model, that estimates the count of people in the United States several years into the future. The first linear model developed in this lesson begins with the following estimates of the count of people in 2010 and 2015 reported by the United Sates Census Bureau:

United States

	2010	**2015**
Total count of people	309,348,193	320,896,618

The above estimates are considered the best estimates of the actual population of the United States at the beginning of 2010 and 2015. As explained in previous lessons, however, they are nonetheless estimates. To make the calculations more manageable, the above Census estimates are further rounded off to the nearest millions of people. The linear model begins with the 2010 and 2015 counts of the United States population:

United States

	2010	**2015**
Total count of people (in millions of people)	309	321

Lesson 8 – Problems

Arithmetic Sequence

A list of numbers is sometimes referred to as a ***sequence***. Each specific number in the list is called an element. Often there exists a specific connection of one element of the list to the next element.

1. Calculate the difference of the 2015 and 2010 population estimates.

2. Add the above difference to the 2015 population. The result is an estimate for the 2020 population assuming the population increases by the same count from 2015 to 2020 as it did from 2010 to 2015. In the same way, add the difference to the 2020 estimate to obtain the 2025 population estimate. Complete the following table by continuing the process:

   United States

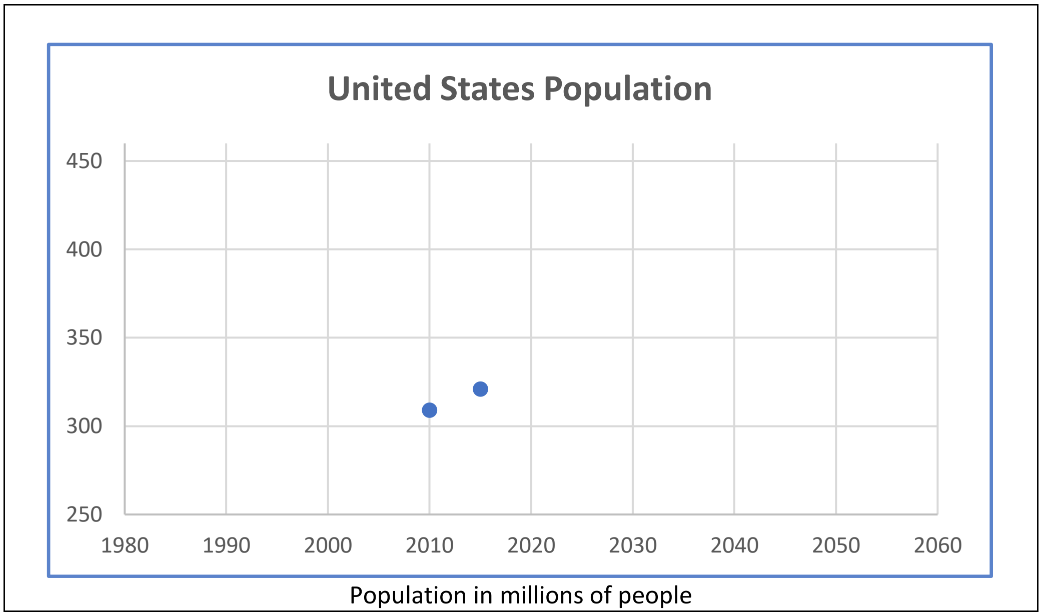
Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	309	321							

3. The above estimates form a list that is described as a ***finite arithmetic sequence***. Based on this example, how would you explain an arithmetic sequence to someone who was unfamiliar with it?

Linear Model

Consider the values from the above table as a collection of coordinate points. f

4. Consider the following coordinate grid in which the x-axis represents the year and the y-axis represents the estimated population values. Plot each of the coordinate points of the sequence in problem 3 on this grid or a grid of this type. The first two points are plotted for you.



5. Connect the coordinate points you plotted on the graph. What do you notice?

A major topic addressed in a study of algebra is deriving a linear equation from two given points. An equation derived from the plotted points forms a linear model of the population estimates of the United States population.

6. There are several procedures designed to derive an equation given two points. Consider the following steps to derive a linear equation by representing the coordinate values (x , y) for 2010 as (x_1 , y_1) and 2015 as (x_2 , y_2).
   a. Calculate the slope or change in population per year (to the nearest tenth) using the two points (2010, 309) and (2015, 321) as represented below:

      Slope = $\frac{y2-y1}{x2-x1}$

      Explain this slope as a change in the population in 1 year.

b. An equation called the "***point-slope***" equation derives the slope based on the two given points. The slope is then used with one of the two points to create a linear model. For this problem, let x represent the year and y represent the population in millions. Use the slope previously calculated from the two points (2010, 309) and (2015, 321). Use the first point (2010, 309) to complete the equation of the linear model. The steps are outlined below:

$y - y_1 = \frac{y2 - y1}{x2 - x1}(x - x_1)$ for (x_1, y_1)

Use the point (2010, 309) and the value of the slope to complete this linear model:

y - ____ = $\frac{(y2-y1)}{(x2-x1)}$ (x - ____)

c. If an estimate of the population for 2020 is derived from this equation, what is the value of x used in this model? Derive an estimate of the population for 2020 using the above equation. Round your estimate of the 2020 population to the nearest millions of people.

d. Rework the above equation into a form called the "***slope - y-intercept***" linear equation. This form is also a representation addressed in a study of algebra. This form is often summarized as **y = mx + b** where **m** is the slope previously calculated in the above problem and **b** is the y-intercept, or the value of y when x =0. (There are, however, other representations of the ***slope - y-intercept*** equation. For example, a statistical representation of this equation is often summarized as $y = b_0 + b_1x$ were b_1 represents the slope and b_0 is the y-intercept.)

7. Answer the following.
   a. Indicate a description of the ***units*** needed to complete the following statement: The slope is "__________" per "_______"
   b. What does a positive slope indicate?
   c. What does a negative slope indicate?
   d. What does a slope that is equal to 0 indicate?

The primary goal of the next several lessons is to estimate future counts of the United States, Kenya, and Japan. The linear equation is used to derive an estimate of the future; however, the equation was derived by looking back at the changes in the population distributions from 2010 to 2015. Will the changes of the past continue into the future? Is it realistic to make an assumption that these changes continue? Is the linear model a good indicator of future population distributions?

Before we answer that last question, let's examine if the linear model provides accurate estimates of past counts?

8. The United States Census estimates for 1980, 1990, 2000, and 2005 are included in the following table. Use the linear model to derive population estimates for each of these years. Record your estimates in the table.

	1980	1990	2000	2005
US Census estimates	227	250	282	296
Linear model estimates				

9. Compare the estimates derived from the linear model to the counts of people reported by the Census Bureau.
   a. Calculate the difference between the linear model estimate and the Census Bureau for 2005. Do you think the linear model provided a good estimate for 2005? Explain your answer.

   b. In the same way, calculate the differences between linear model estimates and the estimates reported by the Census Bureau for 2000? 1990? 1980? Do you think the linear model provided a good estimate for those years? Explain your answer.

   c. Derive the estimates for the years 2010 and 2015 using the linear model. Are the estimates different from the previous estimates? Explain your answer.

10. The Census Bureau estimates that the population of the United States at the time of the signing of the Declaration of Independence in 1776 was approximately 2.5 millions of people. This estimate is not based on any census as the United States Constitution was not completed at that time. What is the linear model's estimate of the population in 1776? Do you think the linear model provided a good estimate for 1776? Explain your answer.

11. Estimate the following future counts of the United States using the linear model:
    a. 2020

    b. 2030

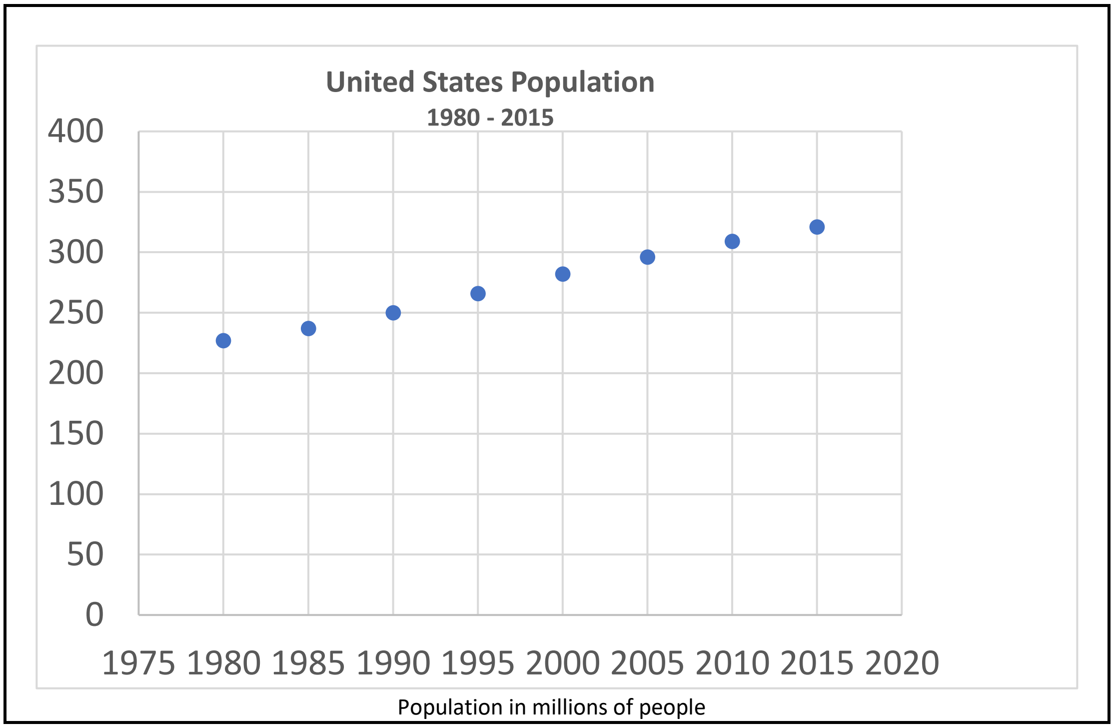
    c. 2050

    d. 2300

12. What years do you think the estimates derived in problem 11 are accurate predictions of the future? What years do you think the estimates derived in problem 11 are not accurate predictions of the future? Why would an estimate from a model like the linear model not be a good estimate?

There are several other models that could be developed to estimate the future. The remaining lessons in this model will analyze a few of them. There are also other linear models. Consider the data collected from the Census Bureau of the United States population from 1980 to 2015 to the nearest millions of people and a scatter plot based on this data. Each of these counts is considered the best estimate of the actual population for the years 1980 to 2015.

Year	**1980**	**1985**	**1990**	**1995**	**2000**	**2005**	**2010**	**2015**
Population (in millions of people) US Census Bureau	227	237	250	266	282	296	309	321



A topic you studied in statistics based on a scatter plot like the one in problem 12 also involved a linear model . Consider drawing a line drawn through the points of the scatter plot described as a ***best-fitting line***. A best-fitting line is a linear model that forms the best estimates of the actual points of the scatter plot. (Sometimes the actual point and the point on the line for a specific value of x are close, and other times they are far apart.)

13. Sketch a line that you think is a best-fitting line of the scatter plot. Identify any two points on the best-fitting line you drew and derive a linear equation based on the two points you selected using the point-slope summary. (The points you select will probably not be points represented by the scatter plot.) Determine the estimates of the United States population for the following years using the equation of the best-fitting line. Estimate each population to the nearest millions of people:

Year	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population estimates (in millions of people)							

14. Compare the estimates derived from the linear model in problem 6 to the estimates derived from the best-fitting line.

Year	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population estimates from model in problem 6 (in millions of people)							
Population estimates from best-fitting line (in millions of people)							

Do the models produce similar estimates? Explain.

Consider the 2010 and 2015 population counts for Kenya and Japan.

15. Complete the following table for Kenya by adding the difference of the 2015 population and the 2010 population to obtain the next population estimates.

Kenya

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
Population (in millions of people)	41	46							

16. Derive a linear model for the above population estimates for Kenya in the same way as the linear model was developed for the United States in problem 6.

17. Complete the following table for Japan by adding the difference of the 2015 population and the 2010 population to obtain the next population estimates.

Japan

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	128	127							

18. Derive a linear model for the above population estimates for Japan in the same way as the linear model was developed for the United States in problem 6.

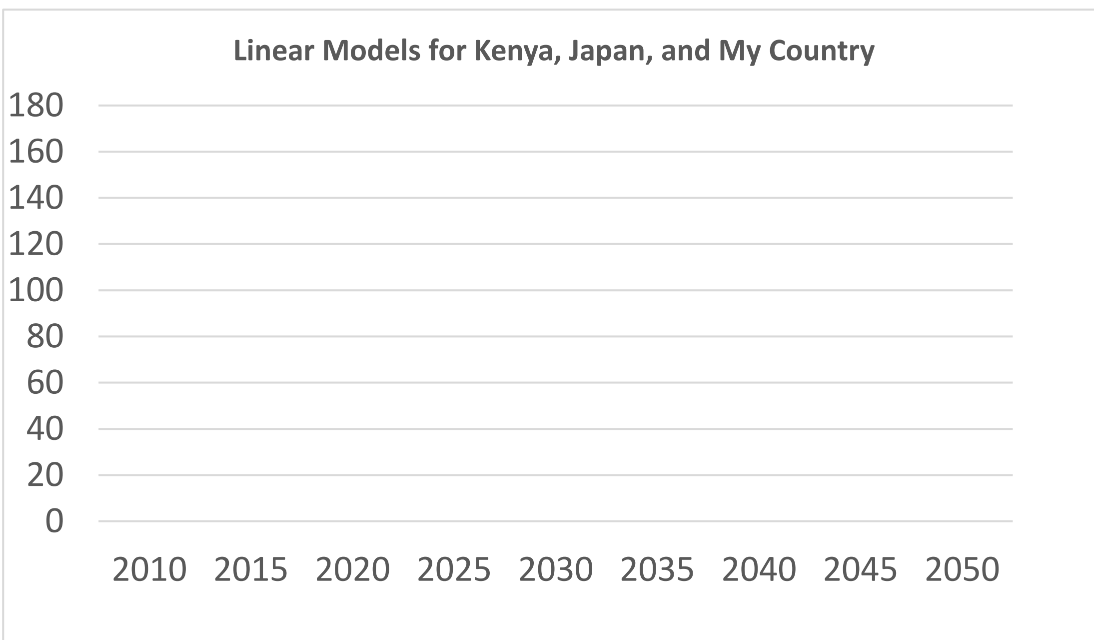
19. Finally, in the same way as you completed the above tables, complete a table for the population you created for your own country, or the My Country data.

My Country

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)									

20. Derive a linear model for the population estimates of My Country in the same way as the linear model was developed for the United States.

21. Use the following grid to record the projected population values for Kenya, Japan, and My Country based on the linear models developed in this lesson.



Population in millions of people

The linear models developed in this lesson provide estimates of the total population in the future. The models do not, however, provide estimates of the age groups that make-up the population. This challenge will be addressed in lessons that follow.

Lesson 9
Looking Forward with a Geometric Sequence and an Exponential Model

Kristin's Story – Chapter 5

Kristin recalled an interesting "word problem" posed by her middle school math teacher. She was not sure if it applied in her problem involving population projections, but she thought there might be a connection. Kristin rarely did her homework in her middle school days, and it bothered her mother. So, her teacher asked Kristin to consider making the following offer to her mother. If she did her homework today, she would receive 1 penny from her mother. If she did her homework the next day, she would receive 2 pennies. And, if she did her homework a 3rd day, she would receive 4 pennies. Could she persuade her mother that for every day she did homework, she would continue to receive double the number of pennies for at least 1 month? If her mother agreed to this arrangement, would Kristin also agree to do her homework every day?

Kristin was unable to convince her mother to agree to this plan as her mother did her own homework regarding this problem. But Kristin distinctly remembers that initially she was not sure it was a good plan. Her initial calculation indicated that after a full week of agreeing to this plan, she would receive $1.27. Hardly a major incentive to do homework for 7 days in a row. She played around with the numbers some more, and it started to strike her as more interesting. If she could stick it out for 14 days, the amount she would receive on the 14th day was $81.92, and the total amount paid to her for the 14 days was $163.83. It started to look worth the effort, but 14 days of homework was also a challenge. She worked with her math teacher and realized that what was happening was certainly worth her effort, but it also explained why her mother would not agree to this plan. They calculated that on the 28th day she would receive $1,342,177.28. The total amount received for the 28 days would be $2,684,354.55. Wow, how did that happen?

Kristin smiled over that memory. Would this same thinking have anything to do with population predictions?

Another model will be developed in this lesson based on a **geometric sequence**. Again, start off with the following estimates of the United States population from the US Census Bureau:

United States	2010	2015
Total count of people	309,348,193	320,896,618

Similar to your previous work with an arithmetic sequence, round off the above population estimates to the nearest millions of people, or:

United States

	2010	**2015**
Total count of people (in millions of people)	309	321

Lesson 9 - Problems

Geometric Sequence

A sequence in which an element is derived by multiplying a ***common ratio*** or constant factor to the prior element of the list is called a ***geometric sequence***. In the following problems, a proportion or common ratio is multiplied to each successive element resulting in a constant percent change in the population for each 5-year interval.

1. Consider the proportion $\frac{321}{309}$ or approximately 1.039 as the decimal representation to the nearest thousandth. What is the result of multiplying this proportion and 309 to the nearest million?

2. For the next estimate of the sequence, multiply 321 million by 1.039. Record your result to the nearest million in the population projection for 2020.

3. Continue multiplying each estimate by $\frac{321}{309}$ or 1.039 to obtain the next estimate until you reach the estimate for the year 2050. Complete the following table (round off your calculations to the nearest millions of people):

United States

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	309	321							
		x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	

4. The counts in the table form a *geometric sequence*. How would you describe the difference of an arithmetic sequence and a geometric sequence to someone unfamiliar with these sequences?

5. Consider a country in which the 2010 population was 309 million people and the 2015 population was also 309 million people. If a geometric sequence is generated to estimate the population in the future, what is the value of the common ratio or constant factor?

6. Consider a country in which the 2010 population was 200 million people and the 2015 population was 100 million people. If a geometric sequence is also generated for this country, what is the value of the common ratio or constant factor? How would you summarize the change in the population for each 5-year period?

7. By multiplying a population estimate by 1.039, what do you know about the next 5-year population estimate? For example, does the population grow, decline or stay the same? Derive the percent change in the population over each 5-year period as a percent of the population.

Exponential Model

The geometric sequence can be used to derive an exponential model to estimate the population for any given year. The exponential model is to the geometric sequence as the linear model was to the arithmetic sequence. The derivation of the exponential model, however, is more difficult to set-up. Consider the following exponential model derived from the above geometric sequence with modifications due to rounding off of key values:

$$y = 309(1.0078)^{x - 2010}$$

8. If the above exponential model is used to estimate the population for a given year, answer the following:
   a. Let x = 2010. What is the value of y?

   b. Let x = 2015. What is the value of y?

   c. What does x represent in the exponential model?

   d. What does 309 represent in the exponential model?

   e. What does y represent in the exponential model?

   f. What does 1.0078 represent?

9. Derive an estimate of the population for the start of the 2022 using the exponential model. Also derive an estimate of the population for the start of the year 2008. Would you have been able to derive these estimates using the values in the geometric sequence? Explain your answer.

10. The constant factor for the 5-year estimates of the geometric sequence was 1.039. The exponential model involves a constant factor of 1.0078. How do you think the constant factor of 1.0078 was derived?

11. Does the exponential model and the geometric sequence derive the same estimates for each 5-year interval? Complete the following table by calculating y from the exponential model. Let y represent an estimate of the population to the nearest millions of people. Recall that 309 million people at the start of 2010 was rounded off to the nearest millions of people from the Census data.

x	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
y	309								

12. Compare the estimates from the exponential model to the estimates from the geometric sequence. Record population estimates from the model to the nearest million. Complete the following table:

Year	**Estimate of population from exponential model**	**Estimate of population based on the geometric sequence:**
2020		
2030		
2040		
2050		

13. The exponential model was derived from the first two values of the sequence. Why are the values in the table for problem 12 not exactly the same? Do you think the estimates are close? Explain your answer.

The International Data Base (IDB) of the Census Bureau indicates that the population for the United States in 1980 was 227,224,681 people, and the population for the United States in 2000 was 282,162,411 people. These counts were based on the census conducted in 1980 and the census conducted in 2000. They represent the best estimates of the actual counts of the United States population. For the following problems, round off each population to the nearest million, or the population in 1980 was 227 millions of people and the population in 2000 was 282 millions of people.

14. Compare the estimates derived from the exponential model to the estimates reported by the Census Bureau for 1980 and 2000. Is the exponential model close to the actual counts? Summarize your comparisons.

15. Do you think the estimates using the exponential model for years after 2015 will be accurate population projections for 2020 to 2050? Explain your answer.

Consider the 2010 and 2015 population counts for Kenya and Japan.

16. Complete the following geometric sequence for Kenya by multiplying each estimate by the common factor based on the 2010 and 2015 counts of people:

Kenya

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	41	46							

An exponential model to estimate the population of Kenya is:

$$y = 41(1.024)^{x - 2010}$$

17. Complete the following geometric sequence for Japan:

Japan

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	128	127							

An exponential model to estimate the population of Japan is:

$$y = 128(0.9984)^{x - 2010}$$

18. In what way is the exponential model for Japan different than the exponential models for the United States and Kenya?

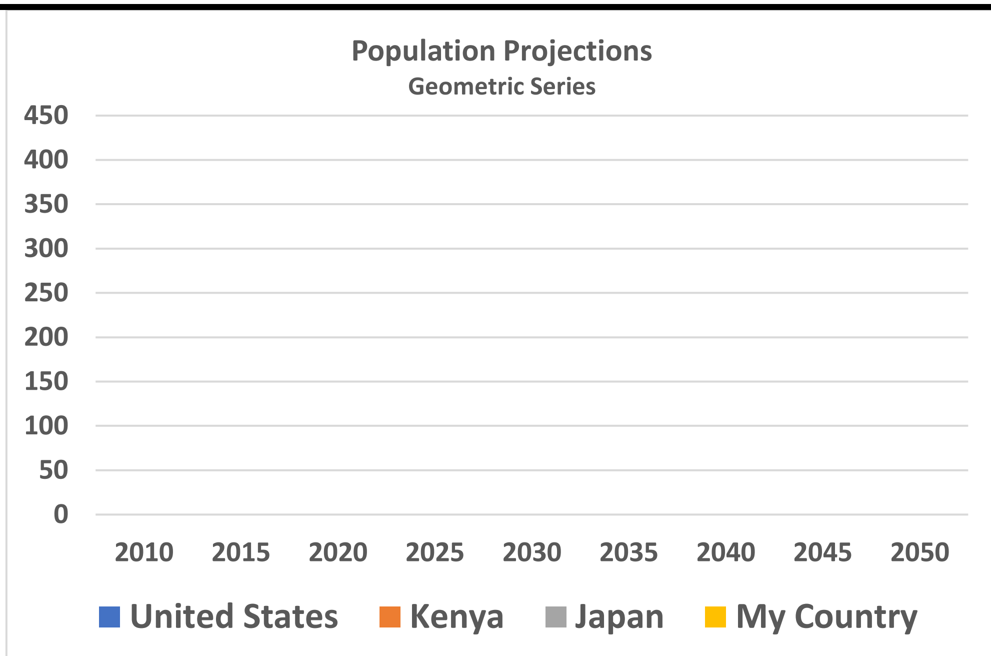
19. Finally, complete a geometric sequence for the population data you created for your own country, or the My Country data in Lesson 5.

