Overview of Module

People Count! (and their data stories) is a collection of 16 lessons for high school or college students, along with data stories and a quilt of countries. This module evolved from several projects originating in the author's computer science and International Baccalaureate (IB) Math Studies classes at Rufus King High School in Milwaukee, Wisconsin. Working with real data, even if the data is not initially of high interest to students, brings out a response that captures students' attention. The initial lessons were expanded based on students' input that ultimately resulted in this module.

People Count! is designed for students enrolled in most high school mathematics classes. Pre-requisite skills are essentially skills learned in a first year algebra course. This module also involves several Microsoft Excel files to complement several lessons and align with the modeling objectives described in this Overview. It is not necessary, however, for students to have access to the files to complete most of the lessons. (It might also be possible to use the files with other spreadsheet applications.) The lessons and files support the main objective of this module, namely modeling with data. Students develop, analyze, and redesign population projection models using past and present population totals by age groups to estimate future population estimates of various countries.

Included in this **Overview** is a general description of the lessons and ideas for using the lessons in a high school or first year mathematics/statistics course. In addition, an explanation of a **Modeling Continuum** is provided to guide the implementation of each lesson. References to the **Continuum** are also included in the **Teaching Notes** for each lesson. Connections to related state mathematics standards and the **Common Core State Standards for Mathematics** (**CCSSM**) are also identified in the section ***Supporting Standards for Mathematics***.

The data used throughout this module were obtained from the International Data Base (IDB) that is a resource of the United States Census Bureau's website (https://www.census.gov/programs-surveys/international-programs/about/idb.html). The incredible depth of the data available has several opportunities for additional projects. Efforts were made to use this data as intended, however, there were times that some liberties were taken. Mostly noticeably, the population totals for a given year are interpreted in this module as the count of people *at the beginning of the stated year*. This interpretation allows a more workable way for students to derive the year of birth of a selected person from a country's population. (The year of birth is important in the data stories highlighted in the lessons.) As reported by the International Data Base, the counts are actually mid-year counts or estimates. This discrepancy (and a few others that are pointed out when using the data) do not result in misleading results. The author and students remain indebted to the US Census Bureau for providing this data.

Ideas for Using this Module

Several ideas emerged as ways to use this module with students in high school mathematics courses.

(1) Stand-alone resource for a unit that involves the study of modeling.
Several courses in high school mathematics directly address modeling. This module provides an opportunity to study modeling within a two to three-week time period. At the time of the writing of this module, mathematics courses that stated modeling as an objective within each course include: Algebra, Algebra 2, Advanced Mathematics, Finite Mathematics, Precalculus, Statistics, and IB Math Studies, as well as several integrated mathematics series.

(2) Supplementary resource that is aligned with the goals of a mathematics course.
Lessons are aligned to several states' mathematics standards for high school students. Selected lessons from the module could be used to complement the textbooks used in these courses. Included in this **Overview** is a section entitled **Implementation Guidelines** that provides a description of each lesson to coordinate with the goals of a specific mathematics course.

(3) Development of student-centered projects.
A specific implementation of this module with students included the following: after a brief introduction of a selected lesson at the beginning of the week, students worked independently or in small groups to complete the lesson and to discuss their answers to the problems at the end of the week. The lessons served the purpose of developing student-centered projects. In addition, the **Teaching Notes** include possible extensions for several lessons that could also be completed as independent projects. Discussions were often led by students in groups as they highlighted the key problems using posters, PowerPoint, or other slide presentation software. See the **Implementation Guidelines** for a description of the lessons that would provide the basis for independent work. The number of lessons involved if implemented in this format will vary.

(4) Intergenerational resource for courses involving parents, guardians, and students.
High schools are frequently looking for ways to involve parents and guardians in the study of mathematics. Several schools have started monthly Parent Evenings in which a specific "mini-course" in mathematics is offered. A mini-course using selected lessons would be of interest to both parents and students. Lessons would be completed and discussed at the Parent Evenings mini-course. The **Implementation Guidelines** identify lessons that provide the best opportunity for an intergenerational study of mathematics.

(5) Summer-school offering
Summer or interim courses are often designed to provide additional opportunities for students to develop their understanding of mathematics. A four or five-week summer course using this material would provide an alternative curriculum for courses offered during the summer or during interim breaks.

(6) Development in a Computer Science course
Several of the lessons could be developed in a computer science course. One particular implementation of this module directed students to develop the recursive models as a programming assignment. The Microsoft Excel files were provided as examples of what the computer programs were expected to accomplish. Modifications of the lessons will be needed to complete this connection.

