Teaching Notes

Lesson 11

The Recursive Model and Falling Dominos

Overview:

This lesson is the first of two lessons that develop the Recursive Model referenced throughout this module. It builds off the ***Population Factors*** derived in the previous lesson. Counts for each of the age groups in 2020 (except the 0 – 4 years old age group) are determined by multiplying the appropriate population factor to the count of the connected age group. This process generates estimates for the 2020 population. These estimates in turn are used to derive the 2025 counts, followed by the 2030 counts, and continued until 2050. It is this process that indicates the recursive or iterative design of this model.

Why is this important? Unlike the linear or exponential models, this model provides students an estimate of the counts of the underlying age groups, and as a result, the new shape of the country after the process is completed. By first looking back, this model also looks forward. This particular model is built on applying the population factors as an indicator of what happened over the last five years that changed the counts of each age group. The model is completed in Lesson 12 with the design of the ***Foundation Factor*** and its role in estimating counts of age groups left blank in this lesson.

This lesson further pushes students' thinking to levels 3 and 4 of the **Modeling Continuum**. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 4	Problems: 2, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16	Problems: 10, 12, 17, 18	Problems: The extension

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent, recursion. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 11 for each student. This lesson also requires 1 additional handout for students to complete the problems, **Handout 5: *Looking Forward for the United States (Student Edition)*.** Provide an electronic or printed copy of the handout.

Launch:

Consider starting this lesson with actually setting up dominos in such a way that when the first domino falls, the next domino falls, and then the next, until the last domino is down. Several online videos of falling dominos are available to visualize this idea. A similar process is set-up in this lesson involving projecting counts in the future that resemble falling dominos, namely, the interconnection of one age group to another age group, to another, etc.

Review with students before they begin the problems the main characters in the story and their ages. Specifically, have a brief discussion that reminds them that at the start of 2015, Kristin is 36 years old, Adeline was 2 years old, Abbey was 26 years old, and Kristin's mother was 66 years old. The problems in this lesson will continue working with the past and future ages of these characters.

Implementation Ideas:

Similar to the previous lesson, the problems are designed to build students' understanding of the lesson's objectives. Note several summaries are provided to organize the objectives in the student lesson. Use these summaries as opportunities to have a brief whole group discussion of the problems. Wrap up this lesson with a discussion of the questions highlighted in the assessment ideas. The suggested assessment task utilizes an Excel file.

Student responses or descriptions

Lesson 11 - Problems

Recall from the Student Edition the following:

(Population of age group 0 – 4 years old 2010)	X	*(Population Factor for connected age groups)*	=>	
(Population of age group 0 – 4 years old 2010)	X	$\frac{Population\ of\ age\ group\ 5-9\ for\ 2015}{Population\ of\ age\ group\ 0-4\ for\ 2010}$	=	*Population of age group 5 – 9 for 2015*

↑

(Population Factor for connected age groups 0 – 4 to 5 – 9 years old)

1. Complete the multiplication indicated in the following table. The first calculation has been completed for the 0 – 4 age group of 2010 to the 5 – 9 age group of 2015 and set-up in the above equation. The population factors listed were derived in the previous lesson.

Population of Age group **2010**	X	Population Factor	=	Population of Connected Age Group **2015**	Census Bureau counts for **2015**
0 – 4 20,189,589	X	1.014	=	**5 – 9** 20,472,243	**5 – 9** 20,481,130
5 – 9 20,331,807	X	1.013	=	**10 – 14** *20,596,120*	**10 – 14** 20,605,579
10 - 14 20,681,214	X	1.020	=	**15 – 19** *21,094,838*	**15 – 19** 21,084,710

2. Why are the answers in problem 1 not equal to the Census Bureau estimates?
   The population factors were rounded off. As a result, any calculations based on these factors will be different from the actual counts of the Census Bureau.

