Focus on Statistics

Investigations for the Integration of Statistics into Grades 9-12 Mathematics Classrooms

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Foreword

I am honored to write the foreword for *Focus on Statistics: Investigations for the Integration of Statistics into Grades 9–12 Mathematics Classrooms,* a collection of data-centric investigations for high-school students.

Statistics is recognized as a fundamental component in the K–12 mathematics curriculum. Examinations of current state standards, many based on the Common Core State Standards (CCSS), and high-stakes national assessments such as the SAT, ACT, and National Assessment of Educational Progress (NAEP) reflect the importance of statistical literacy for all students. Continuing changes in and improvements to standards and assessments call for developing resources such as *Focus on Statistics*.

In the 2018 National Council of Teachers of Mathematics publication *Catalyzing Change in High School Mathematics*, the authors state, "Statistics and probability concepts that are essential for all high-school students support their ability to analyze data, to engage in informal statistical inference, and to understand conditional probability and independence insofar as these relate to statistical thinking. Students should also leave high school with the skills necessary to be quantitatively literate, capable of reasoning with and making sense of quantitative information in order to inform the decisions that they must make now and in the future."

The investigations in *Focus on Statistics* provide classroom teachers and their students with experiences to reinforce the process of statistical reasoning that is so important in making informed decisions.

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Prior to the publication of the CCSS and *Catalyzing Change in High School*

Mathematics, the American Statistical Association (ASA) produced *Guidelines for Assessment and Instruction in Statistics Education (GAISE): A Pre-K–12 Curriculum Framework,* which was approved by the ASA in 2005 (*www.amstat.org/education/gaise*). The ASA/NCTM Joint Committee on Curriculum in Statistics and Probability in 2007 worked with the GAISE Framework authors to incorporate final editing and provide funding for printing the report. GAISE 2 will be released in 2020, keeping the spirit of the original GAISE but updating with respect to advances in technology, the wealth of big data, and the importance of the statistical problem-solving process particularly related to the role of questioning in statistics.

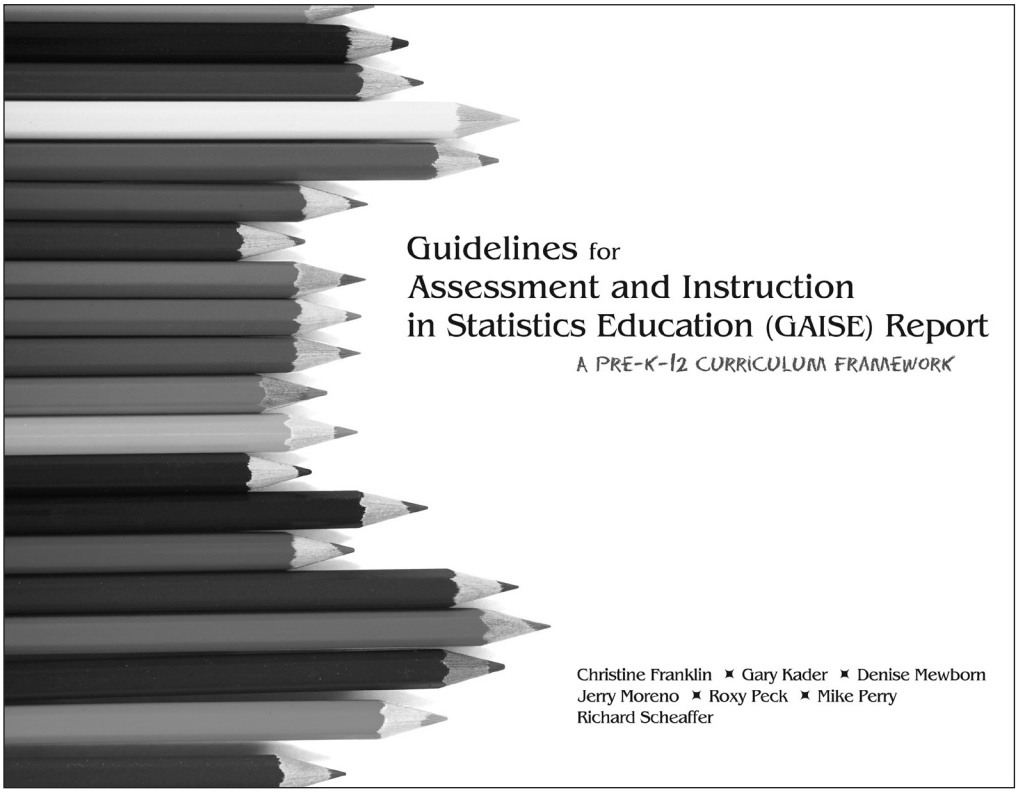
The goals of the GAISE and GAISE 2 Framework are the following:

- Present the statistics curriculum for grades pre-K–12 as a cohesive and coherent curriculum strand (e.g., the progression of the mean from elementary to middle to secondary)
- Promote and develop statistical literacy for all students before graduating secondary school
- Provide links with the NCTM 2000 *Principles and Standards for School Mathematics* and 2018 *Catalyzing Change: Initiating Critical Conversations*
- Discuss differences between mathematical and statistical thinking, particularly the importance of context and variability within statistical thinking
- Clarify the role of probability in statistics
- Illustrate concepts associated with the statistical problem-solving process

The GAISE Framework also reinforced the need for data literacy in K–12:

- Every high-school graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy and productive life.
- Statistics education can promote the 'must-have' competencies for high-school graduates to thrive in this modern world of mass information.
- The importance of student ability to think statistically. The well-known mathematician George Polya said, “Plausible reasoning—the inferential reasoning of science and everyday life by which new knowledge is obtained—is an important part of mathematical reasoning.”

The GAISE document outlines the conceptual structure for statistics education in a two-dimensional framework model with one dimension defined by



The *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework* was the basis for the statistics and probability conceptual category included in the Common Core State Standards in mathematics when they were released in 2010. *Focus on Statistics* follows both the GAISE Framework and the Common Core State Standards grades 9–12.

the four-step problem-solving process (formulate questions, collect data, analyze data, and interpret results), plus the nature of variability. The second dimension is comprised of three levels of statistical development (Levels A, B, and C) that students must progress through to develop statistical understanding. Grade ranges for attainment of each level are intentionally unspecified. Students must begin and master the concepts at Level A before moving on to Levels B and C. It is paramount for students to have worthwhile experiences at Levels A and B during their elementary school years to prepare for development at Level C at the secondary level. Without such experiences, a middle- (or high-) school student who has had no experience with statistics will need to begin with Level A concepts and activities before moving to Level B.

The GAISE Framework has become an instrumental document in providing

guidance to writers of national mathematics documents, state standards, and assessment items; curriculum directors; pre-K–12 teachers; and faculty of teacher preparation colleges on the essential topics and concepts in data analysis and probability for all students as they progress from kindergarten to graduation from high school.

The GAISE Framework has influenced the statistics components of both the Mathematics and Statistics College Board Standards for College Success (2007) and the NCTM document *Focus on High School Mathematics* (2008). The GAISE Framework also influenced the statistics and probability strand of many state mathematics standard revisions (which includes my home state of Georgia). The GAISE Framework was the basis for the statistics and probability conceptual category included in the Common Core State Standards in mathematics when these standards were released in 2010. The GAISE Framework also influenced the LOCUS (Levels of Conceptual Understanding in Statistics), an NSF-funded project that developed assessments to measure students' understanding across levels of development as identified in GAISE. GAISE also influenced the NCTM's *Catalyzing Change in High School Mathematics*. Internationally, the GAISE Framework has been influential. There is now a Spanish version of the GAISE Framework.

Focus on Statistics is an excellent classroom resource that follows both the GAISE Framework and the Common Core State Standards grades 9–12. High school is critical in providing investigations that will further the skills needed for our students to grow and evolve into sound statistical thinkers. The investigations bring the real world to the student and provide the student the opportunity to understand the necessity of statistical reasoning and sense making for everyday life and post-secondary education.