All of the above suggestions are based on the expectation that students are provided opportunities to complete the lessons in small groups Several lessons have components that involve some tedious calculations. Providing opportunities for students to participate in small groups makes these lessons more workable. Composition of groups should consider identifying students with a range of dispositions and abilities in mathematics.

Managing Real-World Mathematics Problems

The challenge with real-world mathematics problems is that they are about the real world. The range of topics that come under the umbrella of real-world problems is wide. Topics involving climate change, energy, animal proliferation or extinction, space travel are just a few topics that require mathematics to understand the problems connected to them. Sometimes these topics, however, result in passions that are not easily expressed in group discussions nor are they easy for you to referee. This challenge is also the case with the **People Count** stories. Immigration, death, birth rates and family planning are topics important in making sense of the mathematics introduced in the models, but these topics may also result in challenges for a meaningful discussion as students struggle to express their interpretation of the problems.

Teenagers are emerging adults. Sometime the lack of maturity is in full display. They do not always listen to each other, they do not always treat each other fairly or with sensitivity, and they do not always allow for other opinions or ideas to be part of their thinking. Then at other times (and often within a short period of time from their less than mature actions), they listen, they balance ideas in their thinking, and they work with each other and consider opposing ideas in forming their own ideas.

A premise in this module is that teenagers should be involved in real-world problems even if there are some possible challenges. Saying that, there is a need for you as their teacher to be prepared to filter the discussions and to weigh to what extent the discussions around immigration or family planning will be received by teenagers or their families that is productive. Consideration of other members of the class must be taken into account. For example, will a personal opinion on immigration from one student make another student uncomfortable or even scared?

This module has been designed to assist you regarding these discussions in at least two ways. First, if you think some of the questions might single out certain students in a way that is uncomfortable for you or the class, focus the discussion on the characters in the **Kristin's**

Stories that are used to introduce the issues mentioned. Rather than open up the discussion in a way that you are not sure where it is headed, focus on the characters they read about in the stories – for example, Raphine who immigrates to the United States and then emigrates, Adeline who is born during one of the stories, Kristin's mother and Hana's grandmother and mother who are senior citizens. Each character is represented as "where do they count" in the graphs and data sets, but they are also characters for discussing the issues of immigration, birth rates, and death. Feel free to use their names and circumstances to further discuss the issues that are asked in the problems.

A second way to allow students to connect with the material and to minimize the possible distraction from the objectives of the lessons is to suggest students write their own data stories previously described in this **Overview of Module**. This approach was used with students in several classes that piloted this module. Their personal stories provided an opportunity to personally connect with these sensitive topics in a positive and more private way.

There are a few problems that have an "*" identification. This is a gentle reminder that this problem might require a little more attention if used in any group discussion. Yet, the implications of these topics are critical for teenagers to address as they move closer into the decision-making age groups of this country. Part of learning to be a problem-solver is using mathematics to understand, and maybe solve, real-world problems.

The Modeling Continuum

High school and college teachers of mathematics are often asked, "What are the important areas of mathematics students should learn?" Generally, the discussion follows with an identification of specific skills or specific standards. This module addresses several of these skills and standards, but the primary answer to that question would be to describe a **process** of solving problems. This module is not solely about developing specific skills or standards; this module is about skills or standards working together to answer important questions. The process in which these components work together is defined as **modeling**.

In a broader sense, modeling is developed in this module in the same way a room in a house might be remodeled or upgraded (for example, a kitchen). The process starts by putting together an initial blueprint, or a sketch, or a "to do" list. Tools and materials are assembled and then used to complete the construction. This module highlights in the students' text a different example that has a similar process - making a quilt. The process starts by organizing tools and materials (needles, cloth, patterns). A sketch or pattern is often designed as a blueprint to complete the next steps. These initial tasks are then followed by assembling and building the quilt. Similar to remodeling a room, it is the final process of building a product that makes each an example of modeling. The tools in this module (i.e., proportions, ratios, median centers, mean centers, sequences) are used to build or rebuild population projection models. If a student does not have the right tools, or if a student does not know what tools are needed, then the process starts with figuring out the tools and the materials. As students grow more comfortable with understanding the tools, the more they are able to participate in one of the most important areas of mathematics - namely modeling.

