Lesson 8
Looking Forward with an Arithmetic Sequence and a Linear Model

Why would estimating the population of a country 10, 20, or 30 years from the present be important? What if the estimates, or ***population projections***, are not accurate? What if the actual count of people turns out to be considerably higher than the estimates projected by the model? What if the actual count of people turns out to be considerably lower than the model estimates?

Kristin's Story – Chapter 4

It was 2018 and a vicious strain of flu was affecting much of the country. Complications from the flu were particularly serious for infants and older people. There was a flu vaccine, but either people did not get the flu shot or it was not as effective as vaccines were in the past. Kristin's clinic was overwhelmed with sick patients. Many other people were also asking for the flu shot, although doctors were not sure it would be effective this late into the flu season.

Kristin's job involved reporting the number of flu cases the clinic treated and the number of flu shots administered during a year. A vaccine for future strains and the next flu season were already in development. Estimating how many people the clinic should be prepared to service for the next several years was her challenge. This was frankly a major responsibility as the vaccines were expensive and often were not adequately covered by the clinic's budget. It was an even more important responsibility, however, to provide adequate care for patients. If an estimate of the number of people needing the vaccine turned out to be too high, the clinic would suffer a major financial loss, and as a result, health care services in other areas would be affected. If an estimate was too low, people would be put at risk of getting the flu, or at least, a more serious case of the flu.

Kristin needed to carefully track the number of flu cases and the number of flu shots administered to make an estimate for the next several years. She also analyzed the ages of the patients serviced during the last few years and their health history. These numbers were then matched with future estimates of the count of people in the age groups targeted for the clinic's services as well as the total population of people in the region. A lot was riding on accurate recordings of the clinic's services in the past and on obtaining reliable estimates of the future counts of people needing services. She wished she had some magical ability to see into the future.

Population models are designed to generate estimates of the future count of people for situations like the one described in Kristin's story. This lesson will focus on a specific type of model, called a linear model, that estimates the count of people in the United States several years into the future. The first linear model developed in this lesson begins with the following estimates of the count of people in 2010 and 2015 reported by the United Sates Census Bureau:

United States

	2010	**2015**
Total count of people	309,348,193	320,896,618

The above estimates are considered the best estimates of the actual population of the United States at the beginning of 2010 and 2015. As explained in previous lessons, however, they are nonetheless estimates. To make the calculations more manageable, the above Census estimates are further rounded off to the nearest millions of people. The linear model begins with the 2010 and 2015 counts of the United States population:

United States

	2010	**2015**
Total count of people (in millions of people)	309	321

Lesson 8 – Problems

Arithmetic Sequence

A list of numbers is sometimes referred to as a ***sequence***. Each specific number in the list is called an element. Often there exists a specific connection of one element of the list to the next element.

1. Calculate the difference of the 2015 and 2010 population estimates.

2. Add the above difference to the 2015 population. The result is an estimate for the 2020 population assuming the population increases by the same count from 2015 to 2020 as it did from 2010 to 2015. In the same way, add the difference to the 2020 estimate to obtain the 2025 population estimate. Complete the following table by continuing the process:

   United States

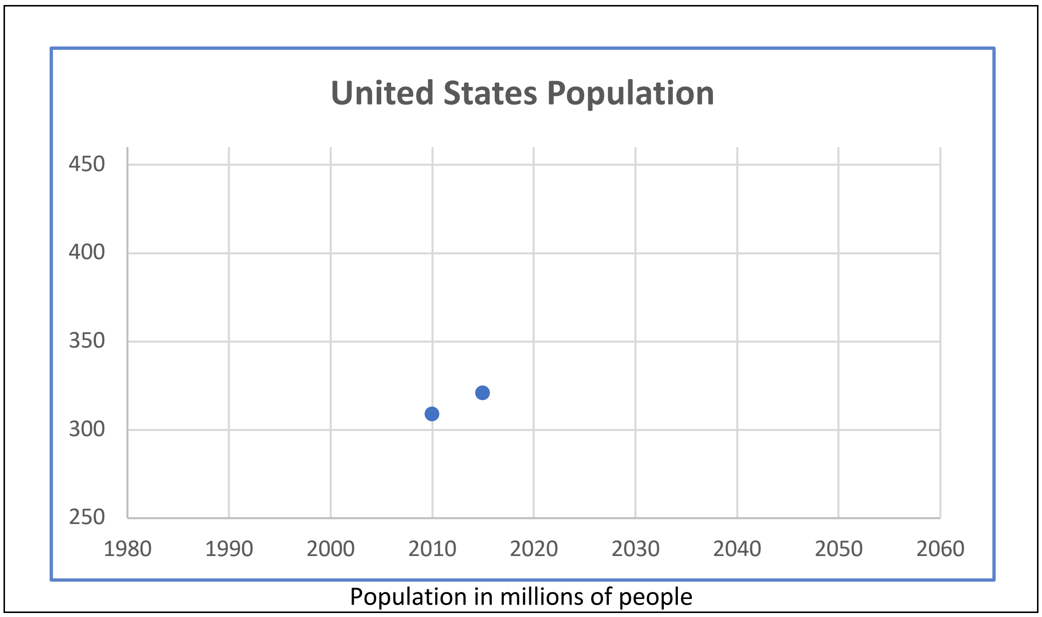
Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	309	321							