My Country

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	163	157							

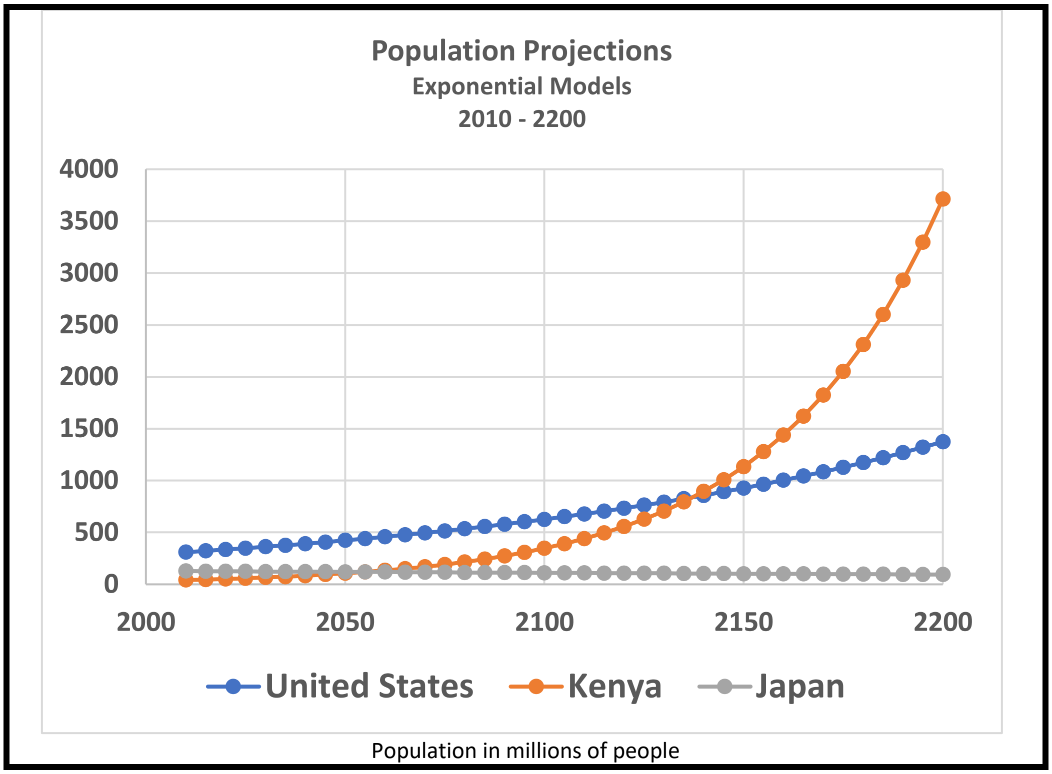
Consider deriving an exponential model for the above sequence of your country.

20. Use the values you derived for each geometric series of the United States, Kenya, Japan, and My Country data to graph their population projections.



Population in millions of people
(Use your own My Country results)

After all is said and done, how do we know if the projections from any model are on target? Some of the ways to check a model is to readjust it over time. The models derived in this lesson and the last lesson might be modified by the results of a 2020 census. Clearly the arithmetic model and the exponential model cannot be used forever. Study the following graph that summarizes the exponential models' projections for the United States, Kenya, and Japan from 2010 to 2200. What is happening to the population in the United States? Kenya? Japan?



Without some adjustments, the above models predict that a country's population in the future will be either unreasonably large or near extinction. What could alter the projections of these models? What events in the previous lessons of **Unit 2: *Looking Back*** might impact the future of the United States, Kenya, and Japan and alter these forecasts?

Recap the exponential model by recording the 2050 estimates to the nearest millions of people for each country . Compare these estimates to the United States Census estimates.

Projections of the 2050 population projects:

	Census Bureau model	Linear model	Exponential model
United States	398,328,349	405 million people	
Kenya	70,755,460	81 million people	
Japan	107,209,536	120 million people	

Similar to the linear models developed in Lesson 8, the exponential models derived in this lesson provide estimates of the ***total*** population for the United States or Kenya, or Japan. In their current form, the models do not provide estimates of the age groups that make-up a population.

Lesson 10
Looking Forward with a Recursive Model
(Present to Past … to Future)

Kristin's Story - Chapter 6

Kristin turned 31 years old in 2010. That year was a memorable year for her as she completed her educational program in health care. She became an aunt for the first time with the birth of her niece Adeline in 2012. Like many women her age, she also thought a lot about whether or not she wanted children. She remembers that at her high school reunion in 2010, several of her friends already had children. Several others were talking a lot about their future plans, including whether or not to get married, or whether or not to have children. Many of her friends were considering career and educational options, including moving to other areas of the country or world.

In 2011, Kristin met Raphine at a medical conference. Raphine is the same age as Kristin. He moved in 2011 to the United States from Kenya. He was accepted at a United States medical school to pursue his goal of becoming a doctor. His plan was to either become a United States citizen after completing his degree or move back to Kenya to practice medicine. He and Kristin planned a 3-week trip in 2015 to Kenya to see Raphine's village, however, a lack of money postponed those plans. They still hope to work out something, but they are not exactly sure when this might happen.

Lesson 10 – Problems

Handouts needed to complete the following problems:

Handout 1: *United States – 2015*

Handout 4: *United States Connected Age Groups*

1. Examine again the **Handout** 1: ***United States - 2015***. In what age group was Kristin and Raphine counted in 2015?

2. Would a summary of the United States population in 2010 include Raphine? Explain your answer.

For the following problems, use ***Handout 4: United States Connected Age Groups.***

3. At the start of 2010, there were 20,189,589 people who were 0 to 4 years old. At the start of 2015, there were 20,481,130 people who were 5 to 9 years old. What is the connection of these two age groups?

4. At the start of 2010, there were 21,983,206 people 15 to 19 years old. In what age group will these people be counted at the start of 2015? How many people were counted in that age group?

5. Adeline was born in 2012. In what age group was Adeline counted in 2010? Explain your answer.

6. In what age group would Adeline be counted in 2015?

Examine the age groups that are described as connected age groups in ***Handout 4***. This handout indicates a connection of the age group 0 – 4 in 2010 to the age group 5 – 9 in 2015. For the recursive model developed in the remaining lessons, these age groups are called ***connected age groups***.

Observe that people like Adeline (or people born after the start of 2010) are counted in the 0 – 4 age group of 2015. Although born prior to 2015, the people counted in the 0 – 4 age group in 2015 have no connected age group to the 2010 age groups. This age group is very important in this model and will be addressed in Lesson 12.

For this recursive model, a ratio is formed of connected age groups. Column 3 represents the ratio of connected groups as a fraction. Column 4 derives a decimal from the fraction and identifies this decimal as a **population factor** for the connected age groups.

7. What is the ratio of the count of people 5 - 9 years old at the start of 2015 to the count of people 0 to 4 years old at the start of 2010?

8. The ratio is represented by a decimal in column 4, or 1.014 to the nearest thousandth. As stated in problem 7, this decimal is defined as the **population factor** for the connected age groups. This population factor is greater than 1 for this example. What does that tell you about the connected age groups?

9. During the 5 years summarized on the table, what is the approximate percent increase of people 5 - 9 years old in 2015 based on the count of people who were 0 – 4 years old in 2010? Is the percent increase also part of the population factor for these connected age groups? Explain.

10. What is the explanation for the growth in the connected age groups with a population factor greater than 1?

11. During the 5 years summarized on the table, what is the approximate percent of change of the count of people 55 – 59 years old in 2015 who were 50 – 54 years old in 2010?

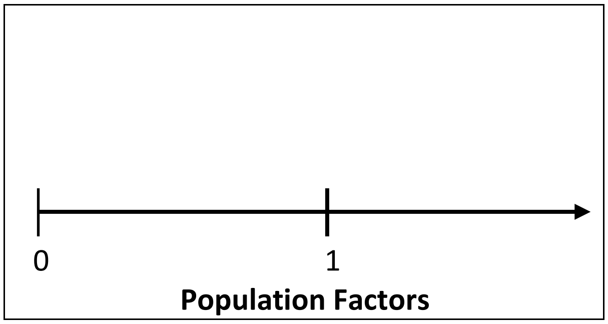
12. What is the explanation of the changes in the connected age groups with a population factor less than 1?

Complete the calculations missing in ***Handout 4***. After you have completed the handout, answer the following questions:

13. What is noticeable about the population factor for the connected age groups of 40 – 44 years old in 2010 to 45 - 49 years old in 2015 when compared to the population factors for younger connected age groups?

14. What happens to the population factors for connected age groups that count people 40 years old or older?

Any change in the count of people counted after the start of the year are a result of people moving into this country (immigrating), people leaving this country (emigrating), and dying. The following problems examine the collection of population factors for the United States.



15. Place a dot for each of the population factors derived on **Handout 4** on the above Population Factors number line. (Stack dots if they are close to each other.)

16. Why is 1 considered an important value in interpreting a population factor? Explain your answer.

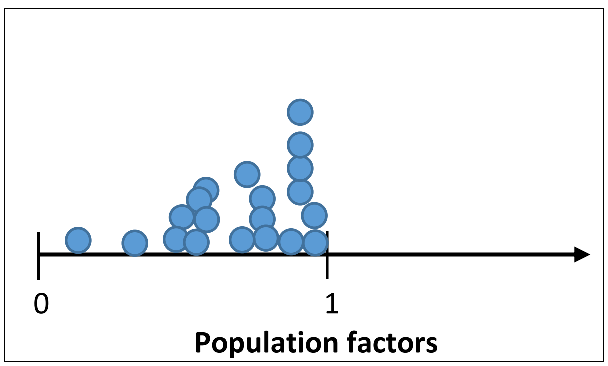
17. Is it possible for the population factor of connecting age groups be equal to 0? Explain your answer.

18. Changes in the count of people over a 5-year period are explained by **birth**, **death**, **immigration**, and **emigration** (people leaving a country). Use the value of the population factor to Identify what changed the counts in the following Connected Age groups from 2010 to 2015. Identify the most dominant explanation for the changes in the connected age groups in the last column. (The first connecting age group is completed for you.)

Connected Age groups from 2010 to 2015	Population Factor for the connected age groups	What could explain the changes in the count of people in the connected age groups?	What was the dominant explanation of the change in the connected age groups?
0 – 4 to 5 – 9	1.014	Immigration, emigration, deaths	Immigration
20 – 24 to 25 – 29			
40 – 44 to 45 – 49			
85 – 89 to 90 – 94			
95 to 99 to 100+			

19. Summarize what a population factor indicates about the connected age groups.

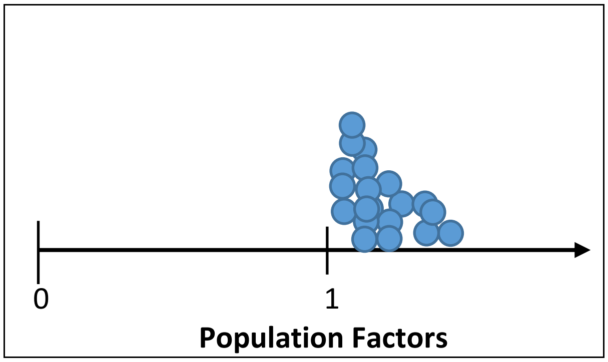
20. Consider the following dot plot of the population factors of a fictitious country:



a. What is the dominant explanation of change in the connecting age groups for a country represented by the above dot plot?

b. Do you think it is possible for a country with the above population factors to have an increase in its total population during a 5-year period? Explain.

21. Consider the following dot plot of the population factors for another fictitious country:



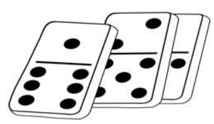
a. What is the dominant explanation of change in the connecting age groups for the above dot plot?

b. Do you think it is possible for a country with the above population factors to have a decrease in its total population during a 5-year period? Explain.

How can the population factors be used to estimate future counts, or population projections? Applying the above population factors to the actual count of people in 2015 will start the recursive model.

Lesson 11
The Recursive Model and Falling Dominos

This lesson derives future counts of connected age groups based on changes from 2010 to 2015. Over the course of 10 years, 20 years, and 30 years, people in each of the connected age groups grow older, leave the country, come into the country, or die. How do these changes affect the overall count and shape of the country in the future? This lesson is similar to what happens when dominos are lined up. When the first domino is knocked down, the next domino (or, the future) falls, followed by the next domino, until the last domino is knocked down. In this lesson, when one age group changes its count in 5 years, it will affect the count of a connected age group in the next 5 years, and that count will affect a connected age group 5 years after it changed. As indicated, the overall effect is similar to what happens with falling dominos.



Lesson 11 – Problems

Handout needed to complete the following problems:

Handout 5: *Looking Forward for the United States (Student Edition)*

A decision regarding the accuracy of the calculations is needed before the population factors are used to estimate the future counts of people. Recall that a ***Population Factor*** is the decimal based on the ratio derived between connected age groups. Similar to Lesson 10, this ratio will be represented to the nearest thousandth. A mathematics equation using a specific population factor is summarized below.

(*Population of age group 0 – 4 years old 2010*) X *(Population Factor for connected age groups)* **=>**

(*Population of age group 0 – 4 years old 2010*) X $\frac{Population\ of\ age\ group\ 5-9\ for\ 2015}{Population\ of\ age\ group\ 0-4\ for\ 2010}$ **=** *Population of age group 5 – 9 for 2015*

↑

(Population Factor for connected age groups 0 – 4 to 5 – 9 years old)

1. Complete the multiplication indicated in the following table. The first calculation has been completed for the 0 – 4 age group of 2010 to the 5 – 9 age group of 2015 as set up in the above equation. The population factors listed were derived in Lesson 10.

Population of Age group in **2010**	X	Population Factor	=	Population of Connected Age Group in **2015**	Census Bureau counts for **2015**
0 – 4 20,189,589	X	1.014	=	**5 – 9** 20,472,243	**5 – 9** 20,481,130
5 – 9 20,331,807	X	1.013	=	**10 – 14**	**10 – 14** 20,605,579
10 - 14 20,681,214	X	1.020	=	**15 – 19**	**15 – 19** 21,084,710

2. Why are the answers derived in problem 1 not equal to the estimates of the Census Bureau?

To make the calculations more manageable, population estimates for 2010 and 2015, along with the population projections for 2020 and beyond, will be represented as millions of people to the nearest hundredth. For example, 20,189,589 will be represented as 20.19 millions of people.

If the population factors derived in **Handout 4** remain constant during the five years from 2015 to 2020, what changes will result for connected age groups? What will be the shape of the United States in 2020? 2025? … 2050? What if the population factors were not constant? The following problems begin answering these questions.

3. Examine the connected age groups of 0 – 4 years old in 2010 to 5 – 9 years old in 2015. A population factor of 1.014 indicates an approximate increase of 1.4% in the 5 – 9 years old age group in 2015 when compared to the 0 – 4 years old age group in 2010.

    a. Using the above summary of the changes in these age groups, is it possible to derive the number of people who moved into the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

b. Is it possible to derive the number of people who moved out of the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

c. Is it possible to derive the number of people who died from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

d. What was the dominate explanation for the change in the number of people from 2010 to 2015 of the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain your answer.

Projection models are based on assumptions to estimate the count of people in the future. At some future time, however, the actual outcomes are then compared to the predicted outcomes to evaluate the projection models. The recursive model designed in this lesson starts with the assumption that the ***population factors remain constant over time***. In other words, the population factors derived in **Handout 4** of the previous lesson are assumed to remain constant from 2020 to 2050.

As previously stated, the population factor 1.014 for the age groups 0 – 4 in 2010 to 5 – 9 in 2015 indicates a 1.4% growth in the count of people. Assume that same rate is used to estimate the count of people 5 – 9 years old in 2020 based on the count of people 0 – 4 years old in 2015. The population factor 1.014 will also be used to derive the count of the 5 – 9 years old age group in 2025 once an estimate of the count of 0 – 4 years old age group is derived for 2020. This population factor of 1.014 is continued to estimate counts of the connected age groups until 2050. In the same way, the other population factors are used to estimate future counts.

The following problem sets up this process for projecting the count of people for each age group in 2020. The model assumes that the population factors derived from the connected age groups of 2010 to 2015 remain constant for the next 5 years.

4. Population factors listed in the following table were recalculated based on the approximations of millions of people in 2010 and 2015. The 2015 population estimates were also rounded off as indicated. Complete the following table. Several age group projections have been completed. (Note: Some of the posted estimates may differ from your calculations due to the software used to implement the recursive model. The differences are minor. Allow a difference of +/- 0.01 of your answers and the answers posted.)

Age group	**Population 2015** (in millions of people to the nearest hundredth)	**Population Factor**	**Population 2020** (in millions of people to the nearest hundredth)
0 – 4	19.91	1.014	
5 - 9	20.48	1.014	20.20
10 - 14	20.61	1.020	20.76
15 - 19	21.09	1.032	21.02
20 - 24	22.69	1.032	21.77
25 - 29	22.40	1.022	
30 - 34	21.62	1.012	22.90
35 - 39	20.31	1.004	
40 - 44	20.16	0.995	
45 - 49	20.80	0.985	20.05
50 - 54	22.29	0.974	20.48
55 - 59	21.77	0.962	
60 - 64	19.04	0.945	20.93
65 - 69	16.05	0.917	
70 - 74	11.48	0.869	
75 - 79	8.12	0.792	
80 - 84	5.80	0.670	
85 - 89	3.86	0.508	3.89
90 - 94	1.85	0.340	
95 – 99	0.50	0.205	
100 +	0.08		

5. Recall that Kristin was 36 years old at the start of 2015. In what age group would she be counted in 2020?

6. One **age group** for 2020 (the shaded cell) is left blank by this process. What age group is left blank? Explain why this age group does not have a population factor for this recursive model. (This particular age group is called the **Foundation Layer** and will be the focus of the next lesson.)

7. One **population factor** also has no entry in the above table. Why is there no population factor entered for this age group?

The Falling Dominos

The recursive process is now ready to begin. The projections for 2025 will be based on the above projections from 2020. In turn, these results will be used to calculate projections for 2030. The process will continue for 2035, 2040, 2045, and 2050. Study **Handout 5: Looking Forward for the United States**. The population factors are provided, along with the estimated population of the United States in 2010 and 2015.

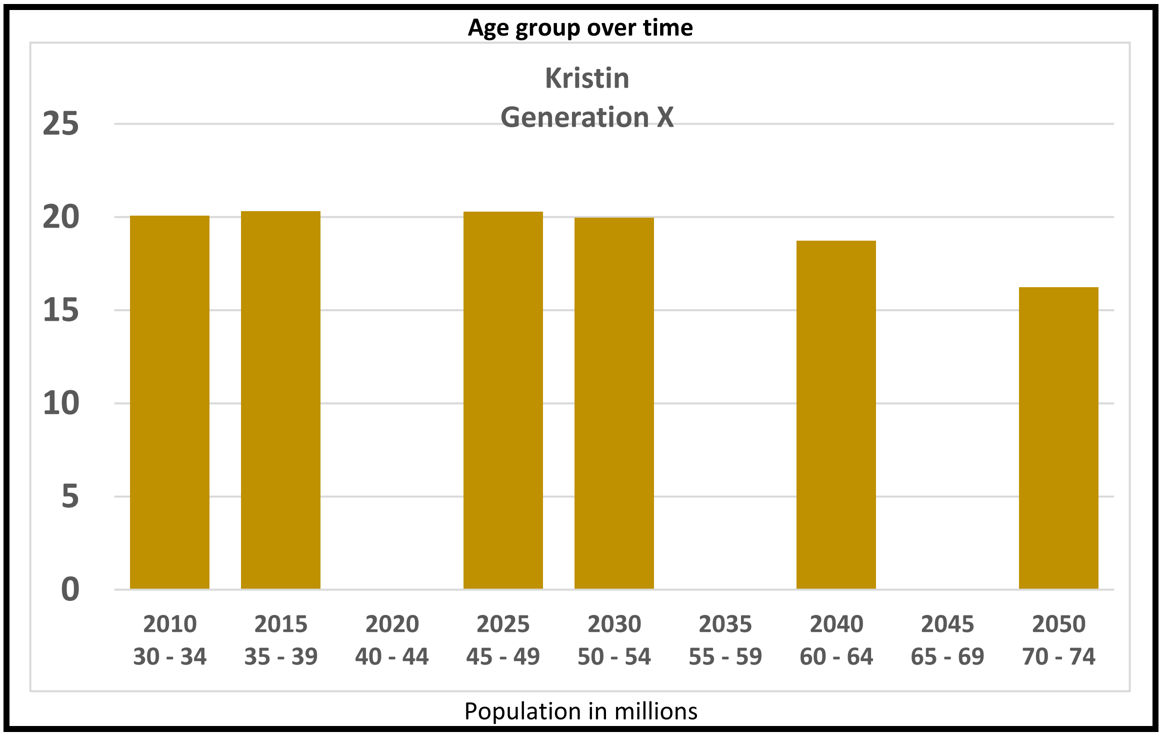
8. Record the missing 2020 projections in the column identified as 2020 in **Handout 5**. Be careful not to provide a projection for the age group 0 - 4 years old in 2020.

9. Use the Population Factors to complete the projections for 2025 from the 2020 projections based on the previous 5-year projections. Several calculations have been completed in the handout. (Remember, round your answers to the nearest hundredth. Each value indicates the count of people in millions.)

10. In 2020, there was one age group that was left blank (the 0 – 4 years old age group). In 2025, there are two age groups that are blank. Why is this second age group (the 5 – 9 years old age group) blank?

11. Complete the missing calculations in **Handout 5** for the years 2030, 2035, 2040, 2045, and 2050. Again, several calculations have been completed. Use the completed calculations to check your work.

12. Use your calculations from **Handout 5** to complete the following bar graph of Kristin's age groups. Recall that Kristin was 36 years old in 2015.



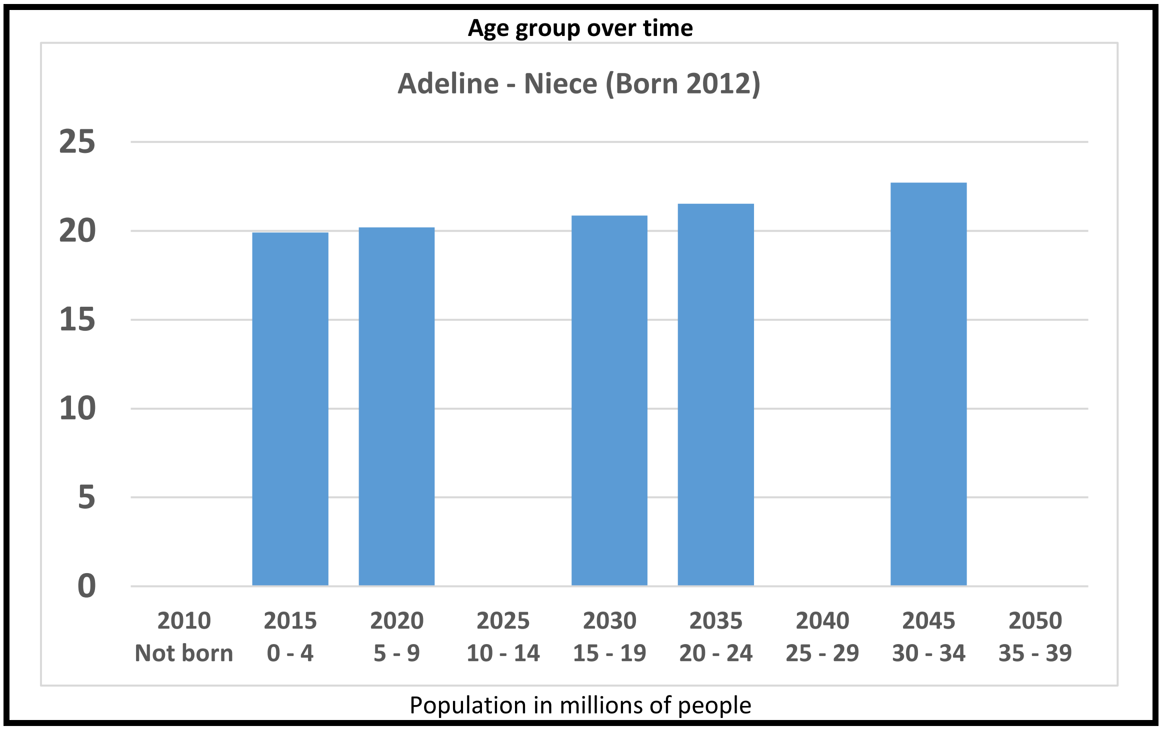
13. Recall that Kristin's niece Adeline was born in 2012. In what age group is she counted at the start of:
    a. 2015?

    b. 2020?