The interplay between skills and a process, however, is complex. Sometimes it is difficult to see where learning a specific skill and developing a process are different (and maybe they are not different). This module provides a guide in sorting through students' work that involves the interplay between skills used in this module and the process to build a population projection model. This guide is called the ***Modeling Continuum***. It has four specific levels that are used to classify problems and analyze students' work. Although the first two levels are not in themselves modeling, they describe the necessary first steps. The last two levels put the skills together to solve special problems and provide evidence of modeling.

Why is important to have a working definition of modeling? Several formal definitions of modeling were considered for this module. In the end, it was difficult to find one that would address the specific process used to solve the problems in this module. Modeling is a major component of several past and current mathematics standards with varying definitions and examples of implementation. Modeling is a process that is difficult to define, although teachers generally know when students are demonstrating its role in learning mathematics. The **Modeling Continuum** designed in this module is intended to provide both teachers and students descriptions of a specific process needed to answer the specific questions in this module.

The **Modeling Continuum** has two basic questions that form umbrellas over the continuum levels. The first umbrella question is simply, "What is ….?" Questions that either directly use that lead or can be reworded to ask "what is" questions are Levels 1 and 2 of the continuum. What is the count of males in the United States in 2015? What is the percent of males in 2015? The first question simply requires students to find and state the information from a table or graph that is part of a lesson. Using a graph to answer that question may involve an approximation, but it is nonetheless directly obtained from a given data set. This process represents a Level 1 of the Continuum. The second question adds one more step, namely, students find the count of males from a table and then derive what percent this count represents of the total population. This question relies on a student's understanding of a proportion (a tool) and how to convert it to a percent. This summary represents an extension of the first level and is classified as a Level 2 of the Continuum.

In a similar way, the umbrella with the lead "What if…?" represents the classification of questions at Levels 3 or 4 of the continuum. What if there are 3 million more males within the next 5 yeas? What proportion of the resulting population are males? What if there are more people moving into our country than moving out? Will the population grow as a result of more people moving into the country? What if we calculate the population factors for Kenya in the same we calculated them for the United States? What do the population factors tell us about Kenya? Each "What if …?" question is followed by a question that requires an interpretation of the answer. Level 3 is a more general interpretation of these questions. It uses the tools of population factors (as defined in this module) based on proportions or percent. Level 4 requires a more robust remake of the projection model, asking students to recreate the conditions summarized in the model. Level 4, as expected, evolves as the lessons progress. The lessons lay the ground work for remodeling the projection models and applying the parameters that were involved in their initial design. The problems often rework the model for use with a country highlighted in the quilt (see cover) after completing the process for the United States. The following summary of the **Modeling Continuum** is used to guide your understanding of the process developed by students in this module to solve problems:

The Modeling Continuum

"What is …?"		"What if …?"	
Level 1	**Level 2**	**Level 3**	**Level 4**
Identifying or extracting data from data sets or projections.	Summarizing data and projections from tables or graphs.	Interpreting the tools (for example, population factors, foundation factors, proportions) that are used to derive projections addressed in the lessons.	Reworking and modifying the tools used to make projections by addressing "What if …?" questions.
Answering questions directly from the presented data. Answering "What is …?" questions.	Summarizing data or outcomes in your own words. Answering "What is …?" questions using proportions, percent, or relative frequencies.	Answering questions or problems that require using the tools discussed in the lessons. Calculating new outcomes of a country's population based on changes in a country's immigration, births, and deaths.	Modifying the tools presented in the lessons that result in new population projections for real or fictitious countries. Answering questions that are a result of the modifications of a country's future population projections.

The lessons summarized in the **Teaching Notes** include suggested levels of the **Modeling Continuum** for each of the problems. The suggestions are not to be interpreted as absolute – with minor tweaking, some of the problems might be classified at a higher level. With assistance provided by the instructor or other students, some problems might be classified at a lower level. The primary role that the Continuum plays is to provide opportunities ***for students*** to self-evaluate their work. It is suggested in several Units 1, 2, and 3 lessons to assign students to complete the **Exit Summary** (**Handout 13**) of their work. If this handout is used for a specific lesson, direct students to select one or two of the problems completed and to identify a description of one of the continuum levels from the **Exit Summary** or **Handout 13** for each problem that they think describes the way they solved the problem. Students proceed to write a brief summary of the process or method they used to solve each problem and why they selected the continuum level and description. An example of a completed **Exit Summary** from a student is provided below. Share this example with students before they attempt their first **Exit Summary**.

People Count! (and their data stories)

Exit Summary

Name: Stephanie

Lesson Number: 1

At your teacher's discretion, identify problems or questions in this lesson that you answered by using one or more of the levels of the Modeling Continuum. Within the column of the level or levels you identified, explain the steps you used to answer the questions or problems.

