Teaching Notes

Lesson 10
Looking Forward with a Recursive Model (Present to Past … to Future)

Overview:

An important tool in designing the ***Recursive Model*** is a population factor. The age group 0 - 4 years old in 2010 is counted to the 5 - 9 years old age group in 2015. The two age groups are defined as connecting age groups. A population factor is calculated by deriving a ratio of connecting age groups and representing that ratio as a decimal. The story behind the population factors is that:

- factors greater than 1 identify connecting age groups in which the change in population during a 5-year period is dominated by immigration,
- factors less than 1 identify connecting age groups in which the change in population during a 5-year period is dominated by death or emigration.
- factors equal to 1 indicate no change in the population of the age groups during a 5-year period.

The problems and questions use the 2010 to 2015 population data to form the ratios from connecting age groups and then summarize what the ratios indicate about the changes in age groups. The subtitle (***Present to Past … to Future***) is based on the connection that the count of people in 2015 (considered the present in this module) is compared to the count of people in 2010 (the past). The ratios are then changed to decimals and percent that will be used to predict the future. Students are continuously asked to explain what these ratios mean about the changes in the count of important age groups.

This lesson further pushes students' thinking to levels 3 and 4 of the **Modeling Continuum**. Students are asked to interpret the implications of the population factors and what they indicate about the changes in the population of a country. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1,7	Problems: 2, 8, 11, 13, 14	Problems: 3, 4, 5, 6, 9, 10, 12	Problems: 15, 16, 17, 18, 19, 20, 21

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 10 for each student. This lesson requires 2 additional handouts for students to complete the problems, **Handout 1: *United States – 2015*** and **Handout 4: *United States Connected Age Groups***. Provide an electronic or a printed copy of the handouts.

Launch:

Begin this lesson by directing students to read ***Kristin's Story – Chapter*** 6. After they have read the story, ask them questions about the characters in the story. Consider the following questions:

- What age was Kristin in 2010?
- What age was Kristin in 2015?
- Was Raphine counted in the 2010 census of the United States? Why or why not?
- In what age group was Adeline counted at the start of 2015?

Work with students who have problems answering these questions by highlighting the details in the story that answer these questions.

Consider forming small groups to work on these problems as discussions among students might help them understand the key points of this important lesson. If the problems are assigned individually, consider providing more completed answers to **Handout 4**. The design of **Handout 4** is to promote small group work.

Implementation ideas:

The problems are designed to build students' understanding of the lesson's objectives. Note several summaries are provided to organize the objectives in the student lesson. Use these summaries as opportunities to have a brief whole group discussion of the problems. Wrap up this lesson with a discussion of the questions highlighted in the assessment ideas.

Student responses or descriptions

Lesson 10 - Problems

1. Examine again the **Handout** 1: ***United States - 2015***. In what age group was Kristin and Raphine counted in 2015?
   Kristin and Raphine are in the 35 – 39 years old age group in 2015.

2. Would a summary of the United States population in 2010 include Raphine? Explain your answer.
   No. Raphine moved to the United States in 2011, therefore, at the start of 2010, he was not counted in the United States population.

For the following problems, use **Handout 4: *United States Connected Age Groups.***

3. At the start of 2010, there were 20,189,589 people who were 0 to 4 years old. At the start of 2015, there were 20,481,130 people who were 5 to 9 years old. What is the connection of these two age groups?
   The people who were 5 - 9 years old in 2015 are for the most part the people who were 0 – 4 years old in 2010. After the start of 2010, the count of people in the 0 – 4 years old age group changed by people moving into the country or moving out of the country or dying. At the start of 2015, the resulting count of people in the age group 5 – 9 years old suggests what happened to the 2010 age group of 0 – 4 years old. Note, there were more people counted in the age group 5 – 9 years old in 2015 than in the 0 – 4 age group at the start of 2010.

4. At the start of 2010, there were 21,983,206 people 15 to 19 years old. In what age group will these people be counted at the start of 2015? How many people were counted in that age group?
   The group of people in the 15 to 19 years old in 2010 will be in the 20 – 24 years old group in 2015. There were 22,693,026 people in that age group.

5. Adeline was born in 2012. In what age group was Adeline counted in 2010? Explain your answer.
   She was not counted at the start of 2010 as she was not born yet.

6. In what age group would Adeline be counted in 2015?
   Adeline would be counted in the 0 – 4 years old age group in 2015. She would turn 3 years old during that year.

