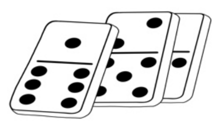
Lesson 11
The Recursive Model and Falling Dominos

This lesson derives future counts of connected age groups based on changes from 2010 to 2015. Over the course of 10 years, 20 years, and 30 years, people in each of the connected age groups grow older, leave the country, come into the country, or die. How do these changes affect the overall count and shape of the country in the future? This lesson is similar to what happens when dominos are lined up. When the first domino is knocked down, the next domino (or, the future) falls, followed by the next domino, until the last domino is knocked down. In this lesson, when one age group changes its count in 5 years, it will affect the count of a connected age group in the next 5 years, and that count will affect a connected age group 5 years after it changed. As indicated, the overall effect is similar to what happens with falling dominos.



Lesson 11 – Problems

Handout needed to complete the following problems:

Handout 5: *Looking Forward for the United States (Student Edition)*

A decision regarding the accuracy of the calculations is needed before the population factors are used to estimate the future counts of people. Recall that a ***Population Factor*** is the decimal based on the ratio derived between connected age groups. Similar to Lesson 10, this ratio will be represented to the nearest thousandth. A mathematics equation using a specific population factor is summarized below.

(*Population of age group 0 – 4 years old 2010*) X *(Population Factor for connected age groups)* **=>**

$$\left(\textit{Population of age group } 0-4 \textit{ years old } 2010\right) \times \frac{Population\ of\ age\ group\ 5-9\ for\ 2015}{Population\ of\ age\ group\ 0-4\ for\ 2010} = \textit{Population of age group } 5-9 \textit{ for } 2015$$

$$\uparrow$$

(Population Factor for connected age groups 0 – 4 to 5 – 9 years old)

1. Complete the multiplication indicated in the following table. The first calculation has been completed for the 0 – 4 age group of 2010 to the 5 – 9 age group of 2015 as set up in the above equation. The population factors listed were derived in Lesson 10.

Population of Age group in **2010**	X	Population Factor	=	Population of Connected Age Group in **2015**	Census Bureau counts for **2015**
0 – 4 20,189,589	X	1.014	=	**5 – 9** 20,472,243	**5 – 9** 20,481,130
5 – 9 20,331,807	X	1.013	=	**10 – 14**	**10 – 14** 20,605,579
10 - 14 20,681,214	X	1.020	=	**15 – 19**	**15 – 19** 21,084,710

2. Why are the answers derived in problem 1 not equal to the estimates of the Census Bureau?

To make the calculations more manageable, population estimates for 2010 and 2015, along with the population projections for 2020 and beyond, will be represented as millions of people to the nearest hundredth. For example, 20,189,589 will be represented as 20.19 millions of people.

If the population factors derived in **Handout 4** remain constant during the five years from 2015 to 2020, what changes will result for connected age groups? What will be the shape of the United States in 2020? 2025? … 2050? What if the population factors were not constant? The following problems begin answering these questions.

3. Examine the connected age groups of 0 – 4 years old in 2010 to 5 – 9 years old in 2015. A population factor of 1.014 indicates an approximate increase of 1.4% in the 5 – 9 years old age group in 2015 when compared to the 0 – 4 years old age group in 2010.

    a. Using the above summary of the changes in these age groups, is it possible to derive the number of people who moved into the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

b. Is it possible to derive the number of people who moved out of the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

c. Is it possible to derive the number of people who died from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

d. What was the dominate explanation for the change in the number of people from 2010 to 2015 of the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain your answer.

Projection models are based on assumptions to estimate the count of people in the future. At some future time, however, the actual outcomes are then compared to the predicted outcomes to evaluate the projection models. The recursive model designed in this lesson starts with the assumption that the ***population factors remain constant over time***. In other words, the population factors derived in **Handout 4** of the previous lesson are assumed to remain constant from 2020 to 2050.