The Modeling Continuum

Level 1	Level 2	Level 3	Level 4
Identifying or extracting data from data sets or projections.	Summarizing data and projections from tables or graphs.	Interpreting the tools (for example, population factors, foundation factors, proportions) that are used to derive projections addressed in the lessons.	Reworking and modifying the tools used to make projections by addressing "What if …?" questions.
Answering questions directly from the presented data. Answering "What is …?" questions.	Summarizing data or outcomes in your own words. Answering "What is …?" questions using proportions, percent, or relative frequencies.	Answering questions or problems that require using the tools discussed in the lessons. Calculating new outcomes of a country's population based on changes in a country's immigration, births, and deaths.	Modifying the tools presented in the lessons that result in new population projections for real or fictitious countries. Answering questions that are a result of the modifications of a country's future population projections.

Level 1	Level 2	Level 3	Level 4
Problem: 3	**Problem:**	**Problem: 14**	**Problem:**
I used the population pyramid graph to answer this question. The longest bar on the male side of the graph was for the 20 – 24 years old males.		This problem was difficult. I knew all of the people 15 – 19 years old would be part of my answer. My question was how to include in my answer the 13 and 14 years old. What if 2/5 of the 10 – 14 years old age group are 13 and 14? So, I took 2/5 of the total count in that age group and added that value to the total of the 15 – 19 years old.	

Supporting Standards of Mathematics

Identifying important mathematics students need to learn is a major challenge for teachers of mathematics to sort out. Most states created standards for mathematics that carefully identify what expectations are important and at what grade levels they should be addressed for students in kindergarten through grade 8. Although standards differ from state to state, overall they address similar expectations and grade level expectations.

The **Common Core State Standards for Mathematics**, or **CCSSM**, is used in this **Overview of Module** to provide a general blueprint of the important mathematics for high school students. Most state standards provide a similar delineation of important mathematics for high school students. Also similar to the **CCSSM**, most state standards for high school students indicate what students should achieve upon the completion of high school and not grade level expectations.

The CCSSM standards supported by this module are outlined below by identifying the important mathematics addressed in each unit. Particularly note the connections in the table to the ***Standards for Mathematical Practice***. The strength of this module is the development of the lessons addressing the ***Standards for Mathematical Practice***.

	Mathematics Standards for High School	**Standards for Mathematical Practice (MP)**
Unit 1: **A Country's Shape**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 1):*** Represent data with plots on the real number line (dot plots, histograms, and box plots). **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 2):*** Use statistics to approximate the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**MP-1: Make sense of problems and persevere in solving them.**

Unit 2: **Looking Back**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 1):*** Represent data with plots on the real number line (dot plots, histograms, and box plots). **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 2):*** Use statistics to approximate the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **Conceptual Category: Modeling**	**MP-1: Make sense of problems and persevere in solving them.** **MP-2: Reason abstractly and quantitatively.** **MP-3: Construct viable arguments and critique the reason of others.**
Unit 3: **Looking Forward**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 6a,6c):*** Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. c. Fit a linear function for a scatter plot that suggests a linear association.	**MP-1: Make sense of problems and persevere in solving them.** **MP-3: Construct viable arguments and critique the reason of others.** **MP-4: Model with Mathematics** Students apply their new mathematical understanding to real-world problems. They also discover mathematics through experimentation and by examining patters in data from real-world contexts.

	Statistics and Probability ***Making Inferences and Justifying Conclusions (S-IC 2):*** Decide if a specific model is consistent with results from a given data-generating process. **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 1):*** Represent data with plots on the real number line (dot plots, histograms, and box plots). **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 2):*** Use statistics to approximate the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **Conceptual Category: Modeling**	
Unit 4: **“What if …?”**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 6a):*** Fit a function to the data. **Statistics and Probability** ***Making Inferences and Justifying Conclusions (S-IC 2):*** Decide if a specific model is consistent with results from a given data-generating process. **Conceptual Category: Modeling**	**MP-1: Make sense of problems and persevere in solving them.** **MP-3: Construct viable arguments and critique the reason of others.** **MP-4: Model with Mathematics** Students apply their new mathematical understanding to real-world problems. They also discover mathematics through experimentation and by examining patters in data from real-world contexts.