3. Examine the connected age groups of 0 – 4 years old in 2010 to 5 – 9 years old in 2015. A population factor of 1.014 indicates an approximate increase of 1.4% in the 5 – 9 years old age group in 2015 when compared to the 0 – 4 years old age group in 2010.

   a. Using the above summary of the changes in these age groups, is it possible to derive the number of people who moved into the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.
   No. The population factor of 1.014 indicates an overall increase but it does not provide a specific breakdown. From 2010 to 2015, the connecting age groups had more people moving into the country than people moving out or dying, however, the specific counts of people moving in compared to the count of people moving out or dying is not possible to determine from the population factor.

   b. Is it possible to derive the number of people who moved out of the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

   No. A student's answer indicates that the overall change does not reflect a specific breakdown of the change in the count of people.

   c. Is it possible to derive the number of people who died from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.
   No. A student's answer indicates that the overall change does not reflect a specific breakdown of the change in the count of people.

   d. What was the dominate explanation for the change in the number of people from 2010 to 2015 of the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain your answer.
   The population factor indicates an overall increase in the count. This increase could only occur through immigration. Immigration is the dominate explanation of the changes in the population, however, emigration and dying also changed the count of people in the 5 – 9 years old age group.

4. Population factors listed in the following table were recalculated based on the approximations of millions of people in 2010 and 2015. The 2015 population estimates were also rounded off as indicated. Complete the following table. Several age group projections have been completed. (Note: Some of the posted estimates may differ from your calculations due to the software used to implement the recursive model. The differences are minor. Allow a difference of +/- 0.01 of your answers and the answers posted.)

Age group	Population 2015 (millions to the nearest hundredth)	Population Factor	Population 2020 (millions to the nearest hundredth)
0 – 4	19.91	1.014	
5 - 9	20.48	1.014	20.20
10 - 14	20.61	1.020	20.76
15 - 19	21.09	1.032	21.02
20 - 24	22.69	1.032	21.77
25 - 29	22.40	1.022	*23.42*
30 - 34	21.62	1.012	22.90
35 - 39	20.31	1.004	*21.88*
40 - 44	20.16	0.995	*20.39*
45 - 49	20.80	0.985	20.05
50 - 54	22.29	0.974	20.48
55 - 59	21.77	0.962	*21.71*
60 - 64	19.04	0.945	20.93
65 - 69	16.05	0.917	*17.99*
70 - 74	11.48	0.869	*14.72*
75 - 79	8.12	0.792	*9.98*
80 - 84	5.80	0.670	*6.43*
85 - 89	3.86	0.508	3.89
90 - 94	1.85	0.340	*1.96*
95 – 99	0.50	0.205	*0.63*
100 +	0.08		*0.10*

5. Recall that Kristin was 36 years old at the start of 2015. In what age group would she be counted in 2020?
   Kristin would be counted in the 40 – 44 years old age group in 2020.

6. One **age group** for 2020 (the shaded cell) is left blank by this process. What age group is left blank? Explain why this age group does not have a population factor for this recursive model. (This particular age group is called the **Foundation Layer** and will be the focus of the next lesson.)
   The age group left blank is the 0 – 4 years old age group for 2020. To determine the count of this age group, we need to know how many people were born during the last 5 years (or from 2015 to 2019), along with how many people moved into the country, how many people died, and how many moved out of the country during the last 5 years who were 0 – 4 years old. This group of people were not previously counted. As a result, there is no connecting age group and no population factor can be set-up.

7. One **population factor** also has no entry in the above table. Why is there no population factor entered for this age group?
   There are no population factors for the age group 100+ years old (or people who will turn 105 or older in the next 5 years). There are definitely people older than 105 years old in the country, but the count is so small in comparison to the total population that we generalize the counts in the 100+ age group.

The Falling Dominos

8. Record the missing 2020 projections in the column identified as 2020 in **Handout 5**. Be careful not to provide a projection for the age group 0 - 4 years old in 2020.
   Check results with the Teacher Edition of Handout 5.

9. Use the Population Factors to complete the projections for 2025 from the 2020 projections based on the previous 5-year projections. Several calculations have been completed in the handout. (Remember, round your answers to the nearest hundredth. Each value indicates the count of people in millions.)
   Check results with the Teacher Edition of Handout 5.