3. The above estimates form a list that is described as a ***finite arithmetic sequence***. Based on this example, how would you explain an arithmetic sequence to someone who was unfamiliar with it?

Linear Model

Consider the values from the above table as a collection of coordinate points. f

4. Consider the following coordinate grid in which the x-axis represents the year and the y-axis represents the estimated population values. Plot each of the coordinate points of the sequence in problem 3 on this grid or a grid of this type. The first two points are plotted for you.



5. Connect the coordinate points you plotted on the graph. What do you notice?

A major topic addressed in a study of algebra is deriving a linear equation from two given points. An equation derived from the plotted points forms a linear model of the population estimates of the United States population.

6. There are several procedures designed to derive an equation given two points. Consider the following steps to derive a linear equation by representing the coordinate values (x , y) for 2010 as (x_1 , y_1) and 2015 as (x_2 , y_2).
   a. Calculate the slope or change in population per year (to the nearest tenth) using the two points (2010, 309) and (2015, 321) as represented below:

      Slope = $\frac{y2-y1}{x2-x1}$

      Explain this slope as a change in the population in 1 year.

b. An equation called the "***point-slope***" equation derives the slope based on the two given points. The slope is then used with one of the two points to create a linear model. For this problem, let x represent the year and y represent the population in millions. Use the slope previously calculated from the two points (2010, 309) and (2015, 321). Use the first point (2010, 309) to complete the equation of the linear model. The steps are outlined below:

$y - y_1 = \frac{y2-y1}{x2-x1}(x - x_1)$ for (x_1, y_1)

Use the point (2010, 309) and the value of the slope to complete this linear model:

$y - \_\_\_\_ = \frac{(y2-y1)}{(x2-x1)}(x - \_\_\_\_)$

c. If an estimate of the population for 2020 is derived from this equation, what is the value of x used in this model? Derive an estimate of the population for 2020 using the above equation. Round your estimate of the 2020 population to the nearest millions of people.

d. Rework the above equation into a form called the "***slope - y-intercept***" linear equation. This form is also a representation addressed in a study of algebra. This form is often summarized as **y = mx + b** where **m** is the slope previously calculated in the above problem and **b** is the y-intercept, or the value of y when x =0. (There are, however, other representations of the ***slope - y-intercept*** equation. For example, a statistical representation of this equation is often summarized as $y = b_0 + b_1x$ were b_1 represents the slope and b_0 is the y-intercept.)

7. Answer the following.
   a. Indicate a description of the ***units*** needed to complete the following statement: The slope is "__________" per "_______"
   b. What does a positive slope indicate?
   c. What does a negative slope indicate?
   d. What does a slope that is equal to 0 indicate?

The primary goal of the next several lessons is to estimate future counts of the United States, Kenya, and Japan. The linear equation is used to derive an estimate of the future; however, the equation was derived by looking back at the changes in the population distributions from 2010 to 2015. Will the changes of the past continue into the future? Is it realistic to make an assumption that these changes continue? Is the linear model a good indicator of future population distributions?

Before we answer that last question, let's examine if the linear model provides accurate estimates of past counts?

8. The United States Census estimates for 1980, 1990, 2000, and 2005 are included in the following table. Use the linear model to derive population estimates for each of these years. Record your estimates in the table.