Writing a Data Story

As your students begin reading **People Count**, they will be introduced to three people – Kristin from the United States, Raphine from Kenya, and Hana from Japan by their ***data stories***. (Interestingly, their names were on the list of the most popular names in their respective countries when they were born.) Each character's age (and the year of their birth) and her or his location on a population pyramid graph are shared in data stories students read in the lessons.

Data stories for this module are short, essentially factual summaries of a person based on age and his or her connection to a larger population. The characters in this module are reflections of people important in understanding a country – they are part of what makes the study of a country's population interesting, and also important.

If students have problems reading the narratives either because of a lack of interest or skills, please read the stories together as a class. They are short and focused on the study of a population. They are also intended to personalize the problems presented in the lesson. The problems are not just about the count of people (or the percent of people) older than 36 years old – they are problems about the count (or percent) older than Kristin and Raphine and Hana and why the percent of older people is important in Kristin's, or Raphine's, or Hana's life. It is also about why Kristin or Raphine or Hana is important in their respective countries. Let the data explain their placement in the population through the stories.

This module provides an opportunity for students to write their own data stories. Consider assisting students to write a story as they progress through the lessons. In particular, this module provides an opportunity for students to communicate with a person of a different generation who Is important in understanding their country.

Consider asking students in what age groups of a population pyramid graph their parents or guardians, or grandparents, or other relatives would be counted, and why a person in that age group is important to study? If students identify a person, work with them (possibly as a class project) in developing 5 to 10 questions to summarize in a short data story. Questions such as: What is your favorite movie? What is your favorite (or least favorite) subject in school? What were the main problems in the country when you were a teenager (assuming the person is older than a teenager)? How did you communicate (e.g., a phone, or a letter) with other people when you were a teenager? Were you an immigrant? Direct students to summarize a person's past by identifying a pyramid graph that summaries the year they were born using the International Data Base (IDB) site that is referenced in the module, along with researching the primary issues in the news when this person was born.

Consider assembling the data stories to share with the class, and also as part of a larger group setting (for example, a parent's night or a newsletter sent home with students). This might also be a project to conduct with another class that also looks at intergenerational topics. The **Teaching Notes** for several of the lessons include ideas that could be considered in a data story.

Implementation Guidelines

The following section summarizes the topics addressed in each unit. This section is followed by a general description of each lesson, a general description of how to implement each lesson, and an estimate of the class time to complete each lesson.

It is assumed that students will have access to a complete copy of each lesson as well as any handouts identified in the **Teaching Notes** and the student's lesson. Copies of the lessons could be printed or electronic. Handouts generally should be provided in print form. Handouts are completed as part of the problems in a lesson and then used in other lessons to interpret and expand students' understanding of the data.

Unit 1: A Country's Shape
This unit defines shape based on the population distribution as displayed by population pyramid graphs and histograms. Specific age groups are analyzed to highlight significant features of each country's population. Summaries include the age groups representing the greatest percent of the population, a ratio of old-to-young, and general descriptions based on the defined shape of a country. The implications of these summaries are also developed in the problems.
Points to consider: In order to develop an understanding of shape as defined in the module, completing Lessons 1 to 3, or Lessons 1 to 4 as a unit is recommended. Lesson 5 is optional, although follow-up work based on the data identified in Lesson 5 is referenced in later units. Consider implementing Lesson 1 for all students as a whole-group discussion, and then assigning Lessons 2 and 3 to small groups of students. If this suggestion is implemented, conduct a whole group discussion of students' work led by the Kenya groups and a whole group discussion led by the Japan groups. Consider directing each group to summarize their work by presenting to class the main results they learned about their country using a PowerPoint or posters. Also consider the optional **Case Study** projects. See Henry's Quilt, **Case Study Project: *Developing a Country Poster,*** after completing Lessons 1 to 3. See Henry's Quilt, **Case Study Project: *Deriving a Country's Centers and Spread,*** after completing Lessons 1 to 4.

	Description	Implementation Design	Anticipated instructional time
Lesson 1: The United States – A Lower Middle-Layered Country	This lesson introduces students to a population pyramid graph of the United States. Using the 2015 pyramid graph, or a histogram of the counts in 5-year age groups or a table prepared from data provided by the United States Census Bureau, students discover the age summaries of a lower middle-layered country. Lesson could stand alone, although comparisons to Kenya and Japan provide a deeper understanding of definition of the shape of a country.	• Provide students: **Handout 1: *United States - 2015*** •Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 1.	One 50-minute class period
Lesson 2: Kenya – A Bottom Layered Country	The 2015 pyramid graph of Kenya, along with a histogram of the counts in 5-year age groups and a table prepared from data provided by the United States Census Bureau, are used to discover the age summaries of Kenya. Lesson could stand alone, although comparisons to United States and Japan provide a deeper understanding of shape.	• Provide students: **Handout 2: *Kenya - 2015*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 2.	One 50-minute class period
Lesson 3: Japan – An Upper Middle-Layered Country	The 2015 pyramid graph of Japan, along with a histogram of the counts in 5-year age groups and a table prepared from data provided by the United	• Provide students: **Handout 3: *Japan - 2015*** • Set up lesson to be completed individually or in small groups.	One 50-minute class period