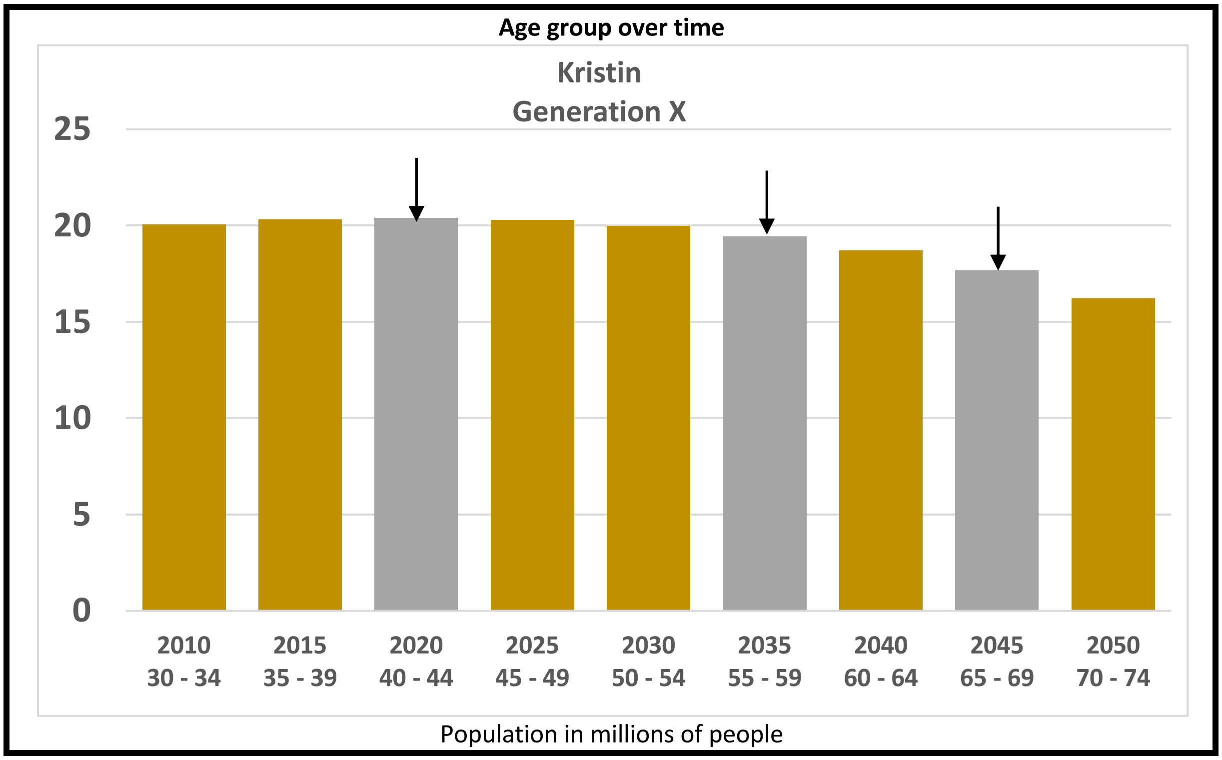
10. In 2020, there was one age group that was left blank. In 2025, there are two age groups that are blank. Why is this second age group blank?

    As there was no estimate of the 0 – 4 years old age group in 2020, there would be no estimate for the 5 – 9 years old age group in 2025. In addition, the count of the 0 – 4 years old age group in 2025 is not connected to any age group, so for the same reasons as indicated previously, that age group also remains blank (for now).

11. Complete the missing calculations in **Handout 5** for the years 2030, 2035, 2040, 2045, and 2050. Again, several calculations have been completed. Use the completed calculations to check your work.
    Check results with the Teacher Edition of Handout 5.