As previously stated, the population factor 1.014 for the age groups 0 – 4 in 2010 to 5 – 9 in 2015 indicates a 1.4% growth in the count of people. Assume that same rate is used to estimate the count of people 5 – 9 years old in 2020 based on the count of people 0 – 4 years old in 2015. The population factor 1.014 will also be used to derive the count of the 5 – 9 years old age group in 2025 once an estimate of the count of 0 – 4 years old age group is derived for 2020. This population factor of 1.014 is continued to estimate counts of the connected age groups until 2050. In the same way, the other population factors are used to estimate future counts.

The following problem sets up this process for projecting the count of people for each age group in 2020. The model assumes that the population factors derived from the connected age groups of 2010 to 2015 remain constant for the next 5 years.

4. Population factors listed in the following table were recalculated based on the approximations of millions of people in 2010 and 2015. The 2015 population estimates were also rounded off as indicated. Complete the following table. Several age group projections have been completed. (Note: Some of the posted estimates may differ from your calculations due to the software used to implement the recursive model. The differences are minor. Allow a difference of +/- 0.01 of your answers and the answers posted.)

Age group	**Population 2015** (in millions of people to the nearest hundredth)	**Population Factor**	**Population 2020** (in millions of people to the nearest hundredth)
0 – 4	19.91	1.014	
5 - 9	20.48	1.014	20.20
10 - 14	20.61	1.020	20.76
15 - 19	21.09	1.032	21.02
20 - 24	22.69	1.032	21.77
25 - 29	22.40	1.022	
30 - 34	21.62	1.012	22.90
35 - 39	20.31	1.004	
40 - 44	20.16	0.995	
45 - 49	20.80	0.985	20.05
50 - 54	22.29	0.974	20.48
55 - 59	21.77	0.962	
60 - 64	19.04	0.945	20.93
65 - 69	16.05	0.917	
70 - 74	11.48	0.869	
75 - 79	8.12	0.792	
80 - 84	5.80	0.670	
85 - 89	3.86	0.508	3.89
90 - 94	1.85	0.340	
95 – 99	0.50	0.205	
100 +	0.08		

5. Recall that Kristin was 36 years old at the start of 2015. In what age group would she be counted in 2020?

6. One **age group** for 2020 (the shaded cell) is left blank by this process. What age group is left blank? Explain why this age group does not have a population factor for this recursive model. (This particular age group is called the **Foundation Layer** and will be the focus of the next lesson.)

7. One **population factor** also has no entry in the above table. Why is there no population factor entered for this age group?

The Falling Dominos

The recursive process is now ready to begin. The projections for 2025 will be based on the above projections from 2020. In turn, these results will be used to calculate projections for 2030. The process will continue for 2035, 2040, 2045, and 2050. Study **Handout 5: Looking Forward for the United States**. The population factors are provided, along with the estimated population of the United States in 2010 and 2015.