	States Census Bureau, are used to discover the age summaries of Japan. Lesson could stand alone, although comparisons to United States and Kenya provide a deeper understanding of shape.	• Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 3.	
Lesson 4: **The Center and Spread of a Country's Shape**	This lesson organizes the 2015 United States population data into tables that students use to calculate the mean age and the median age. Tables are also provided for students to derive an estimate of the spread of ages anchored by the median.	• Set up lesson to be completed Individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 4. • Consider organizing small groups of students to also calculate the mean age, median age, and spread for Kenya and Japan. The templates needed are included in the lesson. Each group is assigned a country and reports to the whole class the results of their work.	One 50-minute class period (An additional 50-minute class period is needed if students include Kenya and Japan.)
Lesson 5: **My Country**	The lesson directs students to create a population distribution by age groups for their own country (referenced in later lessons as ***My Country***). The details students should consider are provided in the lesson. An alternative set-up is to direct students to work with a data set representing a top layered country that is provided in the **Teaching Notes**.	• This lesson could be completed as an independent project. If this suggestion if followed, provide time for students to individually present the pyramid graphs and histograms for the population distributions they created. • The data included in the **Teaching Notes** provide an example of a Top Layered Country that could be used by the whole class. This data set is also referenced in later	One 50-minute class period (if completed during class).

	This lesson is considered optional. It can be used as a way to determine students understanding of the previous 4 lessons, as well as a sense of their disposition in working with this data.	lessons to indicate the unusual features of a top layered country. • If the data provided in the **Teaching Notes** are used, conduct whole class discussions at periodic points in the lesson.	

Unit 2: Looking Back

A country's shape is a result of many factors that emerge by studying the past and present population pyramid graphs and histograms. This unit develops a timeline that analyzes both past distributions and the resulting connection to 2015. In Lesson 6, the timeline identifies the emergence of two significant subgroups (although subjective descriptions are used to define these subgroups) that result in the unique shape of the United States – namely, the Baby Boom Generation and the Millennial Generation. Events that resulted in a change in the count of people over time are identified in the lessons and traced through several decades.

Points to consider:
The primary focus in this unit is developed in Lesson 6. It is important for several reasons that are outlined in the **Teaching Notes**. It is the first lesson that begins to imply what are factors that result in a change of the population over time. It also uses the graphs to indicate why these factors need to be considered in analyzing a country's current distribution.

Lesson 6 builds a foundation for the goal of the next unit, n1amely, to derive population estimates of the future. Lesson 7 continues to connect Kenya and Japan in this study as a comparison of different shapes to the shape of the United States. Lesson 7 is an optional lesson, although it also links Kenya and Japan's current shape to their past.

	Description	Implementation Design	Anticipated instructional time
Lesson 6: Looking Back at the Shapes of the United States	This lesson looks back at the population distributions of the United States using population pyramid graphs and histograms. Links to the past are used to identify the emergence of descriptions of generations (i.e., Baby Boomers or the Millennial Generation).	•Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 6.	One 50-minute class period.

	Lesson could stand alone. The connection to Kenya and Japan adds to an understanding of past distributions to the current shape (as defined in this module) of a country.		
Lesson 7: Looking Back at the Shapes of Kenya and Japan and My Country	This lesson links the past population distributions of Kenya and Japan to their current distributions. This lesson is considered optional.	•Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 7.	One to two 50-minute class periods.

Unit 3: Looking Forward

This unit builds on the analysis of past distributions in deriving estimates of future counts. Students are challenged in each lesson to explain the factors behind each new iteration of a population projection model. The unit begins by developing an arithmetic sequence based on the 2010 and 2015 population totals. Lesson 8 summarizes this sequence with a linear model that is used to estimate the future population totals. A geometric series is used to derive an exponential model in Lesson 9. Each of these lessons, however, do not obtain estimates for the age groups. Although possible to derive age group estimates using these models, the age group analysis begins in Lesson 10 with the special recursive model designed for this module.