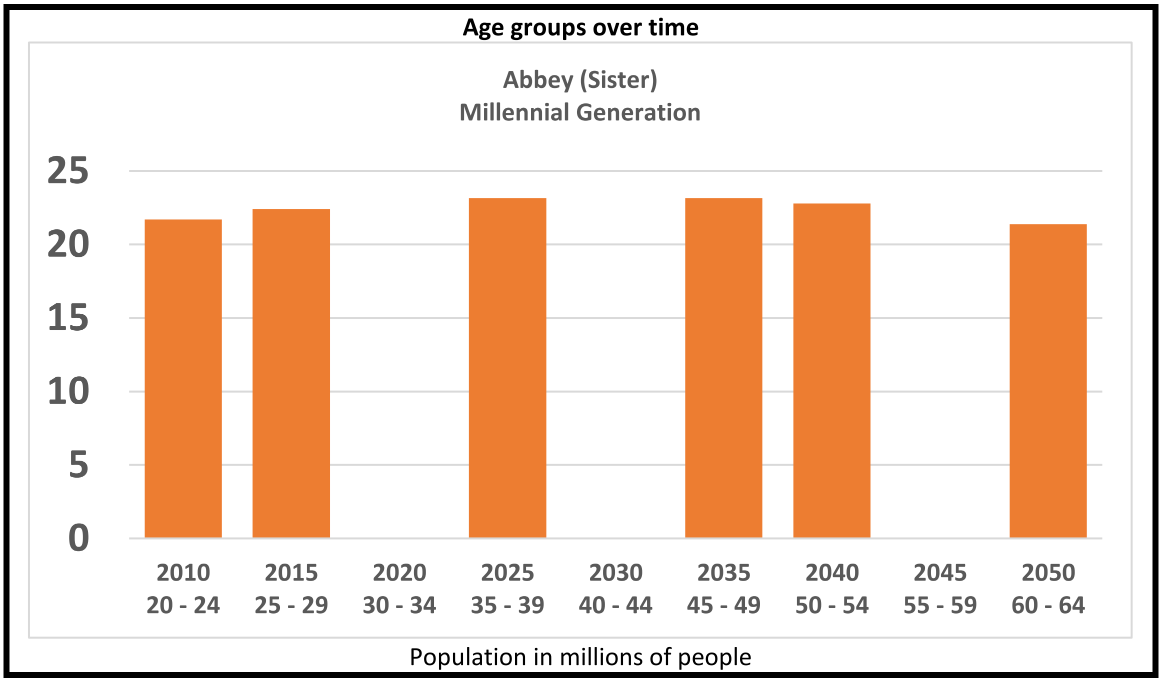
    c. 2035?

    d. 2050?

14. Using **Handout 5,** complete the following bar graph representing Adeline's age groups from 2015 to 2050:

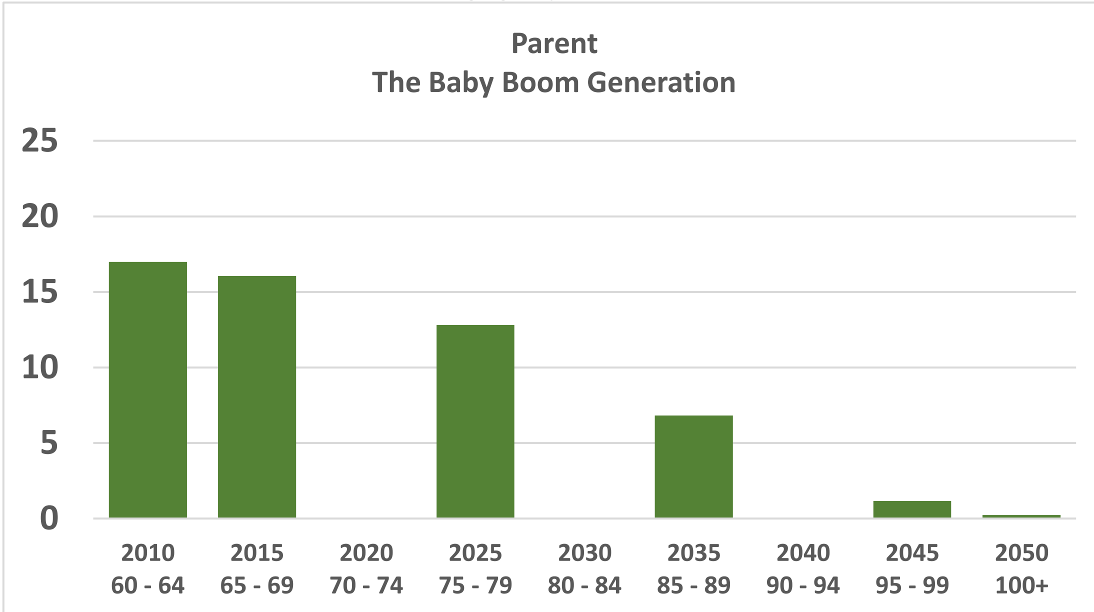


15. Using **Handout 5,** complete the following bar graph representing Kristin's sister Abbey age groups from 2010 to 2050. Recall that Abbey was 26 years old in 2015.



16. Using **Handout 5**, complete the following bar graph representing Kristin's mother's age groups from 2010 to 2015. Recall that Kristin's mother was 66 years old in 2015.

Age group over time



Population in millions of people

17. Examine the completed graph for Adeline. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?

18. Examine the completed graph for Kristin's parent. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?

Extension or Assessment: Explore the following options as directed by your teacher using the Excel file ***"The 1 Country".xls.*** After loading the file, carefully look at the set-up of the country. This file summarizes the population of an imaginary country in which all of the counts in each age group is 1. It also indicates that the population factors for each connecting age groups are 1. The cells with a 0 value will be discussed in the next lesson.

Option 1:

Before you change the values in a specific cell, describe the total population of this imaginary country from 2010 to 2050.

What if it was determined that a person in this country who was 36 years old at the start of 2015 was not counted? Enter the value of 2 in the appropriate cell of the spreadsheet for "1 Country " to add this person to the 2015 census.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

Option 2:

What if it was determined that a person in this country who was 36 years old at the start of 2010 was not counted? Enter a 2 in the appropriate cell of the spreadsheet for "1 Country" to add this person to the 2010 census.

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

Option 3:

What if a person who was 36 years old moved into the country and was counted in the country at the start of 2020? Enter a 2 in the appropriate cell of the spreadsheet for "1 Country".

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

Answer the following question:

- What is the connection of the calculations in this lesson to falling dominos?

Lesson 12
Completing the Recursive Model – The Foundation Layer

Kristin's Story - Chapter 7

It was March 2018. Adeline, Kristin's niece, came home from kindergarten noticeably excited. She shared with her Mom a special notice of a program at her school called "Science Night!"

"I would like to invite Aunt Kristin. She will get to see my classroom, my artwork, and the rocket we are making in our science class."

The Science Night was a big success. Adeline held Kristin's hand as she showed her the many posters and pictures she and her classmates made in their art class. Kristin had a little problem figuring out what each picture was supposed to be, but with a little help from Adeline, she was able to make sense of most of the characters and stories drawn on the posters. The launch of the class rocket was especially exciting. All of the families and guests went outside and watched as a teacher ignited a rocket that Adeline and her classmates helped assemble. With a loud "5..4..3..2..1" by all the students, the rocket lifted off and made a spectacular path upward and then drifted back down to the ground with a parachute.

Kristin was introduced to Adeline's friends. She met Mathew, Jason, Dominic, Paul, Melissa, Natalie, and several others. All of them were showing off their paintings and letter books. Adeline mentioned to Kristin that both Mathew and Dominic were having their 6-year old birthday party later that week, and she was invited. Mathew had to show Kristin his younger sister who was born in August of 2017 and Dominic talked about his younger brother who was born a few months ago.

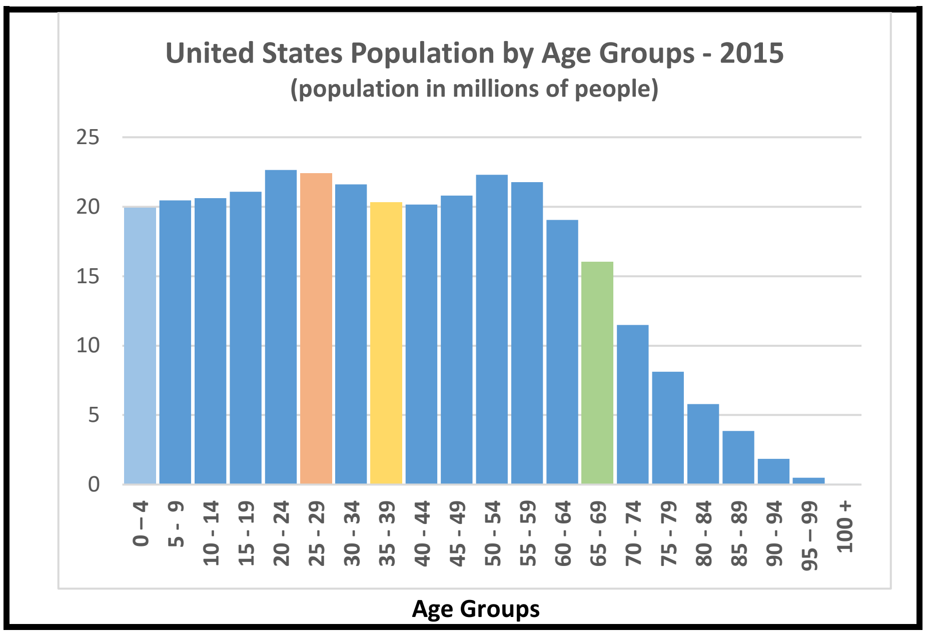
After the Science Night event, Kristin studied again the 2015 histogram of the United States population. She noted that the age group representing Adeline and her friends had approximately 20 million people. She wondered how many of those kids were now in school, and how many teachers and other people involved in education were employed to work with these students. Then she thought of Mathew's younger sister and Dominic's younger brother. She realized as she looked at the graph, they were NOT counted on this graph. They were part of a group of people – new people – who would be counted on the population graphs prepared for 2020. Kristin wondered what that number will be, and if the number of kids in that group will be similar to the number of kids in Adeline's age group.

Lesson 12 – Problems

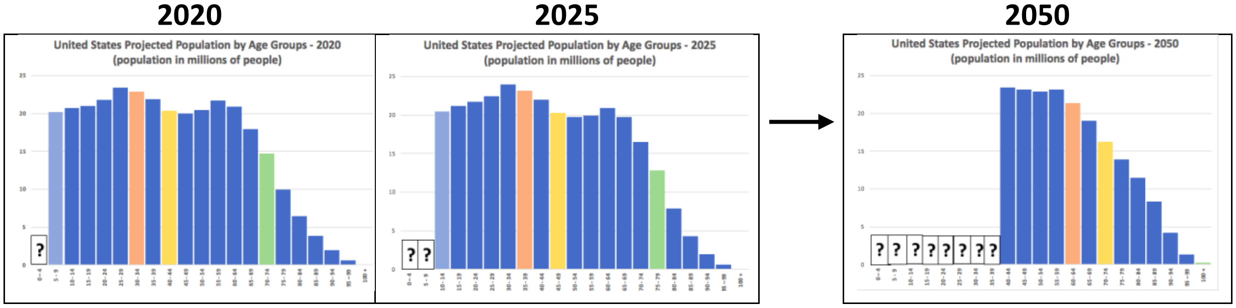
Handout needed to complete the following problems: Handout 6: *United States 2010 - 2050*

Look again at the 2015 histogram and the shape of the United States at the start of 2015 and the estimates derived from the population factors for 2020, 2025, and 2050 using the incomplete recursive model. Certain layers are color coded as we follow the characters in our stories of Adeline, Abbey, Kristin, and Kristin's mother.

Adeline			Kristin	
Abbey			Kristin's mother	



2020 2025 2050



1. Adeline was born in 2012. What is her age at the start of 2015?

2. In what age group would Adeline be counted in the 2015 histogram?

3. Estimate the number of young people counted in that age group.

4. Based on Kristin's story, what do you know about Mathew's young sister and Dominic's younger brother at the start of 2015?

5. Most population graphs prepared by the United States Census Bureau use the 5-year age groups represented in the above histogram. There are people not counted in the 2015 histograms who are part of the population from 2015 to 2019. Describe these people.

6. Assume a histogram of the actual count of people was prepared in 2015 by the United States Census Bureau. If a histogram is prepared every 5 years by the Census Bureau of the actual count, when will the people described in problem 5 be represented on a United States population graph?

There were several blanks cells in **Handout 5** that could not be completed as there was no estimate of the projected number of people born in the 0 – 4 age group after 2015. **Handout 6** begins to fill in these blank cells. This ***new*** group of people, or the ***Foundation Layer***, is the first domino that impacts a country's shape looking forward. A country's shape can drastically change from events that impact the count of people born in a 5-year period.

7. If the actual count of people in the 0 – 4 age group in 2020 turns out to be larger than what people expected, what might explain the larger number of people?

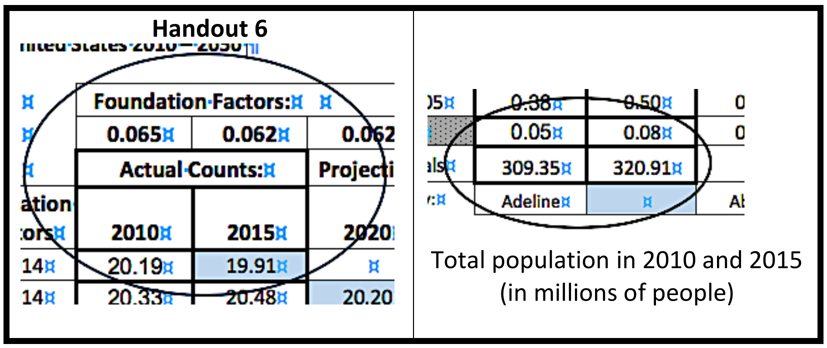
8. In a similar way, if the actual count of people in the 0 – 4 age group in 2020 is less than what people expected, what might explain the smaller number of people?

This lesson begins by deriving an estimate of the number of people in the 0 – 4 years old age group for 2020. This estimate is then used to estimate the number of 5 – 9 years old in 2025. And, just like the last lesson, the dominos begin to fall completing the estimates for all of the blank cells in **Handout** 6. This process will complete the recursive model and set-up options for several "What if …?" problems.

To begin these final steps, consider the following ratio:

$$\frac{Number\ of\ people\ 0\text{–}4\ at\ the\ start\ of\ 2015}{Total\ Number\ of\ people\ at\ the\ start\ of\ 2015} =$$

The decimal value of this fraction estimates the proportion of the population who were 0 – 4 years old at the start of 2015.



9. **Handout 6** indicates that the total number of people in the United States in 2010 was 309.35 million people. It also indicates there were 20.19 million people who were estimated to be 0 – 4 years old. What is the proportion of the population in 2010 who were counted in the 0 – 4 age group? Express this proportion to the nearest thousandth.

10. Based on the above proportion, what is the percent of the people in the United States who were counted in the 0 – 4 age group at the start of 2010?

11. In the same way, use **Handout 6** to derive the proportion of the United States population who were counted in the 0 – 4 age group at the start of 2015.

12. What is the percent of the United States population who were counted in the 0 – 4 age group at the start of 2015?

13. What is the difference in the count of people who were 0 – 4 years old in 2010 to the count of people who were 0 – 4 years old in 2015? What might be an explanation for this difference in the count of people born in the 5 years before 2010 and the count of people born in the 5 years before 2015?

The decimal values of the proportions derived in problems 10 and 12 are called the **Foundation Factors** for this recursive model. Note that the two factors calculated in problems 10 and 12 are similar, although the 2015 factor is slightly smaller. Note where the decimal values of the foundation factors are listed in **Handout 6**. As we look forward, assume that the foundation factor for 2020 is the same as the population factor for 2020.

14. An estimate of the count of people in 2020 who were 5 years old or older to 100+ years was 311.21 million people. Explain how this estimate was determined using **Handout 6**. Is 311.21 million people the total population of the country in 2020? Explain your answer.

15. Consider the following equation:

$$\frac{x}{x+311.21} = 0.062$$

If the above equation is used to determine an estimate of the count of people 0 – 4 years old in 2020, answer the following:

a. What does 0.062 represent in this equation? What does 311.21 represent?

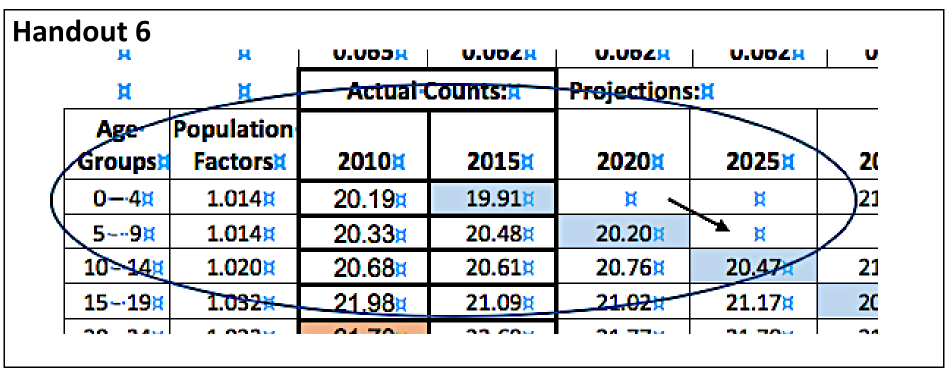
b. What does x represent in this equation?

c. Solve for x to the nearest hundredth.

d. If the count of people in the 5 – 9 years old age group to the 100+ age group is greater than 311.21 millions of people, what happens to the value of x if the population factor stays the same? What happens to the count of the total population?

16. Your solution to the above equation is an estimate of the 0 - 4 years old age group in the 2020 population. Place this value in the appropriate cell of **Handout 6**. What is your estimate of the total population of the United States in 2020? Indicate in the space below how you derived the value of the total population. Also enter this value in the appropriate cell of **Handout 6**.

17. Given your estimate of the 0 - 4 age group for 2020, estimate the number of people in the 5 – 9 age group for the 2025 population group by multiplying the count derived for the 0 – 4 age group by the Population Factor for the connecting age groups of 0 – 4 to 5 – 9 years old. Indicate in the space below how you derived this value. Also enter this value in the appropriate cell of **Handout 6**.



Handout 6

Age Groups	Population Factors	Actual Counts: 2010	2015	Projections: 2020	2025	20
0 – 4	1.014	20.19	19.91			21
5 – 9	1.014	20.33	20.48	20.20		
10 – 14	1.020	20.68	20.61	20.76	20.47	21
15 – 19	1.032	21.98	21.09	21.02	21.17	20

18. Revise the equation in problem 15 to estimate the count of people in the 0 – 4 age group for 2025 population using the same foundation factor of 0.062 for 2015 and 2020. Place your estimate in the proper cell of **Handout 6**. Why was it necessary to derive the estimate for the 5 – 9 age group before you derived the above estimate for the 0 – 4 age group?

19. Derive estimates for the remaining blank cells in **Handout 6**. (Remember your estimates can differ from some of the listed estimates by + 0.01 or - 0.01.)

20. After all blank cells have been filled, Identify the age group with the greatest number of people for each of the following:

Year	Age group with the greatest number of people	Percent of the country within age group (nearest tenth of a percent)
2010		
2015		
2020		
2025		
2030		
2035		
2040		
2045		
2050		

21. Why might a person who was 22 years old at the start of 2015 be highlighted in an online commercial over a person who was 42 years old at the start of 2015?

Lesson 13
"The More Things Change, the More Things Stay the Same"

Kristin's Story - Chapter 8

It was 2018. Kristin friend's Raphine had decided to return to Kenya and rethink his career goals. Kristin also struggled with some of her own career and financial decisions. She thought about buying a house but was nervous about the financial aspects – specifically mortgage payments, insurance, and possible repairs and remodeling expenses. She liked to think that in the future she might move to other cities or even to other countries and owning a house might make moving difficult. There was a sense of freedom she worried would be lost if she owned a home. Several of her friends were also questioning whether or not to buy a house or rent.

The population of the United States was changing. Generational labels were often tossed around, and sometimes Kristin wondered if they made any sense. The Boomers were declining, and the Millennial generation was now called the "power generation". A label of the Z Generation was starting to catch on. She was not sure what that all meant, but she did sense that her own goals were a mixture of her parents' generation and the generations younger than her.

These changes made Kristin often think a lot about her future. Sometimes it made her nervous, and sometimes hopeful. Was this the beginning of new things, or was this more of the same?

Lesson 13 – Problems

Handout needed to complete the following problems: Handout 6: *United States 2010 - 2050*

1. In what way does Raphine's decision to leave the country in 2018 change the count of people in the United States?

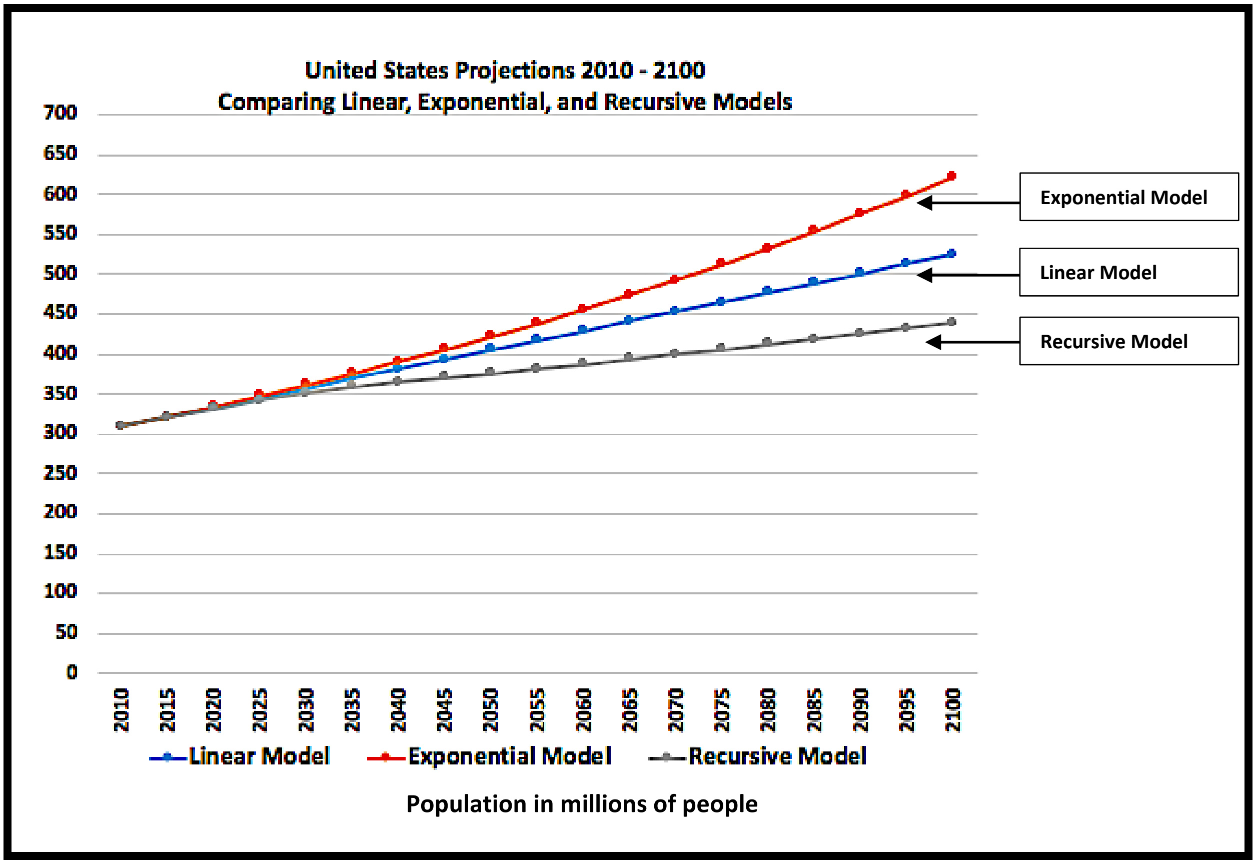
It has been previously mentioned how buying a house impacts a country's economy. Although the specifics of home ownership and its impact on a country's economy are more thoroughly explained in an economics class, buying a house generally involves taking out a loan (called a mortgage) from a banking institution. The mortgage is paid back with interest. The bank also accumulates money by encouraging people to save their money in their bank. Interest is paid to people who save their money with the bank. Saving money and then borrowing money results in economic activity that is linked to a nation's overall economy. The United States has a higher percentage of its population who buy a house than most other countries. The percentage of people who rent rather than own a home is higher in several European countries. Disruption in a country's economy often results when a particular element of the economy, such as home ownership, is impacted by change.