I'm appreciative to the writers of *Focus on Statistics* and the ASA/NCTM Joint Committee for developing this valuable resource in support of the recommendations of GAISE, the recommendations of the Common Core State Standards, *Catalyzing Change in High School Mathematics*, and statistical reasoning in our high-school curriculum.

Christine Franklin
ASA K–12 Statistical Ambassador

Acknowledgments

We are indebted to the ASA/NCTM Joint Committee on Curriculum in Statistics and Probability for its support throughout the process of creating and publishing *Focus on Statistics*. This project is a continuing effort of the Joint Committee to provide classroom-ready investigations written in the spirit of the GAISE framework.

The ASA has been involved in numerous publications, including *Making Sense of Statistical Studies* (15 high-school activities on surveys, observational studies, and experiments) and *Bridging the Gap* (*BTG*), which focused on statistical investigations for grades K–8.

In late 2016, the Joint Committee approved Sara Brown, Pat Hopfensperger, and Henry Kranendonk as the main writers of *Focus on Statistics*. This publication expands on the activities presented in *BTG* and activities found in the Data-Driven Mathematics (DDM) series and Quantitative Literacy Series (QL).

Sincere thanks are extended to Christine Franklin and Anna Bargagoliotti for their support expressed in the foreword and for their section on clarifying what constitutes an investigative question.

This publication expands on the activities presented in *Bridging the Gap* and activities found in the Data-Driven Mathematics series and Quantitative Literacy Series.

Each investigation was reviewed by three high-school teachers and two statistics educators. We are thankful for their excellent comments and suggestions, which improved our writing significantly. The reviewers included Jerry Moreno of John Carroll University; Anna Fergusson of the University of Auckland, New Zealand; Alex Blohm of Loyola University Chicago; Melissa Hongsermeier of South Milwaukee High School; and Patricia Talarczyk of Mentor High School, Mentor, Ohio.

We are deeply in debt to Jerry Moreno, who was our main editor. He spent

countless hours editing and giving excellent suggestions for improving each investigation.

We also extend our gratitude to Anna Fergusson, who did a remarkable job producing graphs of publishable quality for *Focus on Statistics* and providing deep dedication to and support of Joint Committee efforts.

We give special thanks to ASA Editor and Content Strategist Valerie Nirala, whose editorial and design magic brought life to our writings, without which much of what we had to offer would have lacked reader appeal.

And finally, a note of appreciation to ASA Director of Education Rebecca Nichols, whose leadership and direction were much appreciated in helping us achieve our publication goals.

Sara Brown
Pat Hopfensperger
Henry Kranendonk

About Focus on Statistics

Focus on Statistics consists of 19 investigations in statistics for grades 9–12. It is written to help classroom teachers implement key statistical concepts in their classrooms. Each investigation consists of the following headings appropriately written for its specific content:

- Overview (including suggested GAISE level)
- Learning Goals
- Mathematical Practices Through a Statistical Lens
- Materials
- Estimated Time
- Pre-Knowledge
- Instructional Plan (consisting of a brief overview and the four steps of the GAISE process)
- Exit Ticket
- Extensions and Additional Ideas

The 19 investigations are separated into the following topic sections:

- **Section 1:** Getting Started
  Includes one investigation on questioning through the investigative process
- **Section 2:** One-Variable Data Analysis
  Includes four investigations, 1–4
- **Section 3:** Two-Variable Data Analysis
  Includes seven investigations, 5–11
- **Section 4:** Probability
  Includes three investigations, 12–14
- **Section 5:** Inference
  Includes four investigations, 15–18
- **Section 6:** Teacher Resources
  Includes overview of ASA online resources

Included in the 19 investigations are three exploratory investigations that encourage students to collect and analyze their own data. In Investigation 4, students collect data from the Census at School website involving the amount of homework for different grade levels. In Investigation 8, students collect data on two quantitative variables involving the length of time for dominoes to fall. In Investigation 18, students use the Census at School website to collect and compare data pertaining to stress levels for students in the US and New Zealand.