	1980	1990	2000	2005
US Census estimates	227	250	282	296
Linear model estimates				

9. Compare the estimates derived from the linear model to the counts of people reported by the Census Bureau.
   a. Calculate the difference between the linear model estimate and the Census Bureau for 2005. Do you think the linear model provided a good estimate for 2005? Explain your answer.

   b. In the same way, calculate the differences between linear model estimates and the estimates reported by the Census Bureau for 2000? 1990? 1980? Do you think the linear model provided a good estimate for those years? Explain your answer.

   c. Derive the estimates for the years 2010 and 2015 using the linear model. Are the estimates different from the previous estimates? Explain your answer.

10. The Census Bureau estimates that the population of the United States at the time of the signing of the Declaration of Independence in 1776 was approximately 2.5 millions of people. This estimate is not based on any census as the United States Constitution was not completed at that time. What is the linear model's estimate of the population in 1776? Do you think the linear model provided a good estimate for 1776? Explain your answer.

11. Estimate the following future counts of the United States using the linear model:
    a. 2020

    b. 2030

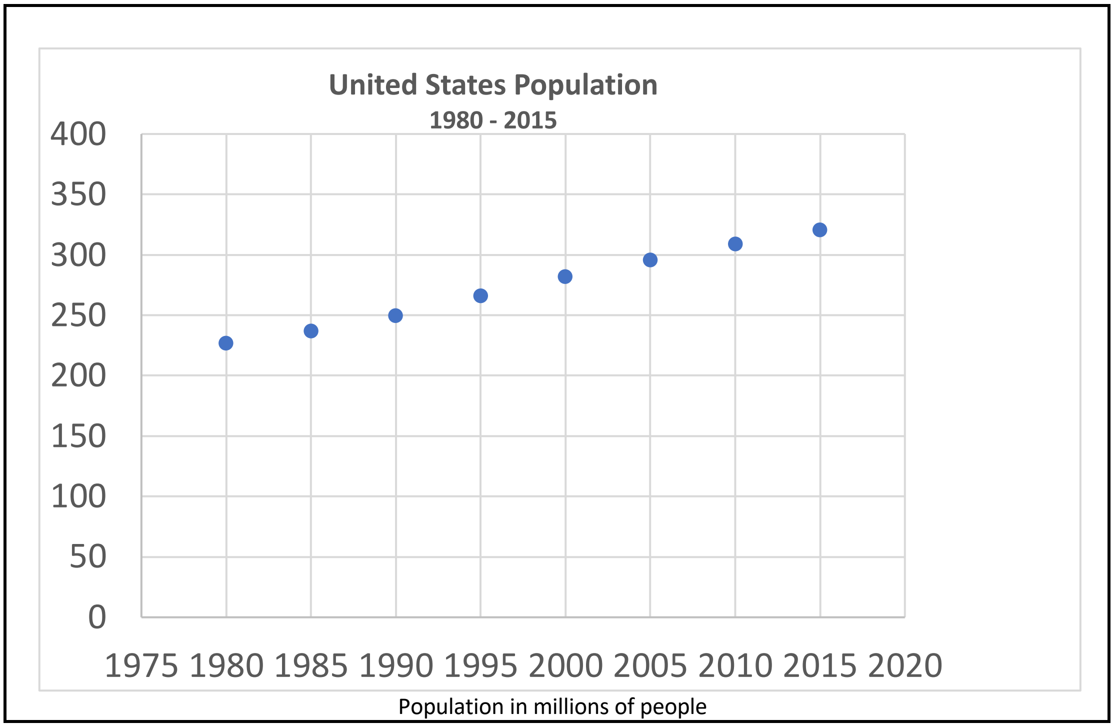
    c. 2050

    d. 2300

12. What years do you think the estimates derived in problem 11 are accurate predictions of the future? What years do you think the estimates derived in problem 11 are not accurate predictions of the future? Why would an estimate from a model like the linear model not be a good estimate?

There are several other models that could be developed to estimate the future. The remaining lessons in this model will analyze a few of them. There are also other linear models. Consider the data collected from the Census Bureau of the United States population from 1980 to 2015 to the nearest millions of people and a scatter plot based on this data. Each of these counts is considered the best estimate of the actual population for the years 1980 to 2015.