Lesson 10 introduces an important feature of the recursive model that is identified as **Population Factors**. Applying these factors to build estimates of future age groups is done in Lesson 11. Lesson 12 completes the recursive model by developing **Foundation Factors** that fill in the “holes” left by the 0 – 4 years old age group in each iteration. As the holes are derived, the resulting projections based on population factors are completed. Lessons 13 and 14 build off of these estimates to summarize the shapes and projected changes in the population of the United States, Kenya, and Japan.

Points to consider:
Lessons 8 and 9 provide students the more familiar models they might have previously studied. The development of a linear model and an exponential model are often used to start a study of modeling. Lesson 10 is an important lesson to launch an understanding of a specific iterative or recursive model. Important features of this model are evident by studying specific

age groups over time, namely the impact of immigration, emigration, and death in population projections. Lessons 10 to 12 should be completed as a unit. Lessons 13 and 14 are optional but add to students' understanding of the population changes over time. In particular, Lessons 13 and 14 compare the linear, the exponential, and the recursive models. Also consider the optional Case Study projects. See Henry's Quilt, **Case Study Project: *Interpreting Population and Foundation Layer Factors,*** **Case Study Project: *Developing Projection Model,*** and **Case Study Project: *A Country Quilt Scavenger Hunt*** after completing lessons 8 to 12. A description of each case study is provided in the introduction to the Case Student Projects.

	Description	**Implementation Design**	**Anticipated instructional time**
Lesson 8: Looking Forward with an Arithmetic Sequence and a Linear Model	This lesson uses the 2010 and 2015 population totals of the United States, Kenya, and Japan to estimate the population of each country in the future by deriving and applying a linear model. The linear model is also used to derive estimates of past years, comparing these estimates to the actual population summaries.	• Lesson could be assigned as an independent project and discussed with all students after they complete the problems. Lesson could also be organized as a class project in which students work independently or in small groups and discuss as a class their answers or responses. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 8.	Two 50-minute class periods.
Lesson 9: Looking Forward with a Geometric Sequence and an Exponential Model	This lesson uses the 2010 and 2015 population totals of the United States, Kenya, and Japan to estimate the population in the future. The exponential model is also used to derive estimates of past years, comparing these estimates to the actual population summaries.	• Lesson could be assigned as an independent project and discussed with all students after they completed the problems. Lesson could also be organized as a class project in which students work independently or in small groups and discuss as a class their answers or responses. • Conduct whole class discussions at periodic points in the lesson – read	One 50-minute class period.

		suggestions in **Teaching Notes** for Lesson 9.	
Lesson 10: Looking Forward with a Recursive Model	The recursive model designed in this module emphasizes the changes in 5-year age groups as a reading of what happened during the last 5 years. Was the count in those age groups changed primarily by immigration, by emigration, or by death? (The impact of births in the country is addressed in Lesson 12.) These changes are examined by forming ratios of ***connected age groups*** that are then represented by decimal values called the **Population Factors.** This lesson is an important lesson for students to complete in order to accomplish the overall goals of this module.	• Provide students **Handout 1: *United States – 2015*** and **Handout 4: *United States Connected Age Groups*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 10. • Highlight during the discussion with students the problems identified in the Assessment section of the **Teaching Notes**, possibly placing selected student responses on a poster for referencing in the next several lessons.	One 50-minute class period.
Lesson 11: The Recursive Model and Falling Dominos	The population factors are applied to obtain future population estimates. The problems in this lesson direct students to summarize these changes over time; for example, if the changes noted in the 2010 to the 2015 connected age groups continue, what is the effect on future counts? A hole is left in the population summaries, however, that is filled in the next lesson.	• Provide students **Handout 5: *Looking Forward for the United States (Student Edition)*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 11.	One 50-minute class period.

	This lesson is an important lesson for students to complete in order to accomplish the overall goals of this module.		
Lesson 12: Completing the Recursive Model with the Foundation Layer	This lesson fills in the holes left in the lesson. A **Foundation Factor** is derived based on the 2015 population. If this factor based on the 0 – 4 years old age group (the only age group that counts the births) stays the same over time, what are the estimates of the future counts in this age group? Filling in this hole will then generate estimates for connected age groups (or, the effect of the falling dominos). Each iteration completes an estimate of the 0 – 4 years old age group that ultimately fills in all holes summarized in the handout. This lesson is an important lesson for students to complete in order to accomplish the overall goals of this module.	• Provide students **Handout 6: *United States 2010 - 2015*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 12.	One 50-minute class period.
Lesson 13: "The More Things Change, the More Things Stay the Same"	This lesson analyzes the recursive model developed in Lessons 10 to 12 in more detail. Over time, the results of applying the population factors derived in Lesson 10 are a growing population and a leveling off	• Provide students: **Handout 6: *United States 2010 – 2015*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points	One 50-minute class period.