In addition, reference to various standards from "Mathematical Practices

Estimated Time Guide

Investigation	Estimated Class Periods (50-minute class period)
Questioning and statistical problem-solving process	1
1. Graphical Displays	1–2
2. Comparing Multiple Groups	1–2
3. Standard Deviation	2
4. Exploratory - Homework	2
5. Scatterplots	1–2
6. Correlation	2
7. Residual Plots	2
8. Exploratory - Dominoes	2
9. Taking a Survey	1
10. Two-Way Tables	2
11. Independent Events	2
12. Simulation	1
13. Expected Value	1
14. Normal Distribution	1–2
15. Sampling Distribution	1–2
16. Testing a Claim	1
17. Bootstrapping	1
18. Exploratory - Difference Between Two Proportions	2

Through a Statistical Lens" is made. Each investigation explicitly contains the four components of the problem-solving process presented in the American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework* (*www.amstat.org/education/gaise*). The GAISE framework emphasizes hands-on learning of statistics by using four steps: formulating a statistical question that can be answered with data; designing and implementing a plan to collect appropriate data; analyzing the collected data by graphical and numerical methods; and interpreting the results of the analysis in the context of the original question. A second component of the GAISE framework is comprised of three levels of statistical development:

A, B, and C. These levels are independent of age and grade level. Students progress through these levels as they continue to have more experiences with concepts of statistics and probability.

Each investigation encourages student involvement through the use of student worksheets. These worksheets provide guidance as students, working in groups, follow the four statistical problem-solving steps. The worksheets are available as a word document at *www.statisticsteacher.org/statistics-teacher-publications/focus*.

A brief overview of each section follows:

Section 1: Getting Started

This investigation introduces the different types of questions used throughout the statistical problem-solving process. The primary focus is exploring what constitutes a well-written investigative question—one that can be answered with data.

Section 2: One-Variable Data Analysis

In this section, dot plots and box plots are used to display data. The standard deviation is explored as a measure of variation.

Section 3: Two-Variable Data Analysis

In the first part of this section, the concepts of correlation and the least squares regression line are developed. In the second part, the concept of association between two categorical variables is developed. In addition, the concept of independent events is investigated.

Section 4: Probability

The GAISE framework views probability as a mathematical model and a tool for statistics. This section develops the concept of a probability distribution, expected value, and the normal distribution.

Section 5: Inference

This section develops the concept of a sampling distribution of sample proportions, testing a claim about a proportion, and using the bootstrapping method to develop a confidence interval.

Section 6: Resources

This section contains teacher resources available on the American Statistical Association website. Statistics Education Web, Statistics Teacher Network, Census at School, statistics education webinars, and publications pertaining to K–12 education are included.

Focus on Statistics is designed so each lesson can stand alone. Our goal is to provide you with a resource to give you and your students data analysis experiences that bring the essential concepts in statistics to life. It also is designed to give you flexibility. Several investigations can be completed

in one to two 50-minute class periods. Many investigations suggest students collect data; if this data collection is done during class time, then additional class time is needed. However, the data collection could be completed outside of class time, as well. For planning purposes, the Estimated Time Guide shows an estimate of the time required to complete each investigation. Exit tickets are provided and could be used at the end of the lesson for formative assessment and/or extra practice. Extension activities would require additional class time.

Linking Investigations

GAISE, CCSS, Statistical Lens, and Catalyzing Change

Linking GAISE Levels and Focus on Statistics

Investigation	Level A	Level B	Level C
Questioning and Statistical Problem-Solving Process		x	
1. Graphical Displays	x		
2. Comparing Multiple Groups	x	x	
3. Standard Deviation		x	
4. Exploratory Lesson		x	
5. Scatterplots	x		
6. Correlation		x	
7. Assessing Linear Fit		x	
8. Exploratory Lesson		x	
9. Analyzing Categorical Data		x	
10. Summarizing Bivariate Categorical Data		x	
11. Comparing Conditional Relative Frequencies		x	x
12. Simulation		x	
13. Expected Value		x	
14. Normal Distribution			x
15. Sampling Distribution			x
16. Testing a Claim			x
17. Bootstrapping			x
18. Exploratory Lesson			x