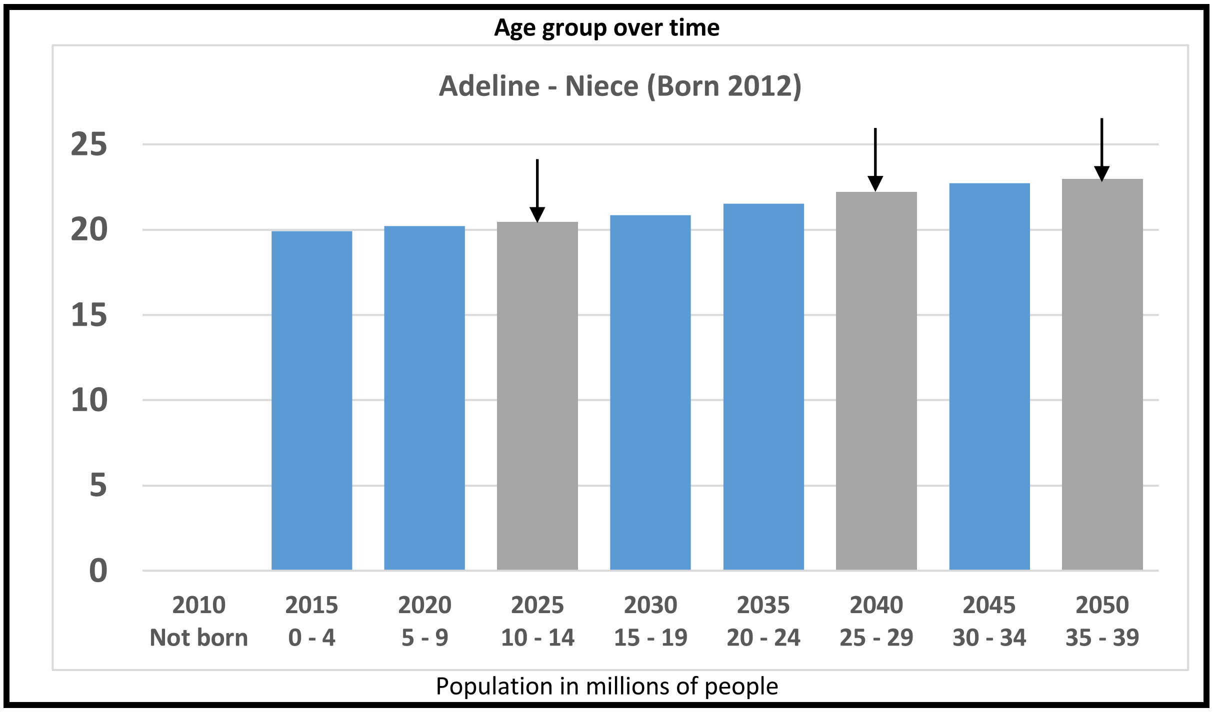
12. Use your calculations from **Handout 5** to complete the following bar graph of Kristin's age groups. Recall that Kristin was 36 years old in 2015.

    Missing age groups 40 – 44, 55 - 59, and 65 – 69 years old are included on the following graph:

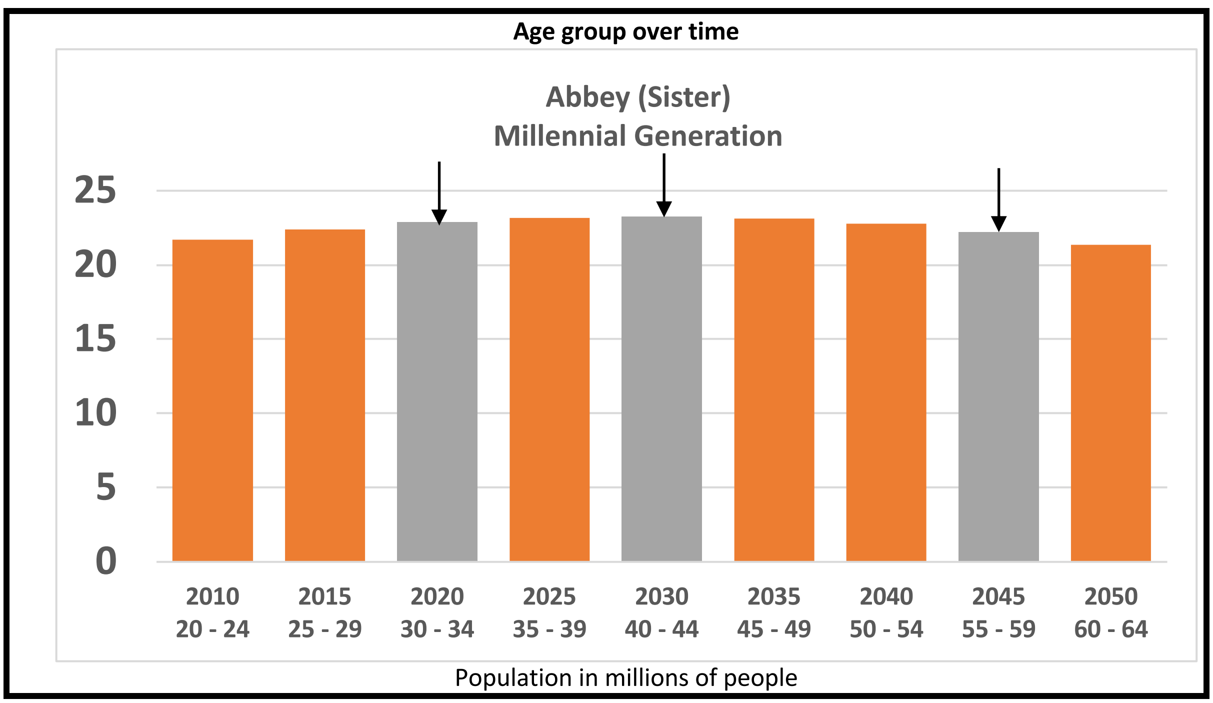


13. Recall that Kristin's niece Adeline was born in 2012. In what age group is she counted at the start of:
    a. 2015?
       Age group 0 – 4 years old
    b. 2020?
       Age group 5 – 9 years old
    c. 2035?
       Age group 20 – 24 years old
    d. 2050?
       Age group 35 – 39 years old

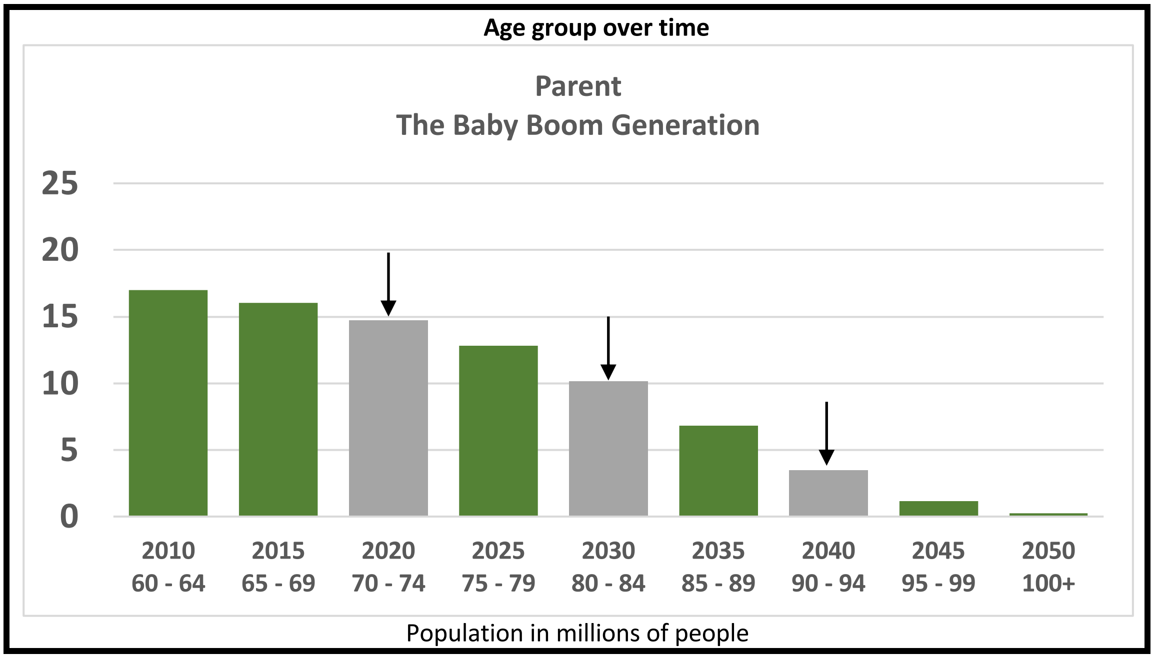
14. Using **Handout 5,** complete the following bar graph representing Adeline's age groups from 2015 to 2050:
    Missing age groups 10 – 14, 20 – 24, and 30 - 34 years old are included on the following graph:



15. Using **Handout 5,** complete the following bar graph representing Kristin's sister Abbey age groups from 2010 to 2050. Recall that Abbey was 26 years old in 2015.
Missing age groups 30 – 34, 40 – 44, and 55 – 59 years old completed on the following graph:



16. Using **Handout 5**, complete the following bar graph representing Kristin's mother's age groups from 2010 to 2015. Recall that Kristin's mother was 66 years old in 2015.
Missing age groups 40 – 44, 50 – 54, and 60 – 64 years old completed on the following graph:



17. Examine the completed graph for Adeline. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?
The counts of the age groups that include Adeline increase during each 5-year period. The first count at the start of 2015 was approximately 20 million people. The count at the start of 2050 for the age group 35 – 39 years old is projected to be approximately 23 million people. The primary reason for the increases is fewer deaths and higher counts of immigration.

18. Examine the completed graph for Kristin's parent. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?
The counts of the age groups that include Kristin increase slightly from the start of 2010 to 2020. After that, each age group that counts Kristin decreases. The count in 2010 was approximately 20 million people for 30 – 34 years old. The count of the age group 70 – 74 years old in 2050 is projected to be approximately 16 million people. The decrease in the count of people in each age group after 2020 is due to the number of deaths.

Assessment Ideas:

Assessment Task:

Consider the following assessment and assessment ideas for this lesson. Students should have access to a computer lab and Excel or a compatible spreadsheet program to complete the rest of this module. Provide time for students to work with the first introduction of the recursive model. An Excel file entitled "The 1 Country.xls" is designed to illustrate the implications of changes in age groups resulting from the population factors over time. This file is included in the Projection Files section of the Teacher Edition.

After providing the file to all students with access to a computer, draw attention to the initial set-up of the imaginary country (the "1 Country"). All population factors and counts for 2010 and 2015 and subsequent years are 1. Ask students to explain what a population factor of 1 means. The only cells that are not 1 are the cells that will be impacted by the Foundation Factor introduced in the next lesson, or age groups that do not have a count in the previous 5-year period. Discuss the following 3 options (all provided on the Student Edition for this lesson) for students to explore with the spreadsheet. Consider assigning a different option to each student or small group of students.

Option 1:

Before you change the values in a specific cell, describe the total population of this imaginary country from 2010 to 2050.

What if it was determined that a person in this country who was 36 years old at the start of 2015 was not counted? Enter the value of 2 in the appropriate cell of the spreadsheet for "1 Country?"

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

The entry of 2 in the 2015 count for the age group 35 – 39 changes the population factor for that age group to 2. This indicates that every previous count for this age group is multiplied by 2 (or population doubles in those cells). The overall population increases to 22 for 2015. Although cells increase as a result of this change over time, the top cells also change to 0 (add 1 to a cell and subtract 1 to another cell). The total population remains at 22 for each 5-year period. The domino effect is visible by observing the changes of the cells based on the change in the population factor. Ask students what this indicates about a country if the 2 represented 2 million people.

Option 2:

What if it was determined that a person in this country who was 36 years old at the start of 2010 was not counted? Enter a 2 in the appropriate cell of the spreadsheet for “1 Country”?

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

The entry of 2 in the 2010 count for the age group 35 – 39 years changes the population factor for that age group to 0.5. The overall population goes down to 1.5 people during each 5 years (not possible unless the values are represented by a different unit – for example, what if the values represented 1 million people?) The domino effect is again visible. The changes would be a result that as the counts moved forward, the population factor indicated that less people were counted due to death or moving out of the country.

Option 3:

What if a person who was 36 years old moved into the country and was counted in the country at the start of 2020? Enter a 2 in the appropriate cell of the spreadsheet for “1 Country?”

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

The entry of 2 in the cell counting the 35 – 39 years old people in 2020 does not change the population factor – only the count in that one cell. The additional person carries through each 5 years, or the count of this person is noted in the next age group. The overall population increased by 1 from the population values before any changes were made. The domino effect is visible with this change.

Wrap up this assessment task with the following question:

- What is the connection of the calculations in this lesson to falling dominos?

Students suggest that the connection to dominos is that as one calculation is completed to estimate the count for an age group, several connected calculations need to be redone (like *falling dominos). The process continues like the impact of one falling domino causing another domino to fall and then another.*

If a spreadsheet application is not available, direct students to complete an **Exit Summary (Handout 13)** for this lesson.