2. If the age groups in 2050 who are under 40 years old generally do not buy a house, do you think the effect on the country's economy would be significant? Why or why not?

3. What products or services might be considered important in 2050 based on the population distribution?

The projection models presented in this module were based on an arithmetic sequence or linear model, a geometric sequence or an exponential model, and a recursive model. The arithmetic sequence was derived by adding a constant to each previous population estimate. The geometric sequence was derived by multiplying a constant factor to each previous population estimate. The arithmetic sequence resulted in a linear model, and the geometric sequence resulted in an exponential model. The recursive model was derived by multiplying a constant list of factors (or population factors) to connected age groups and a constant foundation factor.

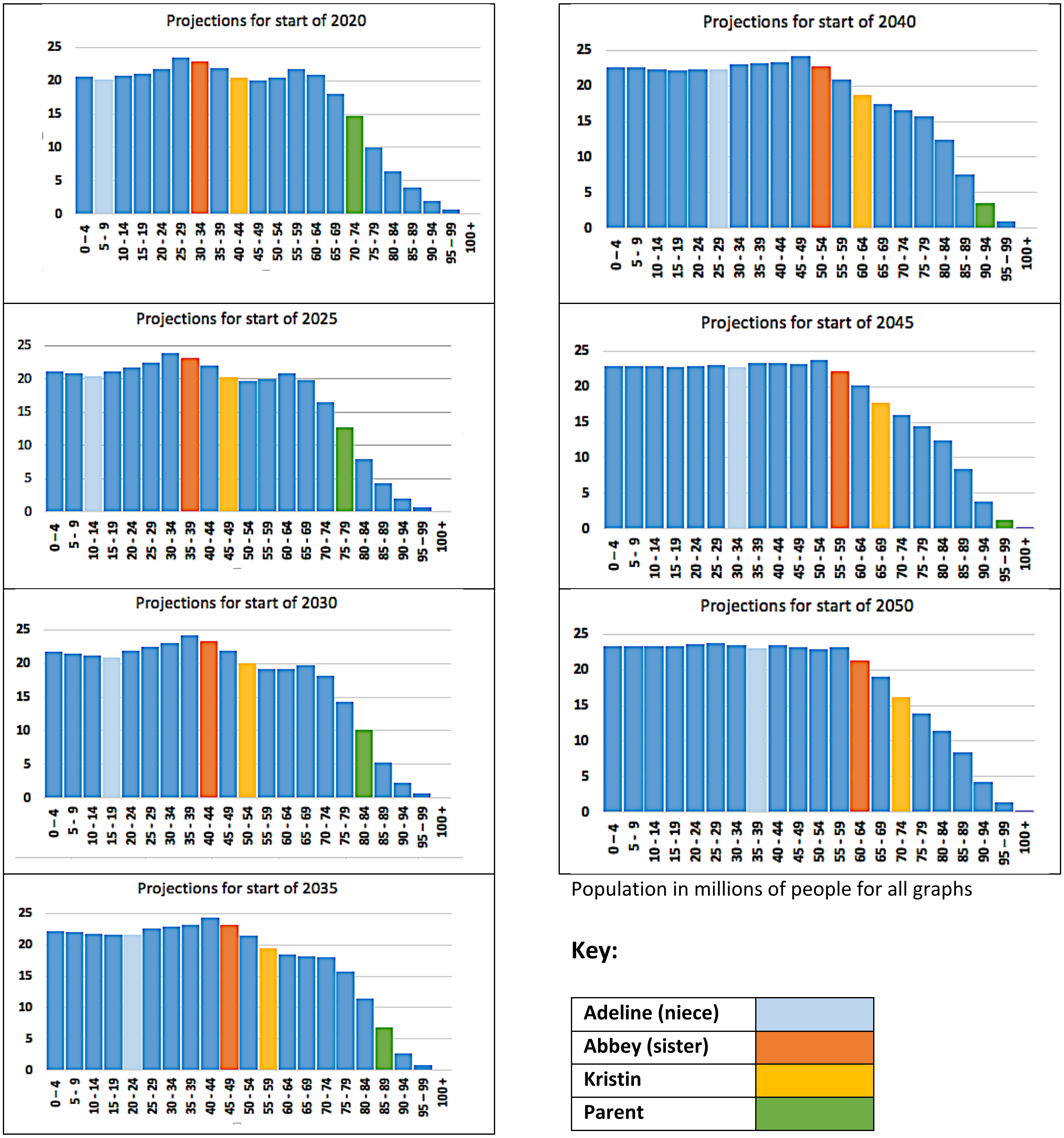
Examine the following graph that compares the estimates of the country's population from 2010 to 2100 for each of these models:



4. The population projections for 5-year intervals are plotted for each of the models from 2010 to 2100. Answer the following:
   a. Which model would result in the greatest change in the total population from 2010 to 2100? Explain your answer.

   b. Which model would result in the least change in the total population from 2010 to 2100? Explain you answer.

The recursive model, like the linear and exponential models, provides a projection of the total population of the country for each 5-year interval. In addition, the recursive model also provides estimates of changes in the shape of the country as visible in histograms by age groups. Examine the following histograms of the country's age groups from 2010 to 2050 based on the recursive model. Note the projections for the age groups that include Adeline, Abbey, Kristin, and her parents as your work through the problems.



Population in millions of people for all graphs

Key:

Adeline (niece)	
Abbey (sister)	
Kristin	
Parent	

5. Answer the following.
   a. Identify at least 2 summaries of the 2025 graph that indicate it followed the 2020 graph.

b. Identify at least 2 summaries of the 2030 graph that indicate it followed the 2025 graph.

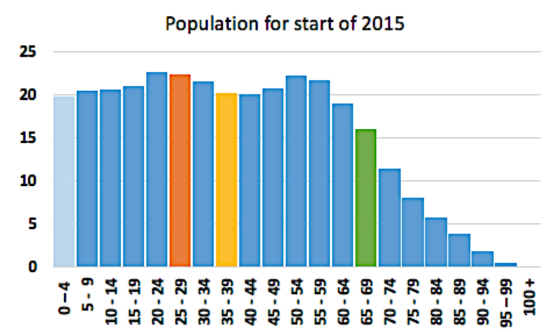
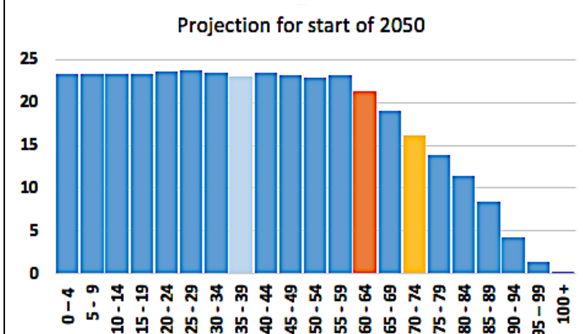
c. In what way is the population distribution as outlined in the histogram for 2035 similar to the population distribution as outlined in the histogram for 2030?

6. Kristin commented that the graphs for 2040, 2045, 2050 begin to look like a rectangle followed by a downward slope at the end.

   a. What summaries of the age groups of the 2050 graph make it look like a rectangle from age groups 0 – 4 years old to 55 – 59 years old?

   b. What do you think is the reason for the downward slope that begins at 60 – 64 years old?

7. The following graphs provide an overall comparison of the changes of the country and the age groups from the start of 2015 to the start of 2050. The counts of the age groups are based on the counts you completed for **Handout 6**. Calculate the percent of the country's population in each of the age intervals requested for 2015 and 2050:

		
Estimated count of people 0 – 24 years old: 104.78 million people Estimated count of people 25 – 49 years old: 105.29 million people Estimated count of people 50 – 74 years old: 90.63 million people Estimated count of people 75 – 100+ years old: 20.21 million people Estimated total population: 320.91 million people		Projected count of people 0 – 24 years old: 116.72 million people Projected count of people 25 – 49 years old: 116.77 million people Projected count of people 50 – 74 years old: 102.64 million people Projected count of people 75 – 100+ years old: 39.53 million people Projected total population: 375.67 million people
Country's Layers Estimated percent of the country who are 0 – 24 years old: Estimated percent of the country who are 25 – 49 years old: Estimated percent of the country who are 50 – 74 years old: Estimated percent of the country who are 75 – 100+ years old:		**Country's Layers** Projected percent of the country who are 0 – 24 years old: Projected percent of the country who are 25 – 49 years old: Projected percent of the country who are 50 – 74 years old: Projected percent of the country who are 75 – 100+ years old:

8. Based on the above percent of the layers, describe the country's shape as bottom-layered, lower middle-layered, upper middle-layered, or top-layered for:
   a. 2015

   b. 2050

9. Use **Handout** 6 to estimate the 5-year age group in which the median age would be located for 2015.

10. Use **Handout** 6 to estimate the 5-year age group in which the median age would be located for 2050.

11. Use **Handout 6** to identify the age group that has the greatest projected increase in the count of people when comparing the 2015 to the 2050 counts. What is the percent increase of the 2050 count in this age group to the 2015 count of this age group?

12. Use **Handout** 6 to identify the age group that has the greatest projected decrease in the count of people when comparing the 2015 to the 2050 counts if any decreases exist. What is the percent decrease of the 2050 count to the 2015 count of this age group?

13. Identify an age group that is projected to have the greatest percent increase of people when comparing the 2015 to the 2050 counts.

As the recursive model continues, the "sameness" of the age groups in which death is not a major change in the count of people displays the least growth. The population factors in these age groups are not that different, resulting in similar projections. The constant foundation factor also results in the count of births to even out.

What if birth rates change during a 5-year interval? What if immigration or emigration rates change for all age groups? What if there is a major health problem that results in higher number of people who die? What if the economy of the country is strong and results in the need for more workers? What if the economy is not strong, with possibly a depression or recession occurring? Each of these possibilities result in changes in the foundation factor or in the population factors during a 5-year interval. The "raggedness" observed in several of the histograms from 2015 to 2030 are a result of conditions prior to 2015 that resulted in noticeable differences in the population factors and the foundation factor. Unit 4 will consider these possible scenarios, and how the shapes and estimates alter the above projections.

Lesson 14
Kenya, Japan, United States – Summing It Up

Lesson 14 – Problems

Handouts needed to complete the following problems:

Handout 6: *The United States 2010 – 2050*

Handout 7*: Kenya 2010 – 2050*

Handout 8*: Japan 2010 – 2050*

The United States

Use completed **Handout 6** to answer the following questions. Remember that the population estimates are in millions of people.

1. What is the count of people who are projected to be 10 – 14 years old in 2025? What is the count of people who are projected to be 15 - 19 in 2030? What is projected to happen during those 5 years to change the count of people in the connected age groups?

2. What is the projected count of people who will be 70 – 74 years old in 2040? What is the projected count of people who will be 75 – 79 years old in 2045? Explain what happened from 2040 to 2045 that changed the count of people in these connected layers?

3. What is the projected count of people 0 – 4 years old in 2030? What is the projected count of people 0 – 4 years old in 2035? Explain why the count of people in this age group is not predicted to stay the same.

Kenya

Use completed **Handout 7** to answer the following questions. Remember that the population estimates of Kenya are in millions of people.

4. The Population Factor for the projected change in the 15 – 19 age group to the 20 – 24 age group is 0.988. The same population factor for the United States is 1.032. What does that indicate is different about the projected count of people in these connected age groups?

5. What is the projected count of people in Kenya in the 80 - 84 age group in 2040? What is the project count of people in the 85 – 89 age group in 2045? Explain what happened during the 5 years to change that count of people.

6. None of Kenya's population factors are projected to be greater than 1.000. What does this indicate is different about the projections for Kenya and the United States?

7. All of the Population Factors in Kenya are less than the Population Factors in the United States (except for the population factor of the age group 90-94 which is due to a round off of the small projected populations). What does the smaller Population Factors indicate about Kenya that is different than the United States?
(Note: A special adjustment to the counts were needed to avoid a division by 0 in the population factors for the oldest 2 age groups. The estimated counts from the Census Bureau indicates a loss of the population during the five-years. Population Factors rounded to the next decimal place would be needed to reflect the small numbers and more accurate proportions.)

Kenya
Handout 7

Age Groups	Population Factors
0 – 4	0.984
5 - 9	0.990
10 - 14	0.987
15 - 19	0.988
20 - 24	0.982
25 - 29	0.973
30 - 34	0.963
35 - 39	0.957
40 - 44	0.951
45 - 49	0.947
50 - 54	0.942
55 - 59	0.926
60 - 64	0.898
65 - 69	0.837
70 - 74	0.742
75 - 79	0.600
80 - 84	0.500
85 - 89	0.250
90 - 94	1.000
95 – 99	0.205
100 +	

8. What is the Foundation Factor used to estimate the count of people in the 0 - 4 age group? Compare this to the Foundation Factor used in the United States. In what way is the projected population of Kenya changed by this different foundation factor?

9. What is the projected count of the 0 – 4 age group in 2030? What is the projected count of people for the 0 – 4 age group in 2035? Explain why the estimates for this age group are different.

10. Determine the projected percent increase in the population of Kenya from 2015 to 2050.

Japan

Use completed **Handout 8** to answer the following questions about Japan. Remember that the population estimates of Japan are in millions of people.

11. The Population Factor for the projected change of the 10 – 14 age group to the 15 – 19 age group is 0.998. The population factor for the same connected age groups in the United States is 1.020. What does that indicate is different about the projected count of people who will be in these connected age groups?

12. None of Japan's population factors are projected to be greater than 1.000. What does this indicate is different about the projections for Japan and the United States?

13. You are able to travel to a city in Japan. Do you expect to meet people who immigrated to Japan? Explain your answer.

14. Identify the age groups in which the population factors for Japan are greater than the population factors for the United States.

Japan

Handout 8

Age Groups	Population Factors
0 – 4	0.996
5 - 9	0.998
10 - 14	0.998
15 - 19	1.000
20 - 24	0.998
25 - 29	0.996
30 - 34	0.996
35 - 39	0.995
40 - 44	0.993
45 - 49	0.989
50 - 54	0.984
55 - 59	0.975
60 - 64	0.965
65 - 69	0.946
70 - 74	0.909
75 - 79	0.845
80 - 84	0.742
85 - 89	0.604
90 - 94	0.427
95 – 99	0.205
100 +	

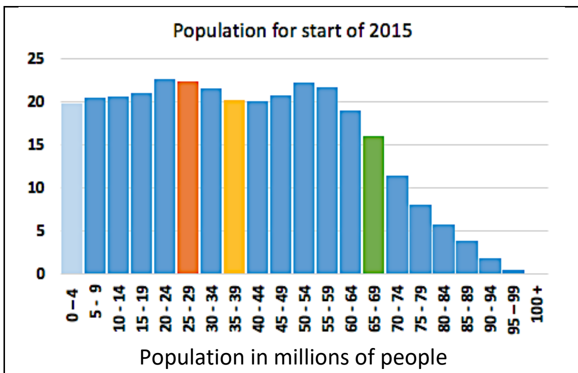
15. What does it indicate about the projected counts of people for connected age groups that have a greater population factor than the United States?

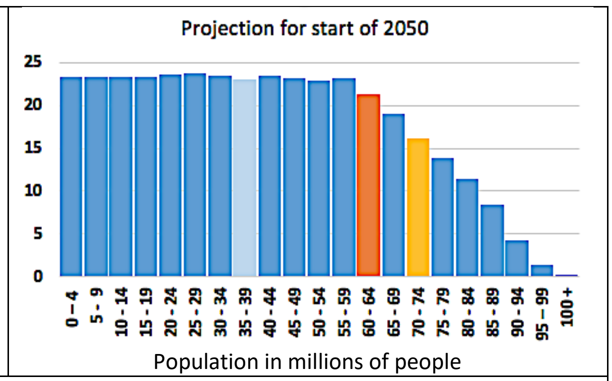
16. What is the Foundation Factor used to project the count of people in the 0 - 4 age group? Compare this to the Foundation Factor used in the United States. What does the different Foundation Factor for Japan indicate when comparing Japan to the United States? Kenya?

17. What is the projected count of the 0 - 4 age group for 2045? What is the projected count of the 0 - 4 age group for 2050? What does the difference in the projected counts tell us about Japan's future?

Summaries

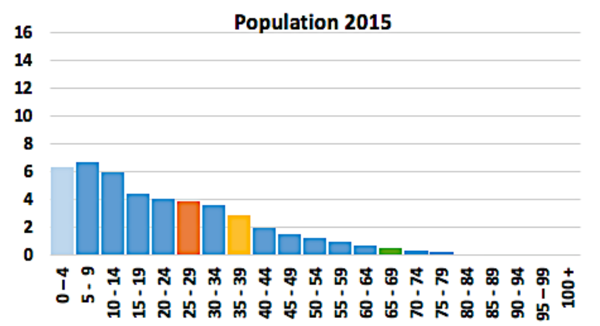
18. The United States



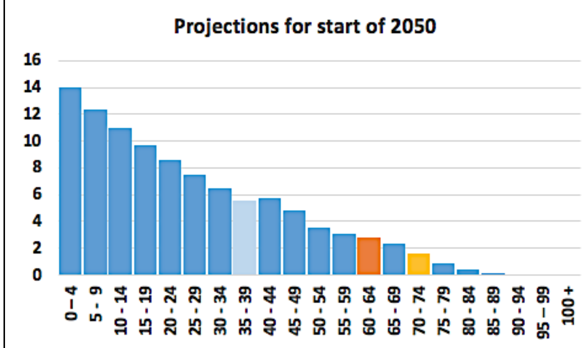


Summarize what happens from 2015 to 2050 if the assumptions about the changes in the population are accurate:

19. Kenya



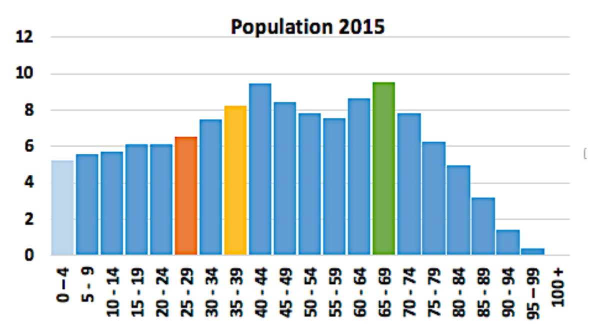
Population in millions of people



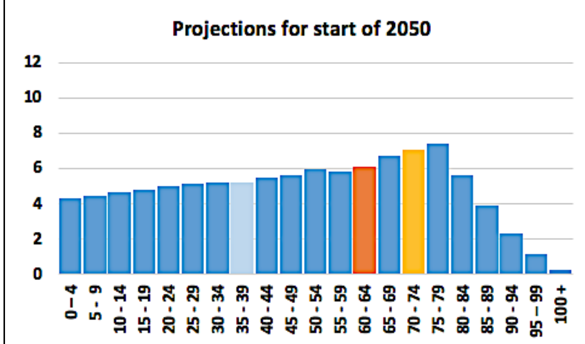
Population in millions of people

Summarize what happens from 2015 to 2050 if the assumptions about the changes in the population are accurate:

20 Japan



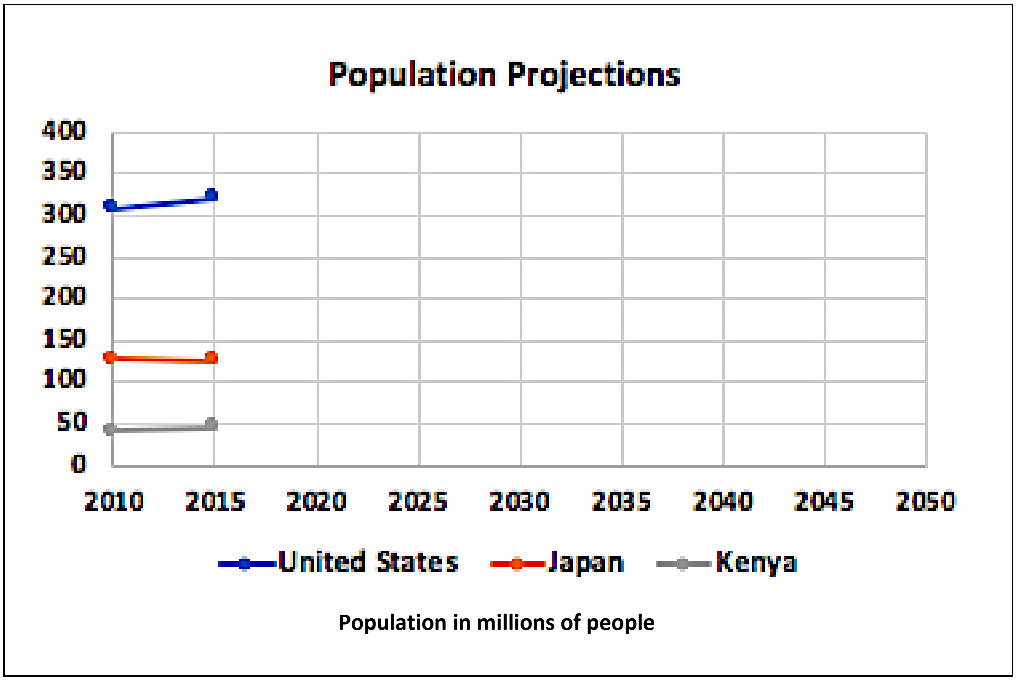
Population in millions of people



Population in millions of people

Summarize what happens from 2015 to 2050 if the assumptions about the changes in the population are accurate:

21 Use **Handouts 6, 7, and 8** to complete the following graph:



State at least two reasons why the recursive model used in making the above projections can not continue without revisions.

Extension

This extension is to be completed using the Excel file **“The 1 Country”.xisx** or ***MyCountry Recursive Model.xisx***. Recall that the file **“The 1 Country”.xisx** implemented the recursive model for a fictitious country, or “The 1 Country”, in which the count for each of the age groups and the foundation and population factors were 1. Replace the counts of the age groups in 2010 and 2015 with the population counts of your country. Make sure you carefully save the revised spreadsheet using a different file name as “The 1 Country” file will also be used in Lessons 15 and 16. If directed by your teacher, use the Excel file ***MyCountry Recursive Model*** to answer the following questions or topics based on the fictitious country introduced in earlier lessons as MyCountry. Describe the results of applying the recursive model to your data or to the MyCountry data by completing the following summaries:

a. Shape of your country or My Country in 2010 (bottom-layered, lower middle-layered, upper middle-layered, top-layered):

b. Shape of your country or My Country in 2050:

c. Description of the histogram for 2015.