Linking Grade Levels and Common Core State Standards

Investigation	Grades 6–8	High School
Questioning and Statistical Problem-Solving Process	6.SP.A.1	
1. Graphical Displays	6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5.C, 6.SP.B.5.D	HSS.ID. A.1, HSS.ID. A.2, HSS. ID. A.3
2. Comparing Multiple Groups	7.SP.B.3, 7.SP.B.4	HSS.ID. A.1, HSS.ID. A.2, HSS. ID. A.3
3. Standard Deviation		HSS.ID. A.2
4. Exploratory Lesson		Review of Standards from Lessons 1 to 3
5. Scatterplots	8.SP.A.1	HSS.ID.B.6
6. Correlation	8.SP.A.1, 8.SP.A.2	HSS.ID.B.6, HSS.ID.C.8
7. Assessing Linear Fit	8.SP.A.3	HSS.ID.B.6.B, HSSS.ID.C.7
8. Exploratory Lesson		Review of Standards from Lessons 5 to 7
9. Analyzing Categorical Data	8.SP.A.4	HSS.IC.B.3, HSSS.IC.A.1
10. Summarizing Bivariate Categorical Data	8.SP.A.4	HSS.ID.B.5
11. Comparing Conditional Relative Frequencies		HSS.CP.A.4, HSS.CP.A.5
12. Simulation	7.SP.C.8.C	HSS.IC.A.2
13. Expected Value		HSS.MD.A.2, HSS.MD.A.4
14. Normal Distribution		HSS.ID.A.4
15. Sampling Distribution	7.SP.A.2	HSS.IC.A.1
16. Testing a Claim		HSS.IC.A.2
17. Bootstrapping		HSS.IC.B.4
18. Exploratory Lesson		HSS.IC.B.5

The Statistical Education of Teachers (SET) report outlines the content and conceptual understanding teachers need to know when assisting their students develop statistical reasoning skills. SET is intended for everyone involved in the statistical education of teachers, both the initial preparation of prospective teachers and the professional development of practicing teachers. PDF download at *www.amstat.org/asa/files/pdfs/EDU-SET.pdf*.

Linking Investigations to Mathematical Practices Through a Statistical Lens

Investigation	Mathematical Practices
Questioning and the Statistical Problem-Solving Process	MP 2. Reason abstractly and quantitatively
1. Graphical Displays	MP 2. Reason abstractly and quantitatively
2. Comparing Multiple Groups	MP 6. Attend to precision
3. Standard Deviation	MP 7. Look for and make use of structure
4. Exploratory Lesson	MP 1. Make sense of problems and persevere in solving them
5. Scatterplots	MP 3. Construct viable arguments and critique the reasoning of others
6. Correlation	MP 7. Look for and make use of structure
7. Assessing Linear Fit	MP 7. Look for and make use of structure
8. Exploratory Lesson	MP 1. Make sense of problems and persevere in solving them
9. Analyzing Categorical Data	MP 2. Reason abstractly and quantitatively
10. Summarizing Bivariate Categorical Data	MP 2. Reason abstractly and quantitatively
11. Comparing Conditional Relative Frequencies	MP 2. Reason abstractly and quantitatively
12. Simulation	MP 5. Use appropriate tools strategically
13. Expected Value	MP 4. Model with mathematics
14. Normal Distribution	MP 4. Model with mathematics
15. Sampling Distribution	MP 8. Look for and express regularity in repeated reasoning
16. Testing a Claim	MP 3. Construct viable arguments and critique the reasoning of others
17. Bootstrapping	MP 5. Use appropriate tools strategically
18. Exploratory Lesson	MP 1. Make sense of problems and persevere in solving them

The NCTM's *Catalyzing Change in High School Mathematics* provides the essential concepts in statistics and probability necessary for all high-school students to be statistically literate. Listed on the following page are the four statistics and probability focus areas presented in *Catalyzing Change in High School Mathematics* and the investigations whose goals match the concepts listed under each focus area.