	of several age groups. Students examine the subtle changes in specific age groups that result in the leveling off of the population. Analyzing changes over time is important, especially when several iterations of this particular model are completed. This lesson is considered optional.	in the lesson – read suggestions in **Teaching Notes** for Lesson 13.	
Lesson 14: Kenya, Japan, and the United States – Summing it Up	Unit 1 launched the importance of examining the populations of the United States, Kenya, and Japan. Unit 2 examined the past populations primarily using graphs. Unit 3 implemented various models to estimate population counts in the future. This concluding lesson of Unit 3 organizes the results of the various models and summaries the different counts for the 3 countries. This lesson provides students an important wrap-up of what has been developed in the module. This lesson builds on the previous lessons from Unit 3 and provides students a better understanding of what the purpose of a model. In particular, students are directed to understand the importance of identifying the	• Provide students the following handouts: **Handout 6: *The United States 2010 – 2015,* Handout 7*: Kenya 2010 – 2050,* and Handout 8*: Japan 2010 – 2050*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 14.	One 50-minute class period.

	assumptions and design of a model when interpreting its results.		

Unit 4: "What if …?"			
The ultimate goal of any model is to provide possible answers to "What if …?" questions. A working model is now in place. What if the population factors and foundation factors noted in the set-up of the recursive model continue? What if they change? What if fewer counts of people move into a country? What if economic conditions require more workers? What if a downturn in the economy results in people having fewer children? These, and several similar questions, are explored in this unit. Students are expected to apply the tools they learned to rebuild the recursive model to answer these types of questions. Students are also expected to be able to articulate what changes they propose to the original model, and why they propose those changes. The two lessons designed in this unit, and the concluding wrap up section, are putting students in control of the model (and the highest level of the **Modeling Continuum**).			
Points to consider: Students complete Lesson 15 with access to the Excel projection files provided with this module. Handouts are provided for students to identify specific factors (or tools) of the recursive model they would alter to address a "What if…?" scenario. They are also expected to explain their decisions based on the impact of what factors they would alter. Lesson 16 is similar in design. Students again rework the recursive model with the goal of matching their results to the projections summarized by the United States Census Bureau. Lesson 16 involves a more precise analysis of the impact of the proposed changes to the model.			
	Description	**Implementation Design**	**Anticipated instructional time**
Lesson 15: "What if …?" Scenarios	Students select a "What if…? scenario from several presented in the lesson. They complete a plan that indicates how they would alter the recursive model based on the selected scenario. In their plan, students indicate which population factors they would alter and what values they would assign to those factors. They also indicate if they would alter the	• Provide students the handouts indicated in the opening section of the lesson. • Provide students access to the projection files indicated in the opening section of the lesson. • Consider organizing students in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in the **Teaching Notes** for Lesson 15.	Two 50-minute class periods. The extension in time is based on students have access to the projection files.

	foundation factors and what values they would assign.		
Lesson 16: The US Census Models and the Recursive Model	Students are presented the 2050 population projections from the Census Bureau. They compare the Census Bureau's result to the results from the 2050 recursive model. They complete a plan that indicates how they would alter the recursive model that would result in a better match to the United States Census projections. In their plan, students indicate the impact of changing specific factors over time. Students also indicate if they would alter the foundation factors and what values they would assign to the foundation factors. Students are again expected to indicate the impact of their changes in the foundation factors over time. This lesson is considered optional.	• Provide students the handouts indicated in the opening section of the lesson. • Provide students access to the projection files indicated in the opening section of the lesson. • Consider organizing students in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in the **Teaching Notes** for Lesson 16.	One to two 50-minute class periods.
Wrap-up of the People Count Stories	Three projects are described in this wrap-up section. Each project involves identifying key components of the recursive model and interpreting their impact on the model.	• Read with students the selected project or projects. • Direct students to complete the project or projects independently or in small groups. • Provide time for students to report their answers to the whole class.	Work is primarily done outside of class. Schedule time for students to summarize their work to

	Students summarize their work by answering questions that require an understanding of the design of the recursive model.	• Other suggestions are provided in the **Teaching Notes**.	the entire class.