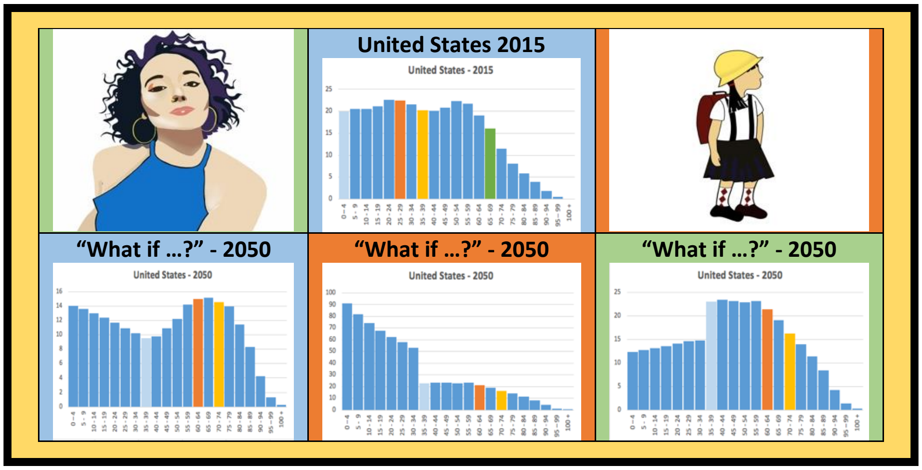
d. General description of the Population Factors (for example, age groups in which the factors were greater than 1, less than 1, or even equal to 1):

e. Value of your foundation factor for 2015. Change the foundation factors for all of the other years (2020, 2025, etc.) to this value.

f. Description of the histogram for 2050.

g. General summary of the population of your country in 2050:

Unit 4
"What if …?"



Introduction

"What if …?" are investigations that think about what might happen to our countries if events change the assumptions used to design our projection models. Each of the countries we studied had a past that resulted in their current shapes. Each of the countries had recent changes in their counts that may or may not continue when future counts are conducted. This unit is about remodeling our projection tools to think about changes in a population based on events that may or may not happen. This unit also evaluates the recursive model by looking at how well it did in estimating the more recent counts of people based on past counts. Should we trust this model?

What if there is a major drought? What if there are changes in laws and policies that limit the count of people coming into a country? What if there is a change in the birth rate? What if a country needs many more workers that require efforts to bring people from other countries into a country? What if …? What may happen to the generations that include Kristin or Raphine or Hana? "What if …?" are stories that are yet to be told.

Lesson 15
"What if …?" Scenarios

The stories are not over. Kristin is moving, Abbey is working, Adeline is learning, and the Baby Boomer parents are retiring. But whatever the present situation, the future is still a question for all of them.

The models developed and analyzed in this module are mathematical designs to think about the future. Will the countries grow or decline as indicated by these models? What are the possible ways these countries may change as a result of shifting demographics? What if you could examine these changes before they happened or did not happen? You know events happened in the past that resulted in population histograms or population pyramid graphs looking "ragged" with certain age groups dominating the profile of a country for decades (for example, the Baby Boom generation). Several of these ragged features were the result of wars or economic challenges or storms or draught.

Up to this point, your connection with the models has been ***static*** – you observed the models through graphs and tables, you examined the assumptions that were made to build the models, and you analyzed the projected results based on the anchor years. For the recursive model, you specifically analyzed the changes in age groups that had a major impact on a country.

For this lesson, you are provided an opportunity to be more involved with the details of the recursive model. Rather than just analyzing a country's population based on the population or foundation factors, you are now able to alter these factors. What if you could examine more closely the impact of death or birth or immigration or the combinations of these factors in the future based on changes not reflected in the 2010 and 2015 counts? This lesson uses the recursive models to examine these "What if …?" possibilities.

The tools for this lesson are several Excel spreadsheet files of the recursive model. The files will allow you to apply the recursive model with altered population factors and altered foundation factors. If you have limited access to Excel, other spreadsheet applications may work with these files, however, the histograms may not display the data as designed in the Excel files.

Lesson 15 – Project

Handouts needed to complete the projects in this lesson:

> **Handout 6: The United States 2010 – 2050**
> **Handout 7: Kenya 2010 – 2050**
> **Handout 8: Japan 2010 – 2050**

Spreadsheet files needed to complete the projects in this lesson:

> ***USA Recursive Model.xlsx***
>
> ***Kenya Recursive Model.xlsx***
>
> ***Japan Recursive Model.xlsx***

Starting the task …

Read through the scenarios provided for the United States, Kenya, and Japan. Each scenario describes an event that could affect a country's population at the start of the year 2020 or later. The scenarios are not always comfortable. Sometimes, as previous lessons indicated, wars, floods, earthquakes, or economic conditions can visibly alter the shape of a country for years after these events. And, as these previous lessons also pointed out, altering the shape can result in new challenges and a new lifestyle within in the country. What if the scenario you selected happened? In what way do you think the population distribution looking beyond this event will change? Will the scenario alter the shape of the country? If yes, how will the altered shape affect the way people make a living, go to school, take care of each other, or use the resources of their country?

The following steps outline the expectations of a task that gives you a chance to alter the future of the countries studied in this module.

Step 1: Read through the scenarios. Select one scenario to complete for this lesson. If time promotes, you may be asked to select another scenario and repeat the process.

Step 2: Review your country's current population distribution using **Handout 6** for the United States, **Handout 7** for Kenya, or **Handout 8** for Japan. Carefully examine the **Scenario Planning Template** for the country you selected that is included at the end of this lesson. Indicate on this template the population factors (if any) you would change, the new values you would assign to these factors, and a brief statement why you would change these factors based on the scenario you selected.

Step 3: In addition to possibly changing the population factors, decide if you would also change the foundation factors for your country and why. Identify your changes and your explanations on the **Scenario Planning Template.** Unlike the proposed changes to the population factors, you may recommend changing the foundation factor for certain periods. For example, you may decide to increase the foundation factor from its current value for the years 2020 to 2040, and then you may decide to decrease it for the years 2045 to 2050.

Step 4: Load the Excel file that matches your country (***USA Recursive Model***, ***Kenya Recursive Model***, and ***Japan Recursive Model***). Make sure you save the revised file to your account or computer by a new name. Directions to save files will be provided by your teachers as computer networks have different procedures regarding saving files. This process will allow you to return to the original values of the recursive model in case of errors or revisions while completing this lesson.

Step 5: Enter the proposed revisions from the **Scenario Planning Template** on the Excel file. Note the outcomes for the age group and for the population totals as a result of your changes to the model. Also note the revised shape of the final histogram.

Step 6: Continue to either revise or alter population and foundation factors on the Excel file if you think your plan needs revisions. Make sure you also record the changes on the **Scenario Planning Template.**

Completing the task …

When you think your scenario has been accomplished with the revisions you entered, complete a **Final Summary Report.** Submit your **Scenario Planning Template** and your **Final Summary Report** to your teacher. Be prepared to summarize the results of your proposed changes to the whole class.

United States

Scenario 1:

It is 2018. The economy in the United States is doing well. The labor market is expanding at a rate that will require more workers than there are currently available in the country. Major companies are focusing are bringing in people from other countries to fill these jobs.

*Scenario 2:

It is 2017. People from most countries are not immigrating into the United States.

Scenario 3:

It is 2025. There is a major recession in the United States. The result is high unemployment and major financial constraints for families across the economic spectrum.

***Scenario 4:**

It is 2018. There is an epidemic that will affect the country for many years. In particular, people 50 and older are dying at higher rate.

Scenario 5:

It is 2018. A major medical breakthrough has been reached in which almost every person will live well into their 100's.

Scenario 6:

It is 2018. The economies in Africa and Europe are doing much better than in the United States. A major effort to recruit workers from the United States to move to countries in Europe and Africa has been launched.

Scenario 7:

It is 2018. Young people indicated that they are not likely to have children.

Scenario 8:

It is 2018. Young people who are 20 – 29 years old have decided to wait and have their families until they are 30 or older.

Kenya

***Scenario 9:**

It is 2018. The Kenyan government has been working with groups across the country to reduce its birthrate.

Scenario 10:

It is 2018. Improvement in health care resulted in a higher life expectancy for people who are 40 years old or older. In addition, people indicated they do not want large families.

***Scenario 11:**

It is 2018. The people of Kenya have problems finding work and are moving to other countries for employment. 25% of the people who are 20 years or older are moving out of the country.

Japan

Scenario 12:

It is 2018. There is a major concern of how to care for older people. The government made an effort to encourage people 25 years or older in other countries to move to Japan to help care for older people.

Scenario 13:

It is 2020. More young people indicate they are planning to have children. Many of them indicate that they would like to have a family of at least 2 children.

Scenario 14:

It is 2018. The government initiated incentives to encourage people from other countries to move and live in Japan. The appeal has attracted many people who are 30 years old or older.

Scenario Planning Template for the United States:

Scenario number: _____

Identify the population factors you would change and why you selected these factors in the following planning template. It is not necessary to change factors if you think the scenario would not require a change to the population factors for an age group:

Age Group	Current Population Factors	Proposed change of Population Factor:	Explanation for changes:
0 – 4	1.014		
4 - 9	1.014		
10 - 14	1.020		
15 - 19	1.032		
20 - 24	1.032		
25 - 29	1.022		
30 - 34	1.012		
35 - 39	1.004		
40 - 44	0.995		
45 - 49	0.985		
50 - 54	0.974		
55 - 59	0.962		
60 - 64	0.945		
65 - 69	0.917		
70 - 74	0.869		
75 - 79	0.792		
80 - 84	0.670		
85 - 89	0.508		
90 - 94	0.340		
95 – 99	0.205		
100 +			

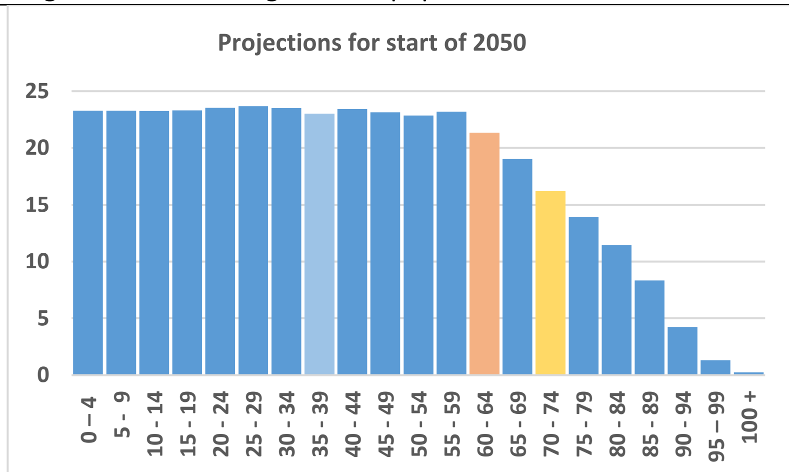
In the following table, indicate any changes to the Foundation Factors and when you would propose making those change. It is not necessary to change factors if you think the scenario would not require a change:

Year	**Current Foundation Factors**	**Proposed changes to Foundation Factors:**	**Explanation for changes:**
2010	0.065	Fixed – cannot change.	
2015	0.062	Fixed – cannot change.	
2020	0.062		
2025	0.062		
2030	0.062		
2035	0.062		
2040	0.062		
2045	0.062		
2050	0.062		

Final Summary Report (USA)

Scenario number: _____

Note the colors used to track Adeline, Abbey, Kristin, and her parents are included in the histograms. Include in your summary of the 2050 histogram what is different about the counts for their age groups.

United States 2050 histogram with no changes to the population and foundation factors:

Population in millions of people

Description of the 2050 histogram with proposed changes (if possible, provide a copy or sketch of the histogram):	
Population in millions of people	**United States 2050**

Describe why the proposed changes were made to address the scenario:

Scenario Planning Template for Kenya:

Scenario number: _____

Identify the population factors you would change and why you selected these factors in the following planning template. It is not necessary to change factors if you think the scenario would not require a change to the population factors for an age group:

Age Group	Current Population Factors	Proposed change of Population Factor:	Explanation for changes:
0 – 4	0.984		
4 - 9	0.990		
10 - 14	0.987		
15 - 19	0.988		
20 - 24	0.982		
25 - 29	0.973		
30 - 34	0.963		
35 - 39	0.957		
40 - 44	0.951		
45 - 49	0.947		
50 - 54	0.942		
55 - 59	0.926		
60 - 64	0.898		
65 - 69	0.837		
70 - 74	0.742		
75 - 79	0.600		
80 - 84	0.500		
85 - 89	0.250		
90 - 94	1.000		
95 – 99	0.205		
100 +			

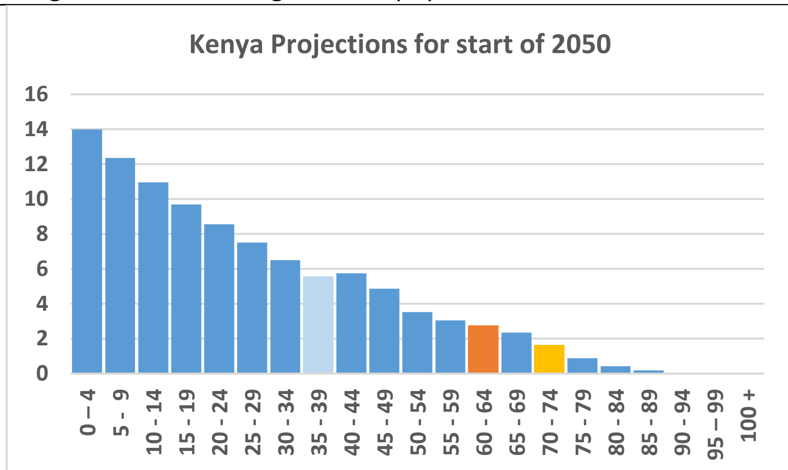
In the following table, indicate any changes to the Foundation Factors and when you would propose making those change. It is not necessary to change factors if you think the scenario would not require a change:

Year	**Current Foundation Factors**	**Proposed changes to Foundation Factors:**	**Explanation for changes:**
2010	0.168	Fixed – cannot change.	
2015	0.139	Fixed – cannot change.	
2020	0.139		
2025	0.139		
2030	0.139		
2035	0.139		
2040	0.139		
2045	0.139		
2050	0.139		

Final Summary Report (Kenya)

Scenario number: _____

Note that the colors used to track Adeline, Abbey, Kristin, and her parents have been included in the histograms. Although they are not counted in Kenya, they represent important age groups to monitor. Include in your summary of the 2050 histogram a description of what is different about the counts for their age groups.

Kenya 2050 histogram with no changes to the population and foundation factors:
 Population in millions of people

Description of the 2050 histogram with proposed changes (if possible, provide a copy or sketch of the histogram):	
	Kenya 2050
Population in millions of people	

Describe why the proposed changes were made to address the scenario:

Scenario Planning Template for Japan:

Scenario number: _____

Identify the population factors you would change and why you selected these factors in the following planning template. It is not necessary to change factors if you think the scenario would not require a change to the population factors for an age group:

Age Group	Current Population Factors	Proposed change of Population Factor:	Explanation for changes:
0 – 4	0.996		
4 - 9	0.998		
10 - 14	0.998		
15 - 19	1.000		
20 - 24	0.998		
25 - 29	0.996		
30 - 34	0.996		
35 - 39	0.995		
40 - 44	0.993		
45 - 49	0.989		
50 - 54	0.984		
55 - 59	0.975		
60 - 64	0.965		
65 - 69	0.946		
70 - 74	0.909		
75 - 79	0.845		
80 - 84	0.742		
85 - 89	0.604		
90 - 94	0.427		
95 – 99	0.205		
100 +			

In the following table, indicate any changes to the Foundation Factors and when you would propose making those change. It is not necessary to change factors if you think the scenario would not require a change:

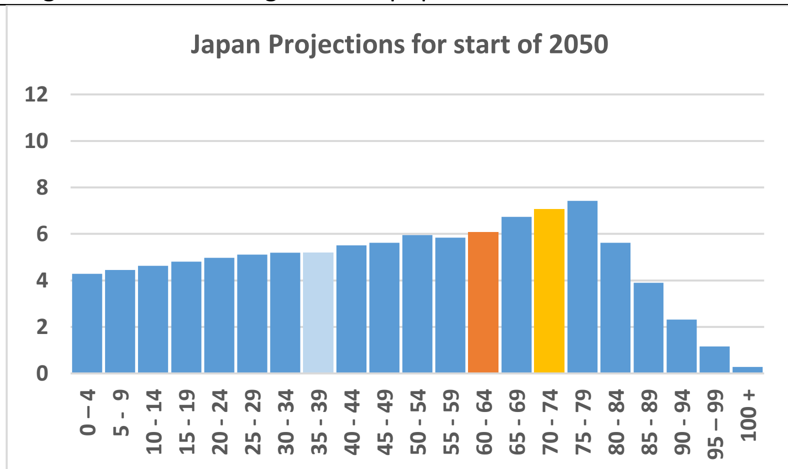
Year	**Current Foundation Factors**	**Proposed changes to Foundation Factors:**	**Explanation for changes:**
2010	0.044	Fixed – cannot change.	
2015	0.042	Fixed – cannot change.	
2020	0.042		
2025	0.042		
2030	0.042		
2035	0.042		
2040	0.042		
2045	0.042		
2050	0.042		

Final Summary Report (Japan)

Scenario number: _____

Note that the colors used to track Adeline, Abbey, Kristin, and her parents have been included in the histograms. Although they are not counted in Japan, they represent important age groups to monitor. Include in your summary of the 2050 histogram what is different about the counts for their age groups.

Japan 2050 histogram with no changes to the population and foundation factors:



Population in millions of people

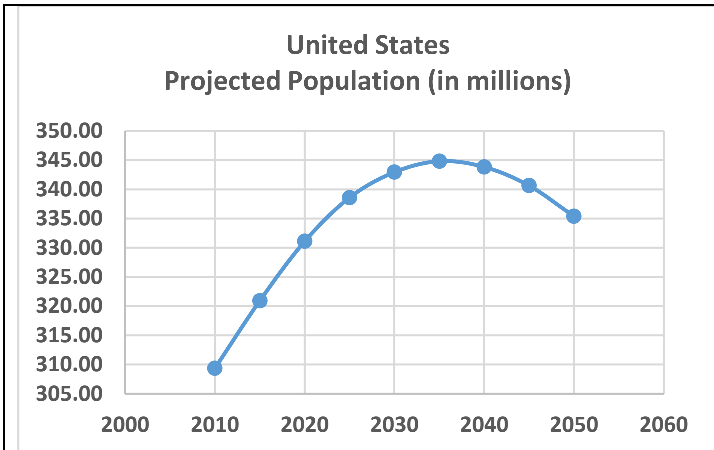
Description of the 2050 histogram with proposed changes (if possible, provide a copy or sketch of the histogram):	
Population in millions of people	**Japan 2050**

Describe why the proposed changes were made to address the scenario:

Extension Problems

Each of the following problems contain graphs of the USA population from 2010 to 2050 as a result of changing the Population Factors for certain age groups, or changing the Foundation Factor for various years, or changing both. What if the following graphs summarize stories of the United States population from 2010 to 2050? What would you change in the recursive model and why you would make those changes to the recursive model to obtain each graph? If you have access to the recursive model for the United States (**USA Recursive Model**), make the changes you identified. Did your changes result in a graph similar to the graph in the problem? If your graph is not the same, what other changes might be considered?

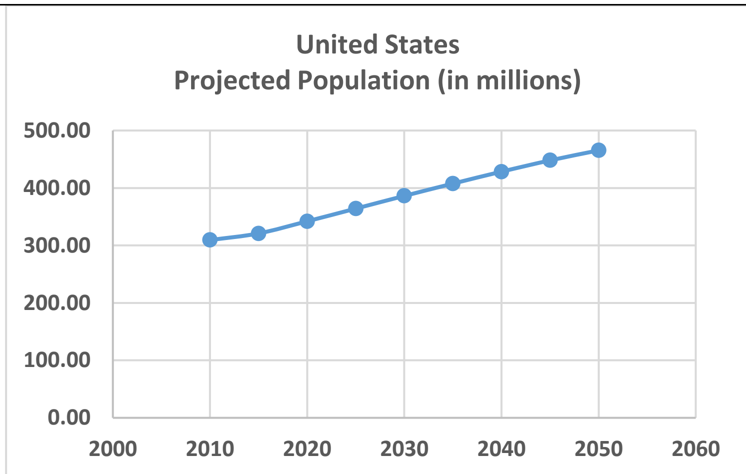
Extension Problem A:



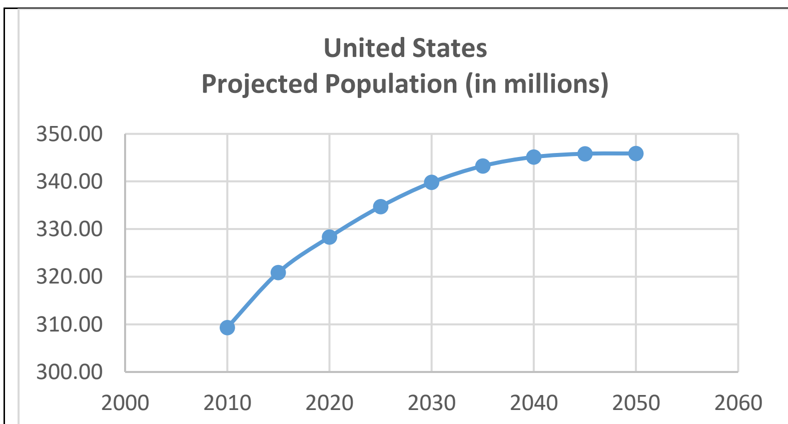
What changes would you make to the United States recursive model that would result in a graph similar to the one presented in this problem?

Summarize the story of the United States if this graph is an accurate summary of the changes in the population from 2010 to 2050.

Extension Problem B:

<table>
<tr><td>
</td><td>What changes would you make to the United States recursive model that would result in a graph similar to the one presented in this problem?</td></tr>
<tr><td colspan="2">Summarize the story of the United States if this graph is an accurate summary of the changes in the population from 2010 to 2050.</td></tr>
</table>

Extension Problem C:

<table>
<tr><td>
</td><td>What changes would you make to the United States recursive model that would result in a graph similar to the one presented in this problem?</td></tr>
<tr><td colspan="2">Summarize the story of the United States if this graph is an accurate summary of the changes in the population from 2010 to 2050.</td></tr>
</table>

Extension Problem D:

For this extension problem, you are in control. Make changes to the recursive model that tells a story of the population changes of the United States from 2010 to 2050. What if your changes were an accurate summary of what might happen in the United States? (If you do not have access the recursive mode spreadsheet file, sketch your graph and tell your story.)

	What changes would you make to the United States recursive model that would result in a graph similar to the one you produced?
Summarize the story of the United States if the graph you produced is an accurate summary of the changes in the population from 2010 to 2050.	

Lesson 16
(Optional Lesson)
The United States Census Models and the Recursive Model

The linear model (Lesson 8)? The exponential model (Lesson 9)? The recursive model (Lessons 10 – 12)? Which model is the most accurate?

Consider models designed to forecast the weather. It is not uncommon that when a major storm is eminent (for example, a hurricane), different weather models are used to forecast the severity and the path of the hurricane, along with the possible impact on people living in its path. Often weather models do not agree on these predictions. For example, a severe hurricane was predicted by one weather model to follow a path into highly populated cities. If accurate, this model indicated major damage to property and danger to people living in its path. Evacuations of people from their homes would be required if this model's forecast was accurate. Another model, however, predicted this same hurricane would follow a path over the ocean and would not require evacuating people. The hurricane did not follow the path that was forecast by the first model. Evaluating the best model, however, was based on the actual path of the hurricane and then looking back at each model's forecast.