Year	**1980**	**1985**	**1990**	**1995**	**2000**	**2005**	**2010**	**2015**
Population (in millions of people) US Census Bureau	227	237	250	266	282	296	309	321



A topic you studied in statistics based on a scatter plot like the one in problem 12 also involved a linear model . Consider drawing a line drawn through the points of the scatter plot described as a ***best-fitting line***. A best-fitting line is a linear model that forms the best estimates of the actual points of the scatter plot. (Sometimes the actual point and the point on the line for a specific value of x are close, and other times they are far apart.)

13. Sketch a line that you think is a best-fitting line of the scatter plot. Identify any two points on the best-fitting line you drew and derive a linear equation based on the two points you selected using the point-slope summary. (The points you select will probably not be points represented by the scatter plot.) Determine the estimates of the United States population for the following years using the equation of the best-fitting line. Estimate each population to the nearest millions of people:

Year	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population estimates (in millions of people)							

14. Compare the estimates derived from the linear model in problem 6 to the estimates derived from the best-fitting line.

Year	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population estimates from model in problem 6 (in millions of people)							
Population estimates from best-fitting line (in millions of people)							

Do the models produce similar estimates? Explain.

Consider the 2010 and 2015 population counts for Kenya and Japan.

15. Complete the following table for Kenya by adding the difference of the 2015 population and the 2010 population to obtain the next population estimates.

Kenya

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
Population (in millions of people)	41	46							

16. Derive a linear model for the above population estimates for Kenya in the same way as the linear model was developed for the United States in problem 6.

17. Complete the following table for Japan by adding the difference of the 2015 population and the 2010 population to obtain the next population estimates.

Japan

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	128	127							

18. Derive a linear model for the above population estimates for Japan in the same way as the linear model was developed for the United States in problem 6.

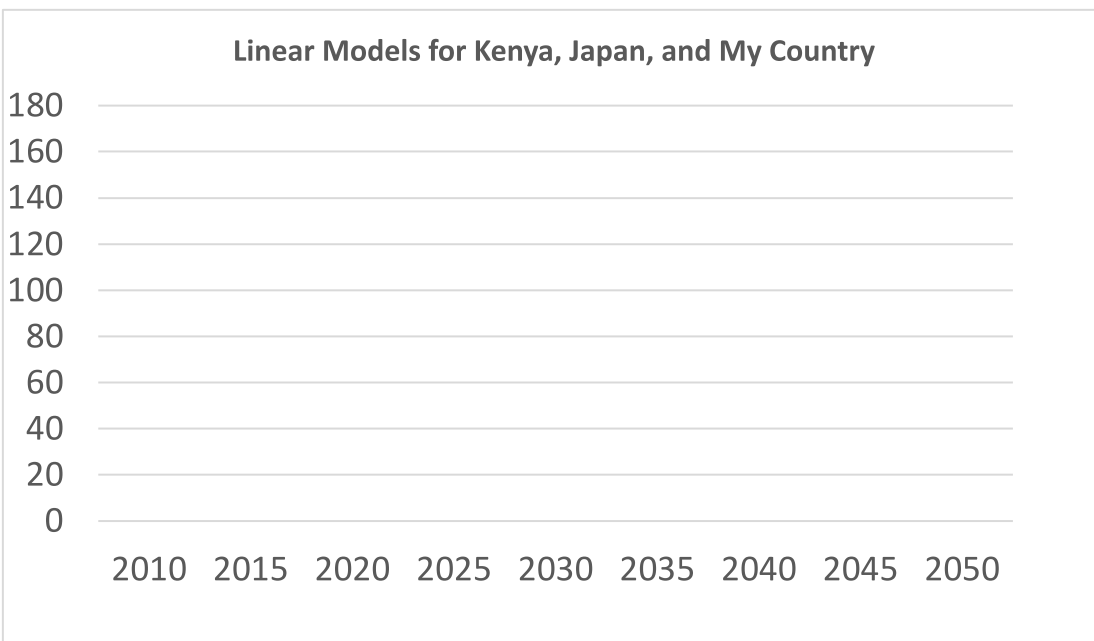
19. Finally, in the same way as you completed the above tables, complete a table for the population you created for your own country, or the My Country data.

My Country

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
Population (in millions of people)									

20. Derive a linear model for the population estimates of My Country in the same way as the linear model was developed for the United States.

21. Use the following grid to record the projected population values for Kenya, Japan, and My Country based on the linear models developed in this lesson.



Population in millions of people

The linear models developed in this lesson provide estimates of the total population in the future. The models do not, however, provide estimates of the age groups that make-up the population. This challenge will be addressed in lessons that follow.