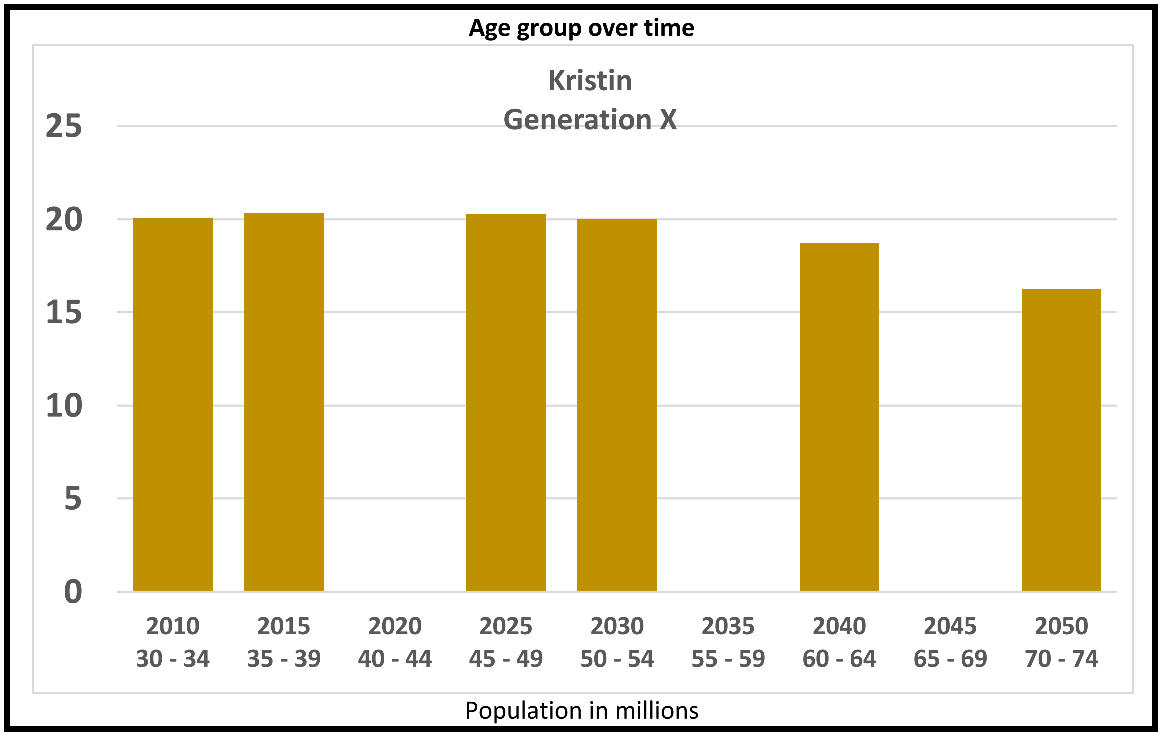
8. Record the missing 2020 projections in the column identified as 2020 in **Handout 5**. Be careful not to provide a projection for the age group 0 - 4 years old in 2020.

9. Use the Population Factors to complete the projections for 2025 from the 2020 projections based on the previous 5-year projections. Several calculations have been completed in the handout. (Remember, round your answers to the nearest hundredth. Each value indicates the count of people in millions.)

10. In 2020, there was one age group that was left blank (the 0 – 4 years old age group). In 2025, there are two age groups that are blank. Why is this second age group (the 5 – 9 years old age group) blank?

11. Complete the missing calculations in **Handout 5** for the years 2030, 2035, 2040, 2045, and 2050. Again, several calculations have been completed. Use the completed calculations to check your work.

12. Use your calculations from **Handout 5** to complete the following bar graph of Kristin's age groups. Recall that Kristin was 36 years old in 2015.



13. Recall that Kristin's niece Adeline was born in 2012. In what age group is she counted at the start of:
    a. 2015?

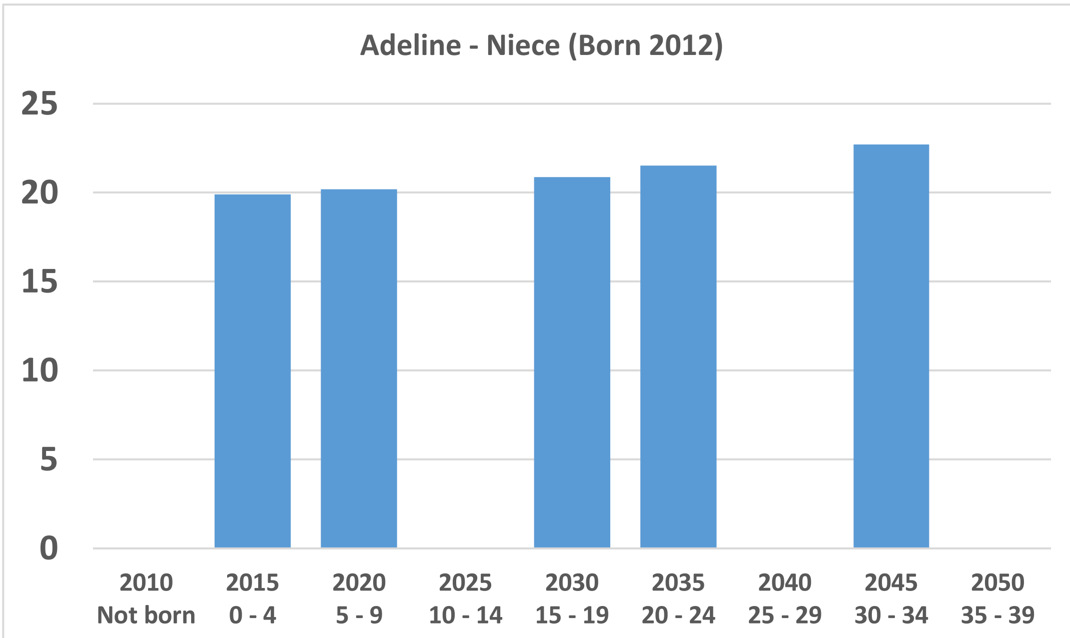
    b. 2020?