Population models are also designed to look into the future. The ultimate evaluation of the best model would be to compare the predicted counts to the actual counts in 2050. This type of evaluation requires us to stick around until 2050 and compare each model's estimates with the actual count in 2050.

The United Census Bureau has several projection models that are different than the models studied in this module. Their models are continuously evaluated and revised over shorter periods of times due to their access to data. The United Census Bureau evaluates projections based on data collected each year regarding immigration, emigration, deaths, and of course, birth rates. It builds these ongoing factors into their models. The Census Bureau also designs and interprets various models at the same time. Some of the models are described as robust as higher birth rates and immigration rates are applied. Other models are less robust as lower birth rates and immigration estimates are applied. Evaluations of the models are then examined when a census has been completed.

Lesson 16 – Problems

Handouts needed to complete the projects in this lesson:

Handout 9: The United States Project Worksheet
Handout 10: Kenya Project Worksheet
Handout 11: Japan Project Worksheet

Spreadsheet files to use for exploring the projects in this lesson:

USA Recursive Model.xlsx

Kenya Recursive Model.xlsx

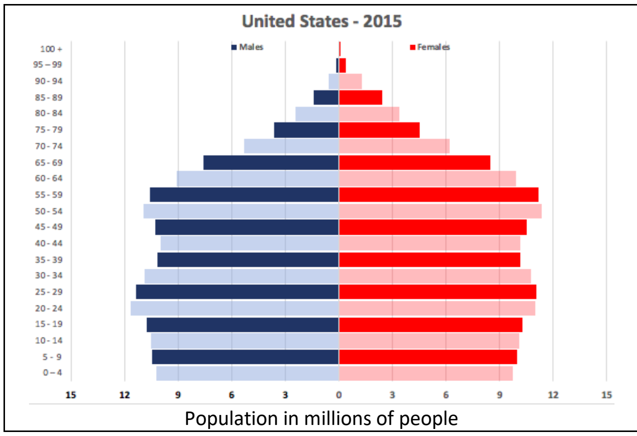
Japan Recursive Model.xlsx

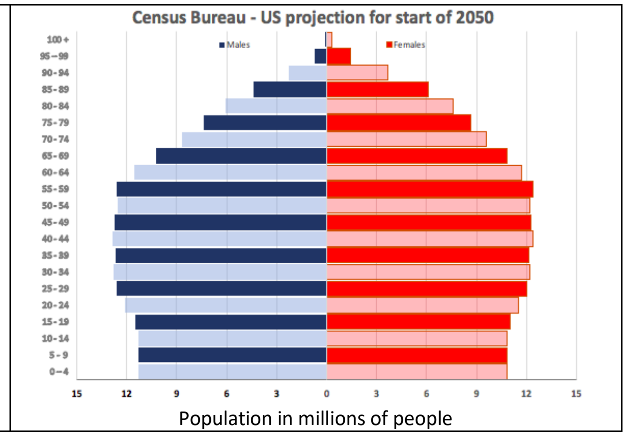
The first lesson of this module included the 2015 population pyramid graph of the United States compiled by the International Data Base (IDB) of the United States Census Bureau. This lesson was followed with problems that also analyzed the 2015 pyramid graphs of Kenya and Japan. The pyramid graphs for these countries are also included in this lesson.

This lesson, however, forecasts the future counts of these countries. The population pyramid graphs for 2015 and 2050 were obtained from the Census Bureau. The first pyramid graph for each country is the familiar one used in several previous lessons that indicates the best estimates of the actual counts of people in 2015. The second pyramid graph is the estimate of the 2050 counts based on a population projection model used by the International Data Base (IDB) of the United States Census Bureau.

The United States

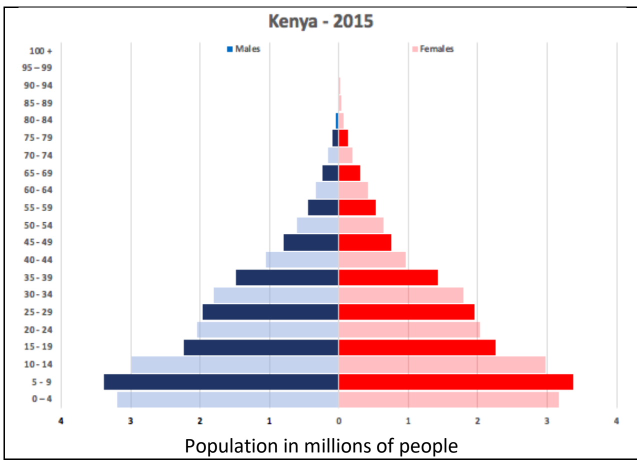
The 2015 United States Population and the 2050 Projections for the United States (data from the IDB):

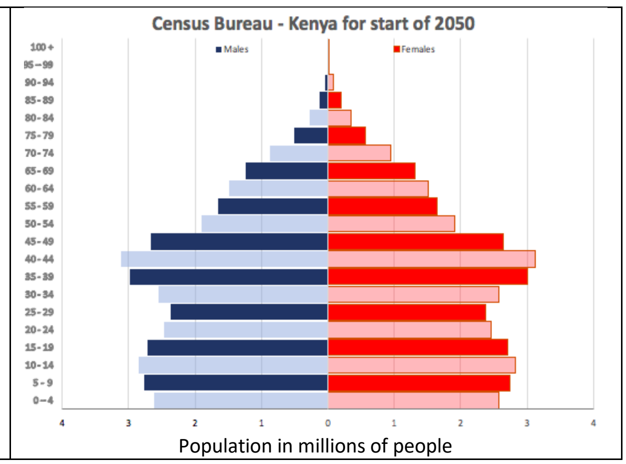




Kenya

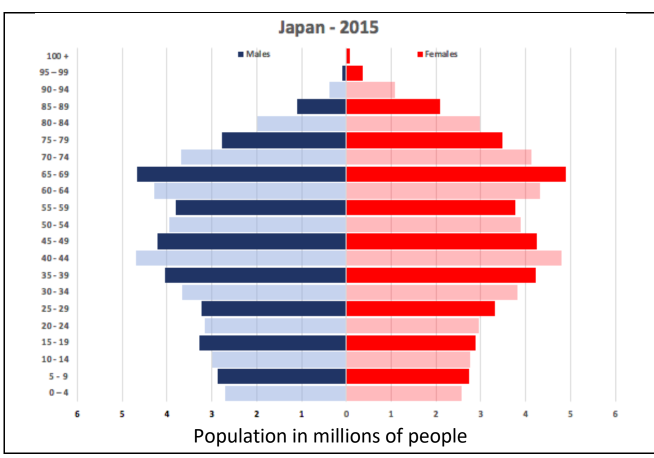
The 2015 Population of Kenya and the 2050 Projections for Kenya (data from the IDB):

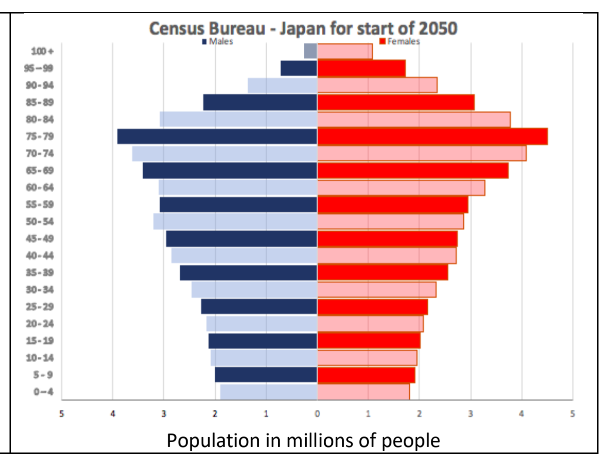




Japan

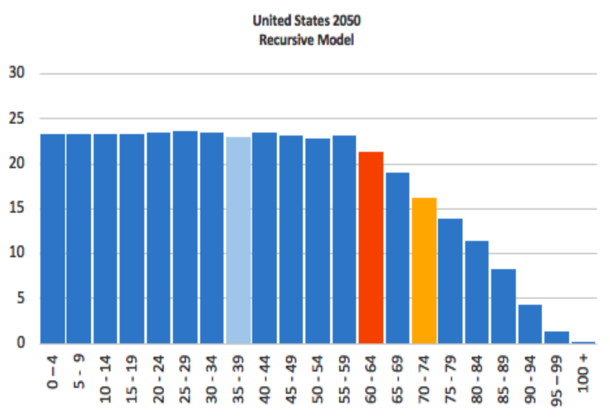
The 2015 Population of Japan and the 2050 Projections for Japan (data from the IDB):





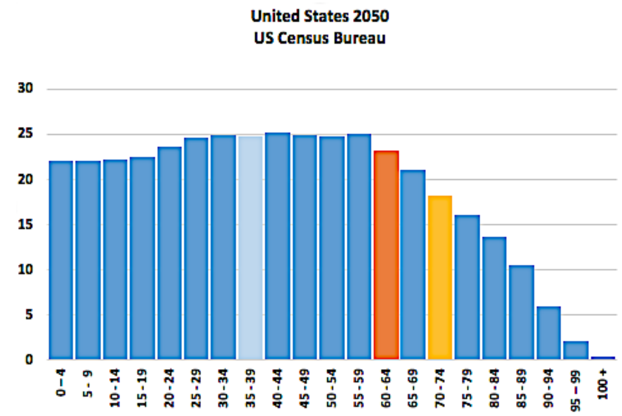
The following histograms were prepared from the recursive model's estimates and the United States Census Bureau's estimates of the 2050 population of the United States ([https://www.census.gov/programs-surveys/international-programs/about/idb.html)](https://www.census.gov/programs-surveys/international-programs/about/idb.html) and are presented below. The histograms combine the counts of males and females in each of the age groups. Are the estimates of the recursive and the Census Bureau models similar? If not, what are the differences in the models' predictions and the implications for the countries? These questions are investigated in this lesson.

Recursive Model Projections for 2050



Total population projected to be 375.67 million people

Census Projections for 2050



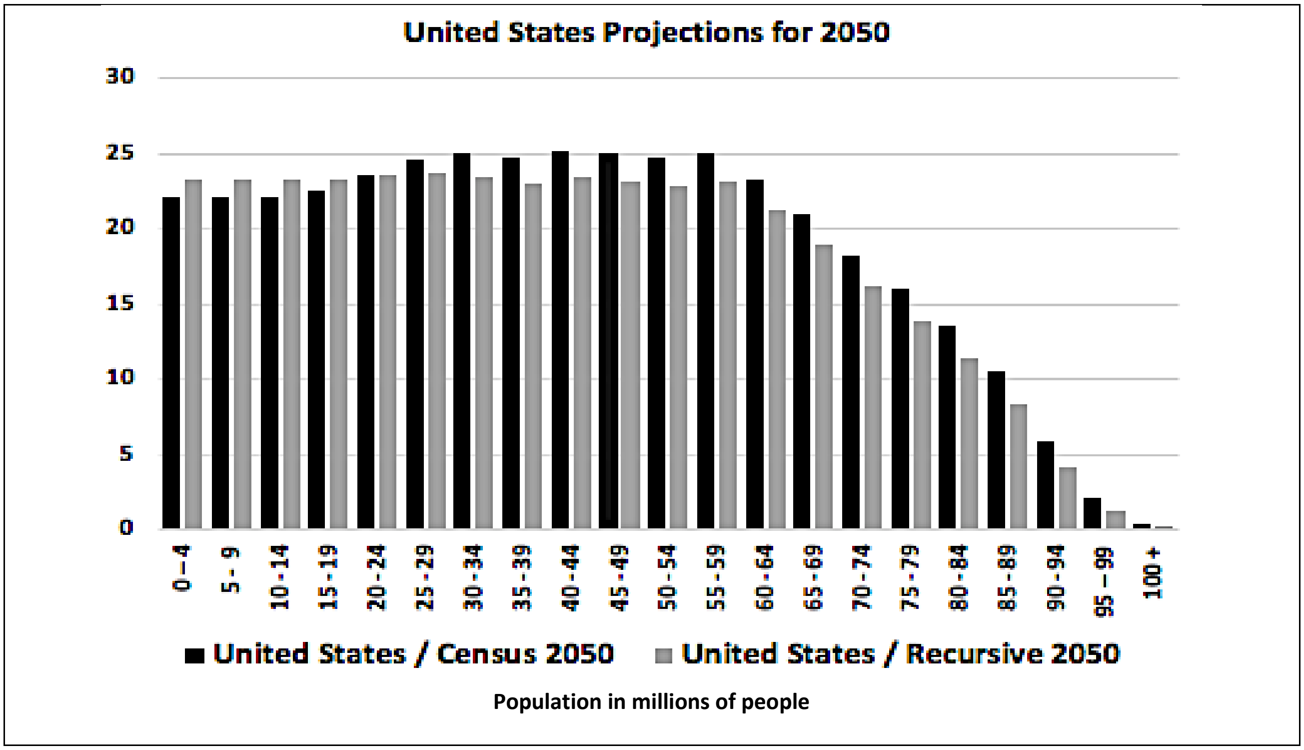
Total population projected to be 398.33 million people

1. Do you think the recursive projections and the Census projections are similar? Explain your answer.

2. Of the two models, which model projects the largest estimated population for the United States in 2050? What is the population projected by that model?

3. What is the percent increase of the Census model's prediction compared to the recursive model's prediction?

The following graph is designed to compare the estimates of each model by age groups:



4. Study the age group estimates of the two models. Identify at least 3 age groups in which the recursive model's projections (color coded in light gray) are greater than the Census Bureau's projections (color coded in black).

5. Identify at least 3 age groups in which the recursive model's projections (color coded in light gray) are less than the Census predictions (color coded in black).

What assumptions did the Census Bureau consider that are different than the assumptions organized in the recursive model? The goal of the next several problems is to revise specific assumptions built into the recursive model and observe if these revisions result in projections that are closer to the projections of the Census model for 2050.

Before making specific changes to the recursive model, analyze the impact on the 2050 projections if **one** of the population factors are increased. Use **Handout 9: The United States Project Worksheet** to complete the following problems**.** This handout has blank cells for each of the projections after 2015 and blank cells for the population and foundation factors. Use this handout to record the domino effect that results when one population factor of an age group is altered.

Note that the recursive model projections for 2050 are less than the Census model projections for several of the middle age groups (25 – 29 years old age group up to the 55 – 59 years old age groups). Consider the following change in the recursive model that might increase the counts in these age groups by 2050. Place a "+" in the cell of the Population Factor for the age group 35 – 39 years old. The "+" indicates that this factor will be increased in the recursive model. How does altering this factor change other cells of this population model?

Age Groups	Population Factors
0 – 4	
5 - 9	
10 - 14	
15 - 19	
20 - 24	
25 - 29	
30 - 34	
35 - 39	+

6. What if we increase the population factor for the 35 – 39 years old age group to the 40 – 44 years old age group? Place a "+" in the location of this population factor on **Handout 9** as illustrated below. How does this change affect the counts in 2020? Answer the following questions related to this change of one population factor:
   a. Why will the estimated count for the 40 – 44 years old increase in 2020? Place a "+" in the 40 – 44 years old age group indicating that the count in this age group will be increased as a result of increasing the population factor.

35 - 39	+	20.08	20.31	
40 - 44		20.91	20.16	+

   b. Why will the total count of the population also increase if we increase the population factor for people 35 – 39 years old? Place a "+" in the 2020 column representing the count of the total population.

7. Why will the estimated count for the 0 – 4 years old also increase in 2020? On **Handout 9**, place a "+" in the age groups representing the 0 – 4 years old. The "+" indicates that the count in this age group will also increase as a result of increasing the population factor of people who are 35 – 39 years old to 40 – 44 years old.

Age Groups	Population Factors	2010	2015	2020
0 – 4		20.19	19.91	+

At the beginning of this lesson, the recursive model's estimate of the count for 0 – 4 years old was greater than the estimate from the Census Bureau. The decision to increase the population factor for the 35 – 39 years old results in an unintended consequence of further increasing the 0 – 4 years old count summarized on the recursive model. In addition, other age groups that also had greater counts than the estimates from the Census Bureau will further increase after 2020 due to this decision. Decreasing the count for the 0 – 4 years age group and other age groups will be addressed in problems 15 to 17.

8. More age groups are affected than just the three identified in 2020. The increases represented by the "+" in 2020 also result in increases in the 2025 counts. Place a "+" in the age groups for 2025 that will increase as a result of the increases predicted in 2020.

9. Continue this process. Place a "+" in the age groups of the table for 2030 to 2050 that will increase in the recursive model as a result of increasing the population factor for the 35 – 39 years old age group.

10. Summarize what age groups are projected to increase in 2050 as a result of the increase in the population factor of the 35 – 39 years old age group to the 40 – 44 years old age group.

11. Will an increase in the age groups identified by the "+" suggest a closer match to the projections reported by the Census Bureau? Identify the age groups in 2050 that will increase in count and as a result more closely match the Census Bureau's projections.

What if an increase in the population factor of the 70 - 74 years old age group is considered?

12 In the same way that a "+" was used to indicate an increase for the 35 – 39 years old age group population factor, place a "+" in the population factor cell for the 70 – 74 years old age group population factor.

13. Why was an increase in the population factor for 70 – 74 considered?

14. What additional age groups are increased as a result of increasing the population factor for the 70 – 74 years old population factor? Place a "+" in the age groups of the handout if the count of the age group will increase. If an age group already has a "+", place a second "+" next to it, or a "++" indicating two changes will increase the count in this cell. Will the increase in the 70 – 74 years old population factor result in a closer match to the Census model? Explain your answer.

15. Several of the younger age groups had a greater count in the recursive model than in the Census model. Let's consider decreasing the Foundation Factor in 2040 from the estimate used in the recursive model.
    a. Place a "-" in the location of the Foundation Factor for 2040. This indicates that a new foundation factor will be considered and that this factor will be less than the 6.2% used in the recursive model to derive the count of 0 - 4 years old.

Foundation Factors:						
0.065	**0.062**					**-**
Actual Counts:		**Projections:**				
2010	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**

    b. By decreasing the Foundation Factor, what age group is affected in 2040?

    c. Although not an age group, what other count decreases as result of decreasing the 2040 foundation factor? Place a "-" in that location.

    d. What additional age groups in 2045 will be impacted by a reduced count of 0 – 4 years old in 2040? Place a "-" in each age group in **Handout 9** that will decrease if the foundation factor for 2040 is decreased. For age groups that were previously identified as "+" or "++", add a "-" next to the list, or "+ -" or "+ + - ".

16. Consider also reducing the Foundation Factors for 2045 and 2050.
    a. Place a "-" in the two locations of the Foundation Factors for 2045 and 2050. Also place a "-" in the age groups of 2045 and 2050 that will decrease as a result of decreasing the foundation factors in 2045 and 2050.

    b. Although not an age group, what else will decrease in count when the foundation factor for 2045 and 2050 is decreased? Place a "-" in each of these locations.

Age groups that are identified in **Handout 9** for 2050 with a "+" or "++" are estimated to increase in counts. Age groups that are identified as "-" are estimated to decrease in counts. Age groups that have a combination, or "+ -" or "+ + -", could increase or decrease depending on the value of the factors entered.

17. Use a completed **Handout 9** to answer the following questions regarding the revised estimates for the 2050 age groups.
    a. What age groups will increase?

    b. What age groups will decrease?

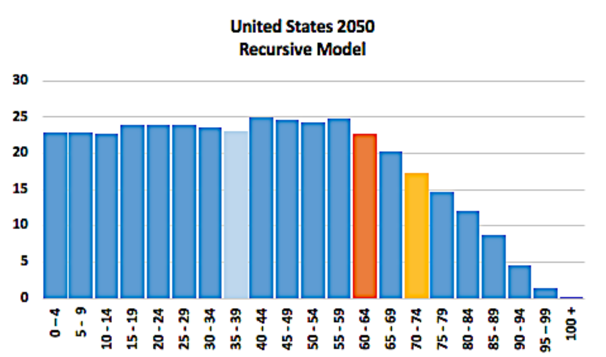
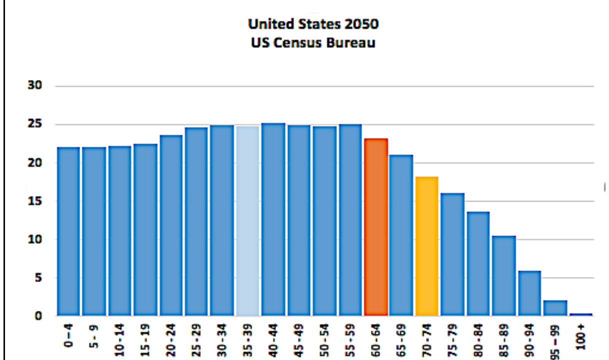
    c. What age groups are unclear whether or not they will increase or decrease?

    d. If an age group remains blank, what does that indicate about the estimated count in 2050?

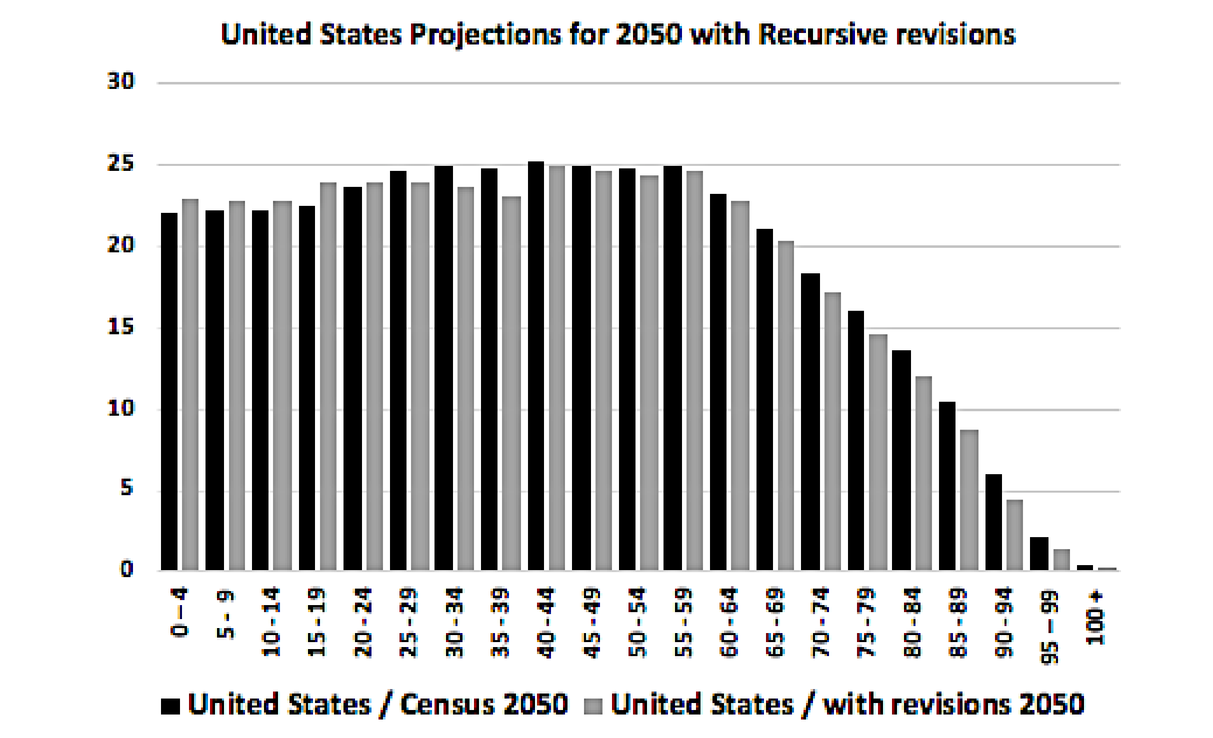
Open up the Excel file ***USA Recursive Model.xlsx,*** enter the following revisions to the recursive model:

Change the Population Factor for the 35 -39 years old to 1.055
Change the Population Factor for the 70 – 74 years old to 0.900
Change the Foundation Factors for 2040, 2045, and 2050 to 0.059.