Linking Investigations to NCTM's Catalyzing Change in High School Mathematics – Essential Concepts in Statistics and Probability

Essential Concepts	Investigation
Focus 1: Essential Concepts in Quantitative Literacy	
Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.	4, 8 and 18
Focus 2: Visualizing and Summarizing Data	
Distributions of quantitative data in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	1, 2, 3, 4, and 14
The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.	9 and 10
Scatterplots can reveal patterns, trends, clusters, and gaps that are useful in analyzing association between two contextual variables.	5
Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.	6, 7, and 8
Focus 3: Statistical Inference	
The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.	15
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic.	15, 16, and 17
Focus 4: Probability	
Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.	11 and 18
Conditional probabilities—that is, probabilities that are "conditioned" by some known information—can be computed from data organized in contingency tables.	10 and 11

Linking Investigations

Standard High-School Mathematics Curriculum

Section 1: Getting Started Investigation on Questioning Through an Investigative Process

This investigation should be used in a first-year high-school course that integrates topics of mathematics and statistics before starting a unit on statistics. The focus is on the four-step statistical problem-solving process and the role of questioning throughout. Criteria for identifying and writing an investigative question are introduced. Each of the following 18 investigations is designed around an investigative question. As a result, students' understanding of what constitutes an investigative question is important as a launch for each investigation.

Section 2: One-Variable Data Analysis Investigations 1–4

These four investigations may be used to teach or review a student's understanding of the skills and concepts used in communicating with data. The development of the investigation process helps students understand how data can be used to answer questions or present convincing arguments.

The investigations may be used after students have studied ratio, proportion, and percent or as part of a beginning unit on statistics. These investigations are also appropriate to use when students are beginning to formalize their work with variables. The investigations could be used as a first or second unit in a first-year high-school course that integrates topics of mathematics and statistics.

Section 3: Two-Variable Data Analysis Investigations 5–11

These investigations are about graphing and assessing the fit of linear functions to bivariate data. The investigations can be used in a first-year high-school course that integrates topics of mathematics and statistics in a variety of ways, most effectively when integrated into the unit on writing the equation of a line. They can also be used after students have completed a section on solving equations in one variable to illustrate how to apply those concepts in real-world contexts and provide investigations into graphical representations of linear relationships.

First-Year High-School Course Integrating Topics of Mathematics and Statistics

Topic	Investigations
Introduction to Statistics	1–4
Linear Functions	5–7 or 5–8
Statistics Unit or after Linear Functions	9 and 10

Second-Year High-School Course Integrating Topics of Mathematics and Statistics

Topic	Investigations
Linear Functions	5–8 or 6–8
Statistics Unit or after Linear Functions	9–11 or 10 and 11
Probability Unit	12–14
Probability Unit after Investigations 11–12	15–18

Investigations 6 to 8 can also be used in a second-year class that expands on the topics of mathematics and statistics when integrated into the unit on the study of functions.

Investigations 9 and 10 can be used in a first-year high-school course that integrates topics of mathematics and statistics after students have studied ratio, proportion, and percent or as part of a unit on statistics.

Investigations 10 and 11 can also be used in conjunction with a probability unit.

Section 4: Probability Investigations 12–14

The concepts on random variables, probability distributions, and expected values require subtle reasoning. It is recommended that these investigations—along with investigations 15–18—be used in a second-year course that expands on the topics of mathematics and statistics, or at least no earlier than late in a first-year high-school course that integrates topics of mathematics and statistics. This is more for the level of reasoning required than for any particular set of algebraic skills needed to do the work.

These three investigations can be used in a second-year course that expands on the topics of mathematics and statistics in conjunction with a probability unit. Investigation 14 can be used after students have studied functions.

Section 5: Inference Investigations 15–18

These four investigations can be used in a second-year high-school course after completing lessons on simulations (Investigation 12) and independent events (Investigation 11). The investigations can be part of a probability unit in a second-year course that expands on the topics of mathematics and statistics or an advanced math course. Again, the level of reasoning required is the reason these lessons are more suited to upper-level courses.