    c. 2035?

    d. 2050?

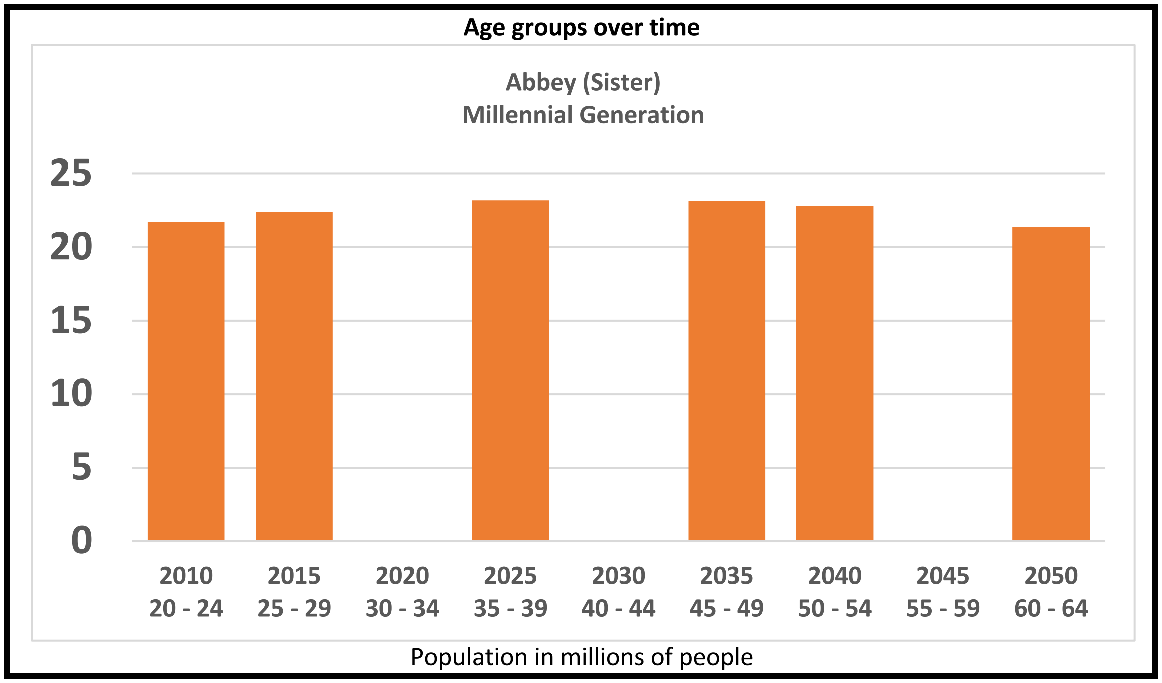
14. Using **Handout 5,** complete the following bar graph representing Adeline's age groups from 2015 to 2050:

Age group over time

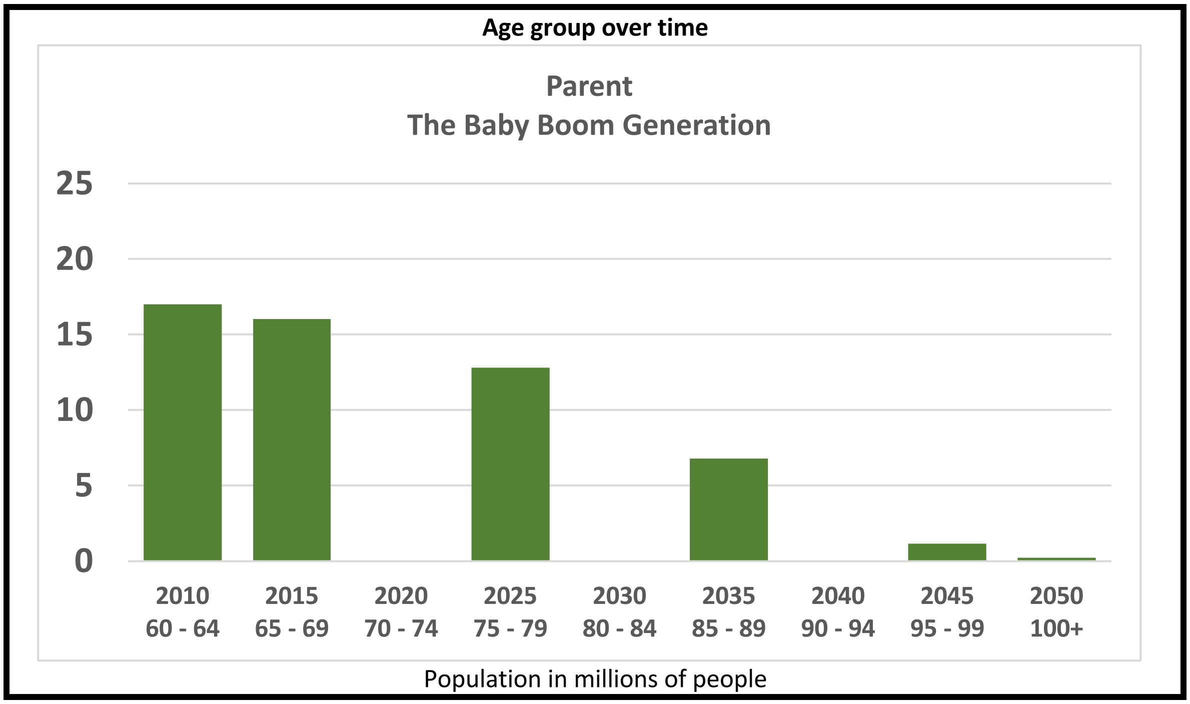


Population in millions of people

15. Using **Handout 5,** complete the following bar graph representing Kristin's sister Abbey age groups from 2010 to 2050. Recall that Abbey was 26 years old in 2015.



16. Using **Handout 5**, complete the following bar graph representing Kristin's mother's age groups from 2010 to 2015. Recall that Kristin's mother was 66 years old in 2015.



17. Examine the completed graph for Adeline. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?

18. Examine the completed graph for Kristin's parent. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?

Extension or Assessment: Explore the following options as directed by your teacher using the Excel file ***"The 1 Country".xls.*** After loading the file, carefully look at the set-up of the country. This file summarizes the population of an imaginary country in which all of the counts in each age group is 1. It also indicates that the population factors for each connecting age groups are 1. The cells with a 0 value will be discussed in the next lesson.

Option 1:

Before you change the values in a specific cell, describe the total population of this imaginary country from 2010 to 2050.

What if it was determined that a person in this country who was 36 years old at the start of 2015 was not counted? Enter the value of 2 in the appropriate cell of the spreadsheet for “1 Country ” to add this person to the 2015 census.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

Option 2:

What if it was determined that a person in this country who was 36 years old at the start of 2010 was not counted? Enter a 2 in the appropriate cell of the spreadsheet for “1 Country” to add this person to the 2010 census.

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

Option 3:

What if a person who was 36 years old moved into the country and was counted in the country at the start of 2020? Enter a 2 in the appropriate cell of the spreadsheet for “1 Country”.

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

Answer the following question:

- What is the connection of the calculations in this lesson to falling dominos?