The 2050 projections resulting from these revisions and the 2050 Census projections are summarized below:

Recursive model projections for 2050 with revisions:	Census model projections for 2050:
	
Total population: 387.5 million people	Total population: 398.33 million people

The following graph combines the above histograms to help us compare the two models. This graph compares the two models **afte**r the revisions were made to the recursive model.



18. Answer the following:
    a. Identify the age groups that have nearly equal projections.

    b. Identify the age groups where the recursive projections (color coded light gray) are noticeable greater than the Census projections (color coded black).

    c. Identify the age groups where the recursive projections are noticeable less than the Census projections.

    d. Did the changes in the population and foundation factors result in a better match of the recursive model to the Census model? Explain your answer.

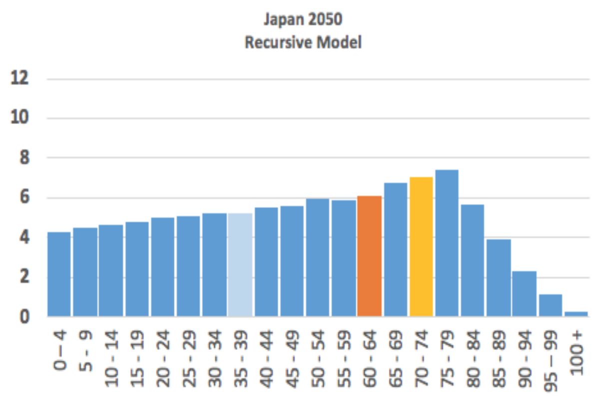
19. Provide an explanation of events in the United States that would result in the changes entered in the recursive model.

Consider other changes to the recursive model that you think will result in a closer match to the Census projections. If you have access to the spreadsheet file, revise either the population factors or the foundation factors that you think will result in a closer match of the two models. Keep track of your changes.

Japan

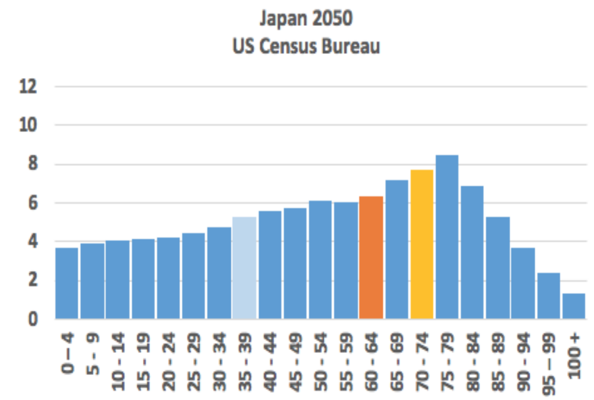
In a similar way, compare the projections from the recursive model to the projections of the United States Census Bureau's model for Japan's population:

Recursive model projections for Japan in 2050:

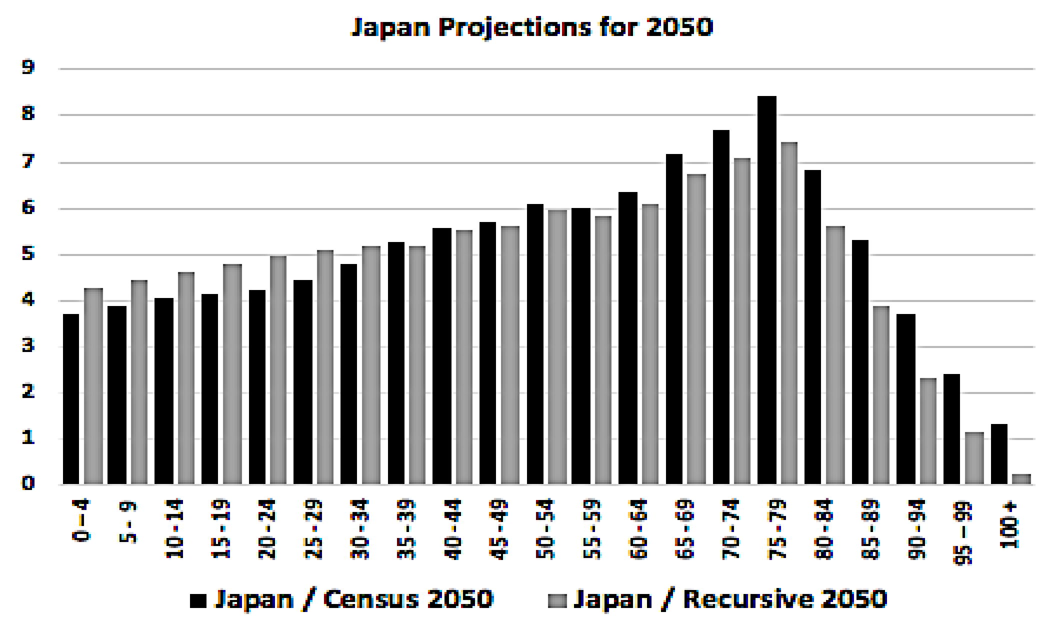


Total population: 102.14 million people

Census model projections for Japan in 2050:



Total population: 107.21 million people



20. Compare the two projection models of Japan's population in 2050 represented in the above histograms. Do you think the recursive projections and the Census projects are similar? Explain your answer.

21. Derive the percent increase of the total population in Japan projected by the Census model to the total population projected by the recursive model.

22. Identify at least 3 age groups in which the recursive model projections (color coded in light gray) are greater than the Census Bureau's projections (color coded in black).

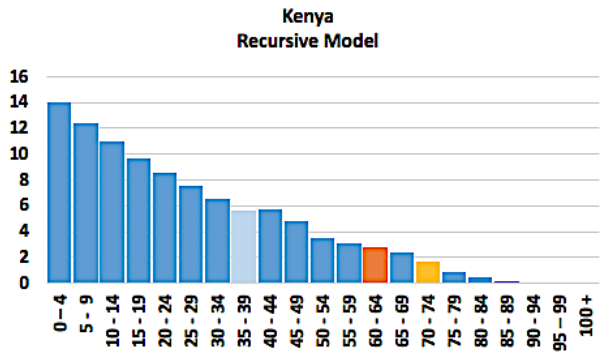
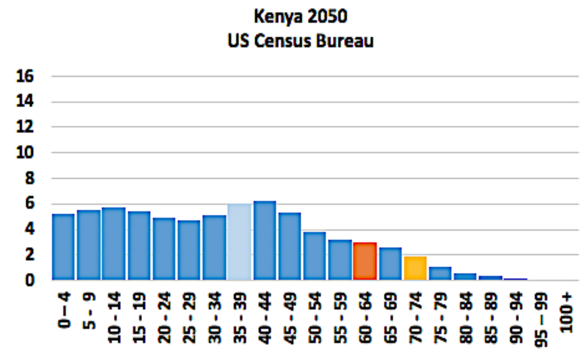
23. Identify at least 3 age groups in which the recursive model projections (color coded in light gray) are less than the Census projections (color coded in black).

24. Given the option to change the population factors or the foundation factors in the spreadsheet **Japan Recursive Model**, identify and enter revisions to the recursive model that you think will result in similar projections to the Census projections.

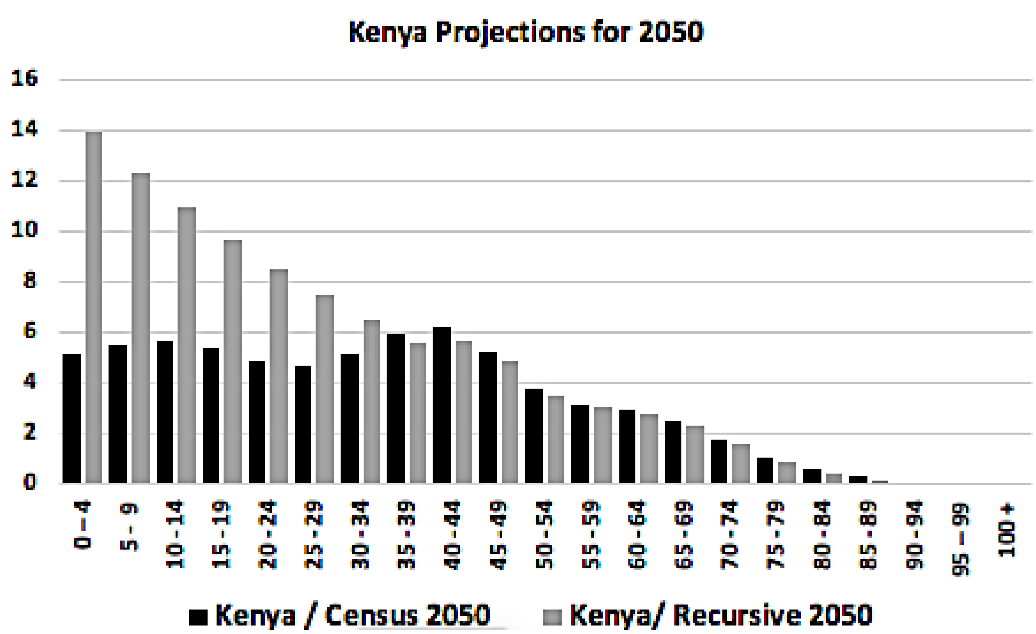
25. Based on the changes you proposed to the recursive model, what assumptions are you altering that you think will result in a better match to the Census Bureau's projections from 2020 to 2050 for Japan?

Kenya

Finally, compare the projections from the recursive model to the projections of the United States Census Bureau's model for Kenya's population:

Recursive model projections for Kenya in 2050:	Census model projections for Kenya in 2050:
	
Total population: 100.62 million people	Total population: 70.76million people

26.



26. Compare the two histograms of Kenya. The first histogram was prepared from the estimates of the recursive model. The second histogram was prepared from the projections of the Census Bureau. Are they similar? Explain your answer.

27. Given the option to change the population factors and the foundation factors in the spreadsheet **Kenya Recursive Model**, what changes do you think will result in a better match of the recursive model and the Census model in 2050?

28. Based on the changes you proposed to the recursive model, what assumptions are you making that are different from the original assumptions in the recursive model from 2020 to 2050?

29. Clearly the recursive model and the Census Bureau model differ the most for the Kenya predictions. Why do you think that the predictions by the Census Bureau for Kenya are different from the recursive model for Kenya?

Population projection models consider several other factors than just the population and foundation factors designed in the recursive model. Data obtained from surveys, death and birth records, health records, and several other data resources are also considered when building a population model. Each year in which the United States Census is conducted, a short survey is also distributed to a sample of households. This additional set of questions is referred to as the long-form. The questions people are asked vary from census to census. In addition, the questions are often met with political controversy.

30. Think of at least two questions you would include on a survey that might impact the assumptions included in a projection model. Indicate why you think the questions are related to projecting the future population counts.

Final problem for this project:

Identify and describe 3 reasons why an accurate population projection is important.

Wrap-up of the People Count Stories

Projects or Investigations

Consider completing one or more of the following projects to wrap-up your work with this module. Access to the spreadsheet files identified in the description of each project is needed to complete the projects. For each project, you will change the factors of the designated spreadsheet file to answer the questions or problems. A written summary of what you changed and why you made those changes is also expected.

Project 1: Estimating the Least Count of Immigration

Spreadsheet file needed to complete this project: USA Recursive Model

How many people during each of the 5-year periods from 2020 to 2050 are counted as immigrants if the recursive model is used to project future counts? Use the recursive model as designed in the spreadsheet ***USA Recursive Model*** to derive estimates of the least count of immigrants for each of the 5-years from 2020 to 2050. For example, what is an estimate of the least number of immigrants at the start of 2020 for the past 5 years if the recursive model is used to estimate future counts? In the same way, what is an estimate of the least number of immigrants at the start of 2025? 2030? … 2050?

Write a summary of how you revised the model and what were the estimates you obtained. Consider developing graphs to display the values over the 5-year intervals.

Project 2: Another Evaluation of the Recursive Model

Handout needed to complete this project: Handout 12: Looking Back to Evaluate the Recursive Model (United States)
Spreadsheet file needed to complete this project: Wrap-up Model.xlsx

Lesson 16 evaluated the recursive model by comparing (and revising) it estimates to the projections of the Census Bureau. The evaluation of which model is more accurate, however, requires waiting until 2050 at which point a census will be conducted.

Is there another way to evaluate the recursive model that would not require waiting until 2050? What if we used past counts provided by the Census Bureau (1980 and 1985), enter these counts in the recursive model, and then compare the projected results from the recursive model to the actual census counts reported in the census of 2010 and 2015?

The above plan provides an evaluation of the recursive model by looking back. Handout 12 provides the US Census counts (with estimates rounded off as indicated) for 1980 and 1985. The Excel spreadsheet **Wrap-up Model.xlsx** provides you the recursive model with columns set-up for the past. Each of these files are tools to assist you with the goals of this project. Enter the data from Handout 12 into the spreadsheet file to derive estimates for the US in 2010 and 2015. Are the estimates a good match to what was reported by the Census Bureau that summarized the census in 2010?

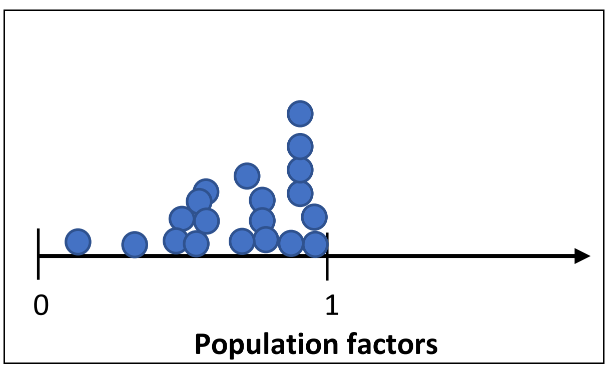
Write a summary that compares the estimates from the recursive model estimates to the reported Census counts. Indicate in your summary what you think happened in the country during those years that required changing the recursive model to match its outcomes to the reported census of 2010 and 2015. (For example, did more people die or leave the country during this time than the model projected? Were there more immigrants than anticipated by the model?)

Project 3: Population Factors are not the Whole Story of a Country

Spreadsheet file needed to complete this project: USA Recursive Model.xlsx

Two problems were presented in Lesson 10 that can be investigated further with the recursive model and the spreadsheet files. The first problem was the following:

1. Consider the following dot plot of the population factors of a fictitious country:



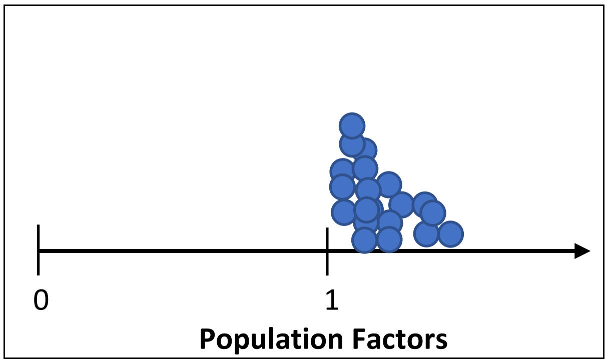
    a. What is the dominant explanation of change in the connecting age groups for a country represented by the above dot plot?

    b. Do you think it is possible for a country with the above population factors to have an increase in its total population during a 5-year period? Explain.

The dot plot indicates that all of the population factors derived in the recursive model were less than 1. Could the total population of country with the above population factors still grow over time? Design a fictitious country or obtain data for a real country to answer that question using the spreadsheet file.

The second problem in that lesson was the following:

2. Consider the following dot plot of the population factors for another fictitious country:



   a. What is the dominant explanation of change in the connecting age groups for the above dot plot?

   b. Do you think it is possible for a country with the above population factors to have a decrease in its total population during a 5-year period? Explain.

The dot plot for the second country indicates that all of the population factors derived were greater than 1. Could the total population of a country with the above population factors decline over time? Design a fictitious country or obtain data for a real country to answer that question using the spreadsheet file.

Write a report that indicates if you think countries could exist for each problem, and if yes, what were the counts and factors you used in setting up the population in these countries.

Teaching Notes

People Count! (and their data stories)

Handouts and Spreadsheet Files

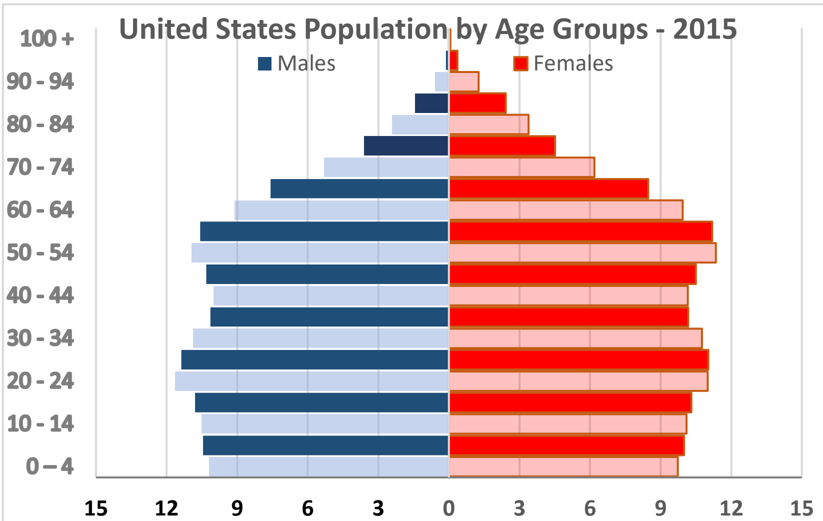
The following handouts are required for students to complete the problems. For each lesson, provide students a copy of the handouts that are identified in the opening sections of the Student Edition and the Teaching Notes.

Handout 1*: United States - 2015*
Handout 2*: Kenya - 2015*
Handout 3*: Japan - 2015*
Handout 4*: United States Connected Age Groups (Student Edition)*
Handout 4*: United States Connected Age Groups (Teacher Edition)*
Handout 5*:* Looking Forward for the United States *(Student Edition*
Handout 5*:* Looking Forward for the United States *(Teacher Edition*
Handout 6*: The United States 2010 – 2050 (Student Edition)*
Handout 6*: The United States 2010 – 2050 (Teacher Edition)*
Handout 7*: Kenya 2010 – 2050*
Handout 8*: Japan 2010 – 2050*
Handout 9*: The United States Project Worksheet*
Handout 10*:*: *Kenya Project Worksheet*
Handout 11: *Japan Project Worksheet*
Handout 12: *Looking Back to Evaluate the Recursive Model (United States)*
Handout 13: *Exit Summary*

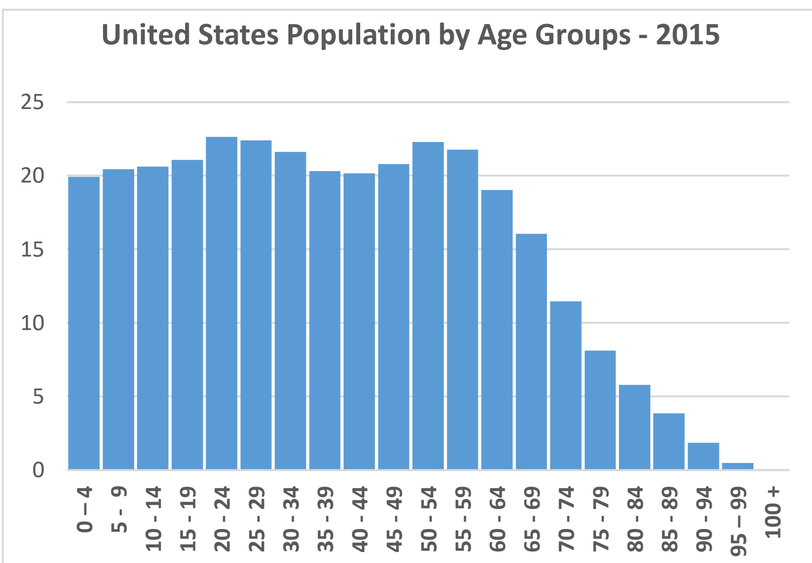
The following files have been created with the spreadsheet program EXCEL and are also identified in the opening sections of the Student Edition and the Teaching Notes for each lesson connected to a file. Students are provided an opportunity to interact with the recursive models and the resulting population projections by revising the data within the spreadsheet. Graphs are also embedded to help students understand the results of their revisions. **The files add opportunities for students as they complete the lessons but are not required to complete most of the lessons.**

“The 1 Country”.xlsx
USA Recursive Model.xlxs
Kenya Recursive Model.xlxs
Japan Recursive Model.xlxs
MyCountry Recursive Model.xlxs
Wrap-up Model.xlxs

Handout 1: United States - 2015



Population in millions of people



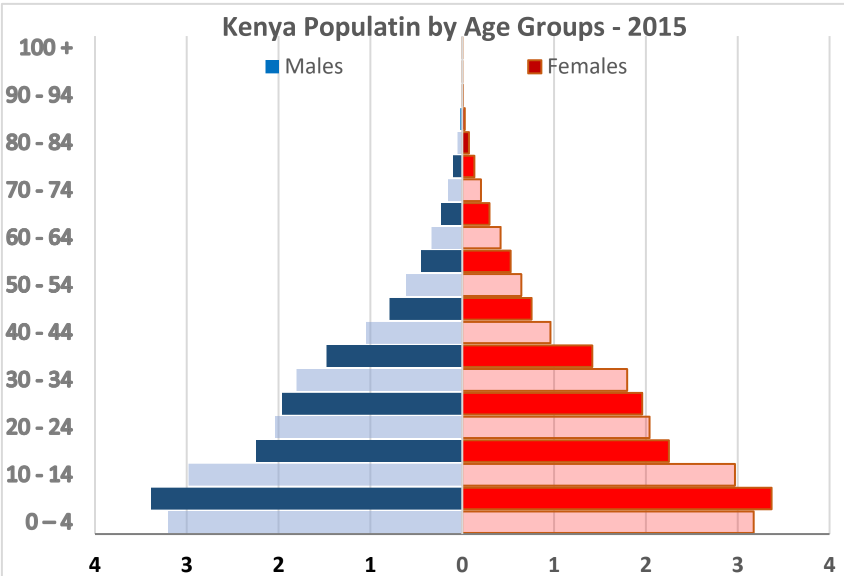
Age Groups

Population in millions of people

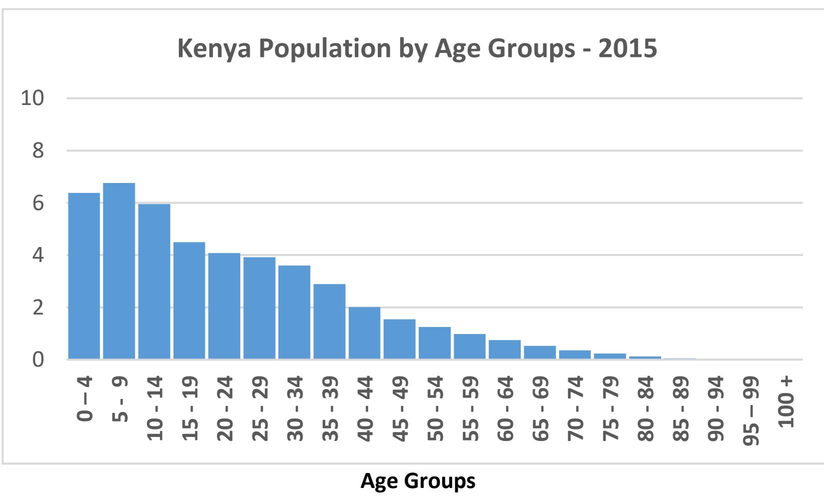
United States 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	10,180,651	9,731,848	19,912,499
5-9	10,455,213	10,025,917	20,481,130
10-14	10,511,459	10,094,120	20,605,579
15-19	10,785,425	10,299,285	21,084,710
20-24	11,644,934	11,048,092	22,693,026
25-29	11,378,910	11,022,258	22,401,168
30-34	10,860,962	10,756,571	21,617,533
35-39	10,142,472	10,170,174	20,312,646
40-44	10,001,650	10,155,086	20,156,736
45-49	10,308,560	10,492,596	20,801,156
50-54	10,941,453	11,348,281	22,289,734
55-59	10,577,216	11,190,639	21,767,855
60-64	9,100,418	9,938,136	19,038,554
65-69	7,585,362	8,463,884	16,049,246
70-74	5,292,456	6,185,320	11,477,776
75-79	3,608,308	4,511,539	8,119,847
80-84	2,411,880	3,387,030	5,798,910
85-89	1,441,268	2,423,021	3,864,289
90-94	588,497	1,263,123	1,851,620
95-99	127,836	367,526	495,362
100+	15,105	62,137	77,242
Total	157,960,035	162,936,583	320,896,618

Handout 2: Kenya - 2015



Population in millions of people



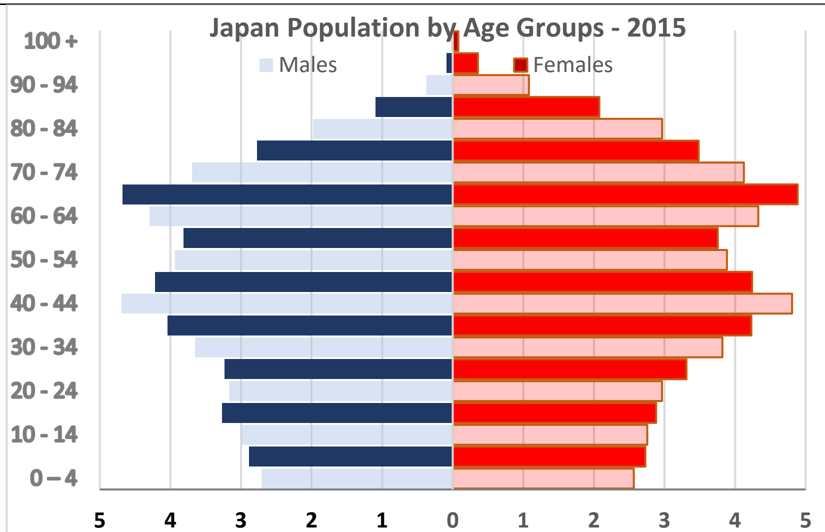
Population in millions of people

Kenya 2015

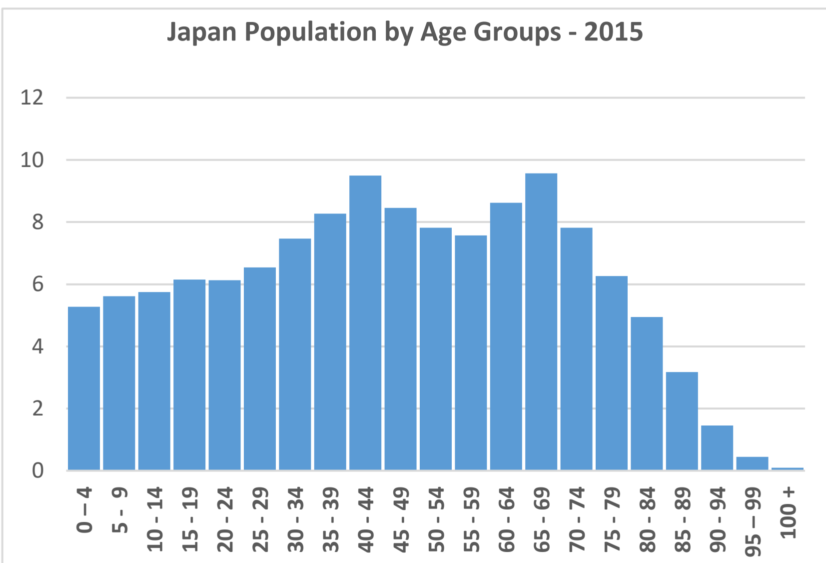
A Bottom-Layered Country

Age Group	Males	Females	Total
0-4	3,202,285	3,173,935	6,376,220
5-9	3,389,526	3,368,650	6,758,176
10-14	2,980,830	2,970,022	5,950,852
15-19	2,243,804	2,250,364	4,494,168
20-24	2,036,695	2,039,596	4,076,291
25-29	1,964,241	1,959,837	3,924,078
30-34	1,806,546	1,795,937	3,602,483
35-39	1,477,051	1,416,238	2,893,289
40-44	1,047,934	960,367	2,008,301
45-49	794,816	754,819	1,549,635
50-54	610,213	643,326	1,253,541
55-59	451,263	526,856	978,119
60-64	333,512	417,185	750,697
65-69	231,712	296,490	528,202
70-74	157,667	203,728	361,395
75-79	100,937	131,786	232,723
80-84	52,960	71,438	124,398
85-89	20,008	28,605	48,613
90-94	4,819	7,488	12,307
95-99	636	1,054	1,690
100+	45	78	123
Total	22,907,500	23,017,801	45,925,301

Handout 3: Japan - 2015



Population in millions of people



Age Groups

Population in millions of people

Japan 2015

An Upper Middle-Layered Country

Age Group	Males	Females	Total
0-4	2,707,598	2,565,400	5,272,998
5-9	2,882,367	2,729,721	5,612,088
10-14	2,992,683	2,756,585	5,749,268
15-19	3,269,467	2,881,921	6,151,388
20-24	3,167,481	2,964,887	6,132,368
25-29	3,231,011	3,311,576	6,542,587
30-34	3,650,718	3,821,036	7,471,754
35-39	4,037,243	4,230,960	8,268,203
40-44	4,692,392	4,807,537	9,499,929
45-49	4,216,126	4,241,648	8,457,774
50-54	3,936,931	3,885,016	7,821,947
55-59	3,810,051	3,755,442	7,565,493
60-64	4,294,784	4,328,875	8,623,659
65-69	4,679,676	4,887,715	9,567,391
70-74	3,693,158	4,124,955	7,818,113
75-79	2,775,712	3,484,536	6,260,248
80-84	1,980,330	2,966,549	4,946,879
85-89	1,094,666	2,077,183	3,171,849
90-94	373,121	1,079,680	1,452,801
95-99	84,247	357,337	441,584
100+	12,901	78,437	91,338
Total	61,582,663	65,336,996	126,919,659

Handout 4: United States Connected Age Groups (Student Edition)

Age group 2010 (Counted at the start of 2010)	**Connected Age Group in 2015** (Counted at the start of 2015)	**Population Factor Based on the Ratio of connected age groups**	**Decimal equivalent or Population Factor** (to the nearest thousandth)
0 – 4 20,189,589	5 – 9 20,481,130	$\frac{20{,}481{,}130}{20{,}189{,}589}$	1.014
5 – 9 20,331,807	10 – 14 20605579	$\frac{20{,}605{,}579}{20{,}331{,}807}$	1.013
10 – 14 20,681,215	15 – 19 21,084,710	$\frac{\quad}{20{,}681{,}215}$	1.020
15 – 19 21,983,206	20 – 24 22,693,026	$\frac{22{,}693{,}026}{\quad}$	1.032
20 – 24 21,704,549	25 – 29 22,401,168	$\frac{22{,}401{,}168}{21{,}704{,}549}$	1.032
25 – 29 21,145,232	30 – 34 21,617,533	$\frac{21{,}617{,}533}{21{,}145{,}232}$	
30 – 34 20,070,096	35 – 39 20,312,646	$\frac{20{,}312{,}646}{20{,}070{,}096}$	1.012
35 – 39 20,079,840	40 – 44 20,156,736	$\frac{\quad}{20{,}079{,}840}$	
40 – 44 20,905,848	45 – 49 20,801,156	$\frac{20{,}801{,}156}{20{,}905{,}848}$	0.995
45 – 49 22,637,291	50 – 54 22,289,734	—	
50 – 54 22,353,471	55 – 59 21,767,855	$\frac{\quad}{22{,}353{,}471}$	0.974
55 – 59 19,795,182	60 – 64 19,038,554	$\frac{19{,}038{,}554}{19{,}795{,}182}$	0.962
60 – 64 16,990,224	65– 69 16,049,246	$\frac{16{,}049{,}246}{16{,}990{,}224}$	0.945
65 – 69 12,521,439	70 – 74 11,477,776	$\frac{11{,}477{,}776}{12{,}521{,}439}$	0.917

70 – 74 9,336,583	75 – 79 8,119,847	$\frac{8,119,847}{}$	0.870
75 – 79 7,320,106	80– 84 5,798,910	$\frac{5,798,910}{7,320,106}$	
80 – 84 5,759,428	85 – 89 3,864,289	$\frac{3,864,289}{5,759,428}$	0.671
85 – 89 3,640,827	90– 94 1,851,620	$\frac{1,851,620}{}$	0.509
90 – 94 1,471,494	95 – 99 495,362	$\frac{495,362}{1,471,494}$	0.337
95 – 99 376,356	100 + 77,242	$\frac{}{376,356}$	0.205

Handout 5: Looking Forward for the United States
Student Edition

		Actual Counts:		Projections:						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	1.014	20.19	19.91							
5 - 9	1.014	20.33	20.48	20.20						
10 - 14	1.020	20.68	20.61	20.76	20.47					
15 - 19	1.032	21.98	21.09		21.17	20.88				
20 - 24	1.032	21.70	22.69	21.77	21.70	21.86	21.55			
25 - 29	1.022	21.15	22.40	23.42	22.47	22.40	22.56	22.25		
30 - 34	1.012	20.07	21.62	22.90	23.94	22.97	22.90	23.06	22.74	
35 - 39	1.004	20.08	20.31	21.88	23.17		23.25		23.34	
40 - 44	0.995	20.91	20.16	20.39		23.26		23.34	23.26	23.43
45 - 49	0.985	22.64	20.80	20.05			23.14	24.20	23.22	23.14
50 - 54	0.974	22.35	22.29	20.48	19.74	19.97	21.51	22.78		22.86
55 - 59	0.962	19.80	21.77	21.71	19.95	19.23	19.45	20.95		23.20
60 - 64	0.945	16.99	19.04	20.93		19.18	18.49	18.71	20.15	21.34
65 - 69	0.917	12.52	16.05	17.99	19.78	19.72		17.47	17.67	19.03
70 - 74	0.869	9.34	11.48	14.72	16.49	18.13	18.08	16.61	16.02	
75 - 79	0.792	7.32	8.12	9.98	12.79		15.76		14.44	13.93
80 - 84	0.670	5.76	5.80	6.43	7.91	10.14	11.36	12.49	12.46	11.45
85 - 89	0.508	3.64	3.86		4.31	5.30	6.79	7.61	8.37	
90 - 94	0.340	1.47	1.85	1.96	1.98	2.19	2.69		3.87	4.25
95 – 99	0.205	0.38	0.50	0.63	0.67	0.67	0.75	0.92	1.17	1.32
100 +		0.05	0.08	0.10	0.13	0.14	0.14	0.15	0.19	0.24

Key

Adeline	
Abbey	

Kristin	
Kristin's mother	

Handout 6: The United States 2010 – 2050
Student Edition

		Foundation Factors:								
		0.065	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	1.014	20.19	19.91			21.75	22.23	22.63	22.97	23.29
5 - 9	1.014	20.33	20.48	20.20			22.07	22.55	22.95	23.30
10 - 14	1.020	20.68	20.61	20.76	20.47	21.15	21.80	22.37	22.86	23.27
15 - 19	1.032	21.98	21.09	21.02	21.17	20.88	21.57	22.23	22.81	23.31
20 - 24	1.032	21.70	22.69	21.77	21.70	21.86	21.55		22.95	23.55
25 - 29	1.022	21.15	22.40	23.42	22.47	22.40	22.56	22.25	22.99	
30 - 34	1.012	20.07	21.62	22.90	23.94	22.97	22.90	23.06	22.74	23.50
35 - 39	1.004	20.08	20.31	21.88	23.17	24.23	23.25	23.17	23.34	23.02
40 - 44	0.995	20.91	20.16	20.39	21.97	23.26		23.34	23.26	23.43
45 - 49	0.985	22.64	20.80	20.05	20.28	21.85	23.14		23.22	23.14
50 - 54	0.974	22.35	22.29	20.48	19.74	19.97	21.51	22.78		22.86
55 - 59	0.962	19.80	21.77	21.71	19.95	19.23	19.45	20.95	22.19	23.20
60 - 64	0.945	16.99	19.04	20.93	20.88	19.18	18.49	18.71	20.15	21.34
65 - 69	0.917	12.52	16.05	17.99	19.78	19.72	18.12	17.47	17.67	19.03
70 - 74	0.869	9.34	11.48	14.72	16.49	18.13	18.08	16.61	16.02	16.20
75 - 79	0.792	7.32	8.12	9.98	12.79	14.34	15.76	15.72	14.44	13.93
80 - 84	0.670	5.76	5.80	6.43	7.91	10.14	11.36	12.49	12.46	11.45
85 - 89	0.508	3.64	3.86	3.89	4.31	5.30	6.79	7.61	8.37	8.35
90 - 94	0.340	1.47	1.85	1.96	1.98	2.19	2.69	3.45	3.87	4.25
95 – 99	0.205	0.38	0.50	0.63	0.67	0.67	0.75	0.92	1.17	1.32
100 +		0.05	0.08	0.10	0.13	0.14	0.14	0.15	0.19	0.24
	Totals	309.35	320.91		341.87	350.87	358.55	364.94	370.45	375.67
	Key:	Adeline		Abbey		Kristin		Parent		

Handout 7: Kenya 2010 - 2050

		Foundation Factors:								
		0.168	0.139	0.139	0.139	0.139	0.139	0.139	0.139	0.139
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	0.984	6.87	6.38	7.17	8.04	9.01	10.08	11.25	12.55	13.99
5 - 9	0.990	6.01	6.76	6.28	7.05	7.91	8.86	9.91	11.07	12.35
10 - 14	0.987	4.55	5.95	6.69	6.22	6.98	7.83	8.77	9.81	10.96
15 - 19	0.988	4.13	4.49	5.87	6.60	6.13	6.89	7.73	8.66	9.69
20 - 24	0.982	3.99	4.08	4.44	5.80	6.52	6.06	6.81	7.64	8.55
25 - 29	0.973	3.70	3.92	4.01	4.36	5.70	6.41	5.95	6.69	7.50
30 - 34	0.963	3.00	3.6	3.81	3.90	4.24	5.54	6.24	5.79	6.51
35 - 39	0.957	2.10	2.89	3.47	3.67	3.76	4.08	5.34	6.01	5.58
40 - 44	0.951	1.63	2.01	2.77	3.32	3.52	3.60	3.91	5.11	5.75
45 - 49	0.947	1.32	1.55	1.91	2.63	3.16	3.34	3.42	3.72	4.86
50 - 54	0.942	1.04	1.25	1.47	1.81	2.49	2.99	3.17	3.24	3.52
55 - 59	0.926	0.81	0.98	1.18	1.38	1.71	2.35	2.82	2.98	3.05
60 - 64	0.898	0.59	0.75	0.91	1.09	1.28	1.58	2.17	2.61	2.76
65 - 69	0.837	0.43	0.53	0.67	0.82	0.98	1.15	1.42	1.95	2.34
70 - 74	0.742	0.31	0.36	0.44	0.56	0.68	0.82	0.96	1.19	1.63
75 - 79	0.600	0.20	0.23	0.27	0.33	0.42	0.51	0.61	0.71	0.88
80 - 84	0.500	0.10	0.12	0.14	0.16	0.20	0.25	0.30	0.37	0.43
85 - 89	0.250	0.04	0.05	0.06	0.07	0.08	0.10	0.13	0.15	0.18
90 - 94	1.000	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.04
95 – 99	0.205	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03
100 +		0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	Totals	40.84	45.93	51.57	57.85	64.80	72.48	80.96	90.32	100.62

Handout 8: Japan 2010 - 2050

Age Groups	Population Factors	2010	2015	2020	2025	2030	2035	2040	2045	2050
	Foundation Factors:									
		0.044	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042
		Actual Counts:		**Projections:**						
0 – 4	0.996	5.63	5.27	5.26	5.15	5.01	4.84	4.66	4.47	4.29
5 - 9	0.998	5.76	5.61	5.25	5.24	5.13	4.99	4.82	4.64	4.46
10 - 14	0.998	6.16	5.75	5.60	5.24	5.23	5.12	4.98	4.81	4.63
15 - 19	1.000	6.13	6.15	5.74	5.59	5.23	5.22	5.11	4.97	4.81
20 - 24	0.998	6.55	6.13	6.15	5.74	5.59	5.23	5.22	5.11	4.97
25 - 29	0.996	7.50	6.54	6.12	6.14	5.73	5.58	5.23	5.22	5.11
30 - 34	0.996	8.30	7.47	6.51	6.10	6.12	5.71	5.56	5.20	5.19
35 - 39	0.995	9.55	8.27	7.44	6.49	6.07	6.09	5.69	5.54	5.19
40 - 44	0.993	8.52	9.50	8.23	7.40	6.46	6.04	6.06	5.66	5.51
45 - 49	0.989	7.91	8.46	9.43	8.17	7.35	6.41	6.00	6.02	5.62
50 - 54	0.984	7.69	7.82	8.36	9.33	8.08	7.27	6.34	5.93	5.95
55 - 59	0.975	8.84	7.57	7.70	8.23	9.18	7.95	7.15	6.24	5.84
60 - 64	0.965	9.92	8.62	7.38	7.51	8.03	8.95	7.75	6.98	6.08
65 - 69	0.946	8.27	9.57	8.32	7.12	7.24	7.75	8.64	7.48	6.73
70 - 74	0.909	6.89	7.82	9.05	7.86	6.73	6.85	7.32	8.17	7.07
75 - 79	0.845	5.86	6.26	7.10	8.22	7.14	6.12	6.22	6.65	7.42
80 - 84	0.742	4.27	4.95	5.29	6.00	6.95	6.03	5.17	5.26	5.62
85 - 89	0.604	2.40	3.17	3.67	3.93	4.46	5.16	4.48	3.84	3.90
90 - 94	0.427	1.03	1.45	1.92	2.22	2.37	2.69	3.12	2.71	2.32
95 – 99	0.205	0.33	0.44	0.62	0.82	0.95	1.01	1.15	1.33	1.16
100 +		0.05	0.09	0.09	0.13	0.17	0.19	0.21	0.24	0.27
	Totals	127.56	126.91	125.24	122.63	119.22	115.22	110.88	106.46	102.14

Handout 9: The United States Project Worksheet
(Planning worksheet)

		Foundation Factors:								
		0.065	**0.062**							
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4		20.19	19.91							
5 - 9		20.33	20.48							
10 - 14		20.68	20.61							
15 - 19		21.98	21.09							
20 - 24		21.70	22.69							
25 - 29		21.15	22.40							
30 - 34		20.07	21.62							
35 - 39		20.08	20.31							
40 - 44		20.91	20.16							
45 - 49		22.64	20.80							
50 - 54		22.35	22.29							
55 - 59		19.80	21.77							
60 - 64		16.99	19.04							
65 - 69		12.52	16.05							
70 - 74		9.34	11.48							
75 - 79		7.32	8.12							
80 - 84		5.76	5.80							
85 - 89		3.64	3.86							
90 - 94		1.47	1.85							
95 – 99		0.38	0.50							
100 +		0.05	0.08							
	Totals	309.35	320.91							
	Key:	Adeline		Abbey		Kristin		Parent		

Handout 10: Kenya Project Worksheet
(Planning worksheet)

		Foundation Factors:								
		0.168	0.139	0	0	0	0	0	0	0
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4		6.87	6.38							
5 - 9		6.01	6.76							
10 - 14		4.55	5.95							
15 - 19		4.13	4.49							
20 - 24		3.99	4.08							
25 - 29		3.70	3.92							
30 - 34		3.00	3.6							
35 - 39		2.10	2.89							
40 - 44		1.63	2.01							
45 - 49		1.32	1.55							
50 - 54		1.04	1.25							
55 - 59		0.81	0.98							
60 - 64		0.59	0.75							
65 - 69		0.43	0.53							
70 - 74		0.31	0.36							
75 - 79		0.20	0.23							
80 - 84		0.10	0.12							
85 - 89		0.04	0.05							
90 - 94		0.01	0.01							
95 – 99		0.01	0.01							
100 +		0.00	0.01							
	Totals	40.84	45.93							

Handout 11: Japan Project Worksheet
(Planning worksheet)

		Foundation Factors:								
		0.044	0.042							
		Actual Counts:		Projections:						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4		5.63	5.27							
4 - 9		5.76	5.61							
10 - 14		6.16	5.75							
15 - 19		6.13	6.15							
20 - 24		6.55	6.13							
25 - 29		7.50	6.54							
30 - 34		8.30	7.47							
35 - 39		9.55	8.27							
40 - 44		8.52	9.50							
45 - 49		7.91	8.46							
50 - 54		7.69	7.82							
55 - 59		8.84	7.57							
60 - 64		9.92	8.62							
65 - 69		8.27	9.57							
70 - 74		6.89	7.82							
75 - 79		5.86	6.26							
80 - 84		4.27	4.95							
85 - 89		2.40	3.17							
90 - 94		1.03	1.45							
95 – 99		0.33	0.44							
100 +		0.05	0.09							
	Totals	127.56	126.91							

Handout 12: Looking Back to Evaluate the Recursive Model (United States)
Wrap-Up – Looking Back

		Foundation Factors:								
		Actual Counts:		Projections:						
Age Groups	Population Factors	1980	1985	1990	1995	2000	2005	2010	2015	2020
0 – 4		16.45	17.84							
5 - 9		16.60	16.66							
10 - 14		18.24	17.03							
15 - 19		21.11	18.73							
20 - 24		21.39	21.26							
25 - 29		19.69	21.67							
30 - 34		17.74	20.03							
35 - 39		14.08	17.60							
40 - 44		11.73	14.09							
45 - 49		11.05	11.61							
50 - 54		11.69	10.85							
55 - 59		11.61	11.23							
60 - 64		10.14	10.91							
65 - 69		8.81	9.34							
70 - 74		6.84	7.52							
75 - 79		4.83	5.51							
80 - 84		2.96	3.38							
85 - 89		1.58	1.77							
90 - 94		0.56	0.70							
95 – 99		0.12	0.18							
100 +		0.02	0.03							
	Totals	227.24	237.94							

Handout 13: People Count! (and their data stories)

Exit Summary

Name:

Lesson Number:

At your teacher's discretion, identify problems or questions in this lesson that you answered by using one or more of the levels of the Modeling Continuum. Within the column of the level or levels you identified, explain the steps you used to answer the questions or problems.

The Modeling Continuum

Level 1	Level 2	Level 3	Level 4
Identifying or extracting data from data sets or projections.	Summarizing data and projections from tables or graphs.	Interpreting the tools (for example, population factors, foundation factors, proportions) that are used to derive projections addressed in the lessons.	Reworking and modifying the tools used to make projections by addressing "What if …?" questions.
Answering questions directly from the presented data. Answering "What is …?" questions.	Summarizing data or outcomes in your own words. Answering "What is …?" questions using proportions, percent, or relative frequencies.	Answering questions or problems that require using the tools discussed in the lessons. Calculating new outcomes of a country's population based on changes in a country's immigration, births, and deaths.	Modifying the tools presented in the lessons that result in new population projections for real or fictitious countries. Answering questions that are a result of the modifications of a country's future population projections.

Level 1	Level 2	Level 3	Level 4
Problem:	**Problem:**	**Problem:**	**Problem:**