Examine the age groups that are described as connected age groups in ***Handout 4***. This handout indicates a connection of the age group 0 – 4 in 2010 to the age group 5 – 9 in 2015. For the recursive model developed in this module, these age groups are called ***connected age groups***.

7. What is the ratio of the count of people 5 to 9 years old at the start of 2015 to the number of people 0 to 4 years old at the start of 2010?
   The ratio is 20,481,130 people to 20,189,589 people.

8. The ratio is represented by a decimal in column 4, or 1.014 to the nearest thousandth. As stated in problem 7, this decimal is defined as the **population factor** for the connected age groups. This population factor is greater than 1 for this example. What does that tell you about the connected age groups?
   A population factor greater than 1 indicates that the count of people in the 5 – 9 years old age group in 2015 is greater than the count of people in the 0 – 4 years old age group in 2010. The increase is explained by immigration as that is the only way that people in the connected age groups could increase the counts of people.

9. During the 5 years summarized on the table, what is the approximate percent increase of people 5 - 9 years old in 2015 based on the count of people who were 0 – 4 years old in 2010? Is the percent increase also part of the population factor for these connected age groups? Explain.
   First step is to determine the difference of the counts, or:
   20,481,130 – 20,189,589 = 291,541 people.
   Divide the above difference by the people in the 0 – 4 years old age group, or
   291,541 people / 20,189,589 = 0.014 to the nearest thousandth.
   Convert to a percent, or 1.4%
   Note the population factor for these age groups is 1.014. This factor also indicates the count of people in the connected age groups will increase by 1.4%.

10. What is the explanation for the growth in the connected age groups with a population factor greater than 1?

    Connected age groups with a population factor greater than 1 indicate that the increase in the counts were based on immigration or people moving into the country.

11. During the 5 years summarized on the table, what is the approximate percent of change of the count of people 55 – 59 years old in 2015 who were 50 – 54 years old in 2010?
    Count of people 55 – 59 years old in 2015 is 21,767,855, and the count of people 50 – 54 years old in 2010 is 22,353,471. The difference is:
    (21,767,855 – 22,353,471) = - 585618
    The rate of change: -584618/22,353,471 which is approximately -0.026 to the nearest thousandth. The percent change is approximately – 2.6%. Note, the percent change is negative. The population factor would be (1 – 0.026) or 0.974.

12. What is the explanation of the changes in the connected age groups with a population factor less than 1?
    The changes in the count of the connected age groups are a result of people moving out of the country or dying during the 5 years from 2010 to 2015.

Complete the calculations missing in ***Handout 4***. After you have completed the handout, answer the following questions:

To check students' work, see **Handout 4 *(Teacher Edition).***

13. What is noticeable about the population factor for the connected age groups of 40 – 44 years old in 2010 to 45 - 49 years old in 2015 when compared to the population factors for younger connected age groups??

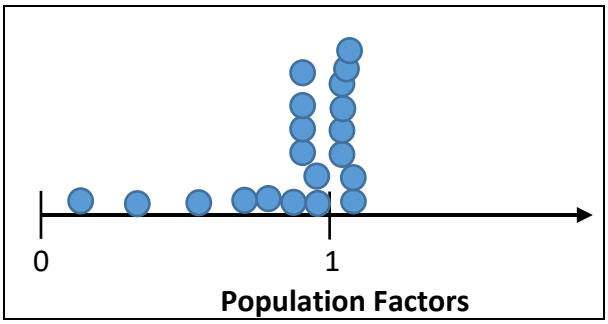
    The population factor of these two connected age groups is less than 1 (the first population factor less than 1).

14. What happens to the population factors for connected age groups that count people 40 years old or older in 2015?

    The population factors for connected age groups in which people are 40 years old or older are close to 1 or less than 1.

Any change in the count of people counted after the start of the year are a result of people moving into this country (immigrating), people leaving this country (emigrating), and dying. The following problems examine the collection of population factors for the United States.

15. Place a dot for each of the population factors derived on **Handout 4** on the above Population Factors number line. (Stack dots if they are close to each other.)



(Essentially look for 8 dots slightly greater than 1, 6 dots slightly less than 1, and 6 dots less than 1 as estimated in the above example.)

16. Why is 1 considered an important value in interpreting a population factor? Explain your answer.
A population factor of 1 indicates the count of the age group did not change during 5-years. This result suggests that no one died, or the number of deaths was equal to the number of people moving into the country. Any factor greater than 1 indicates the population increased. Any factor less than one indicates the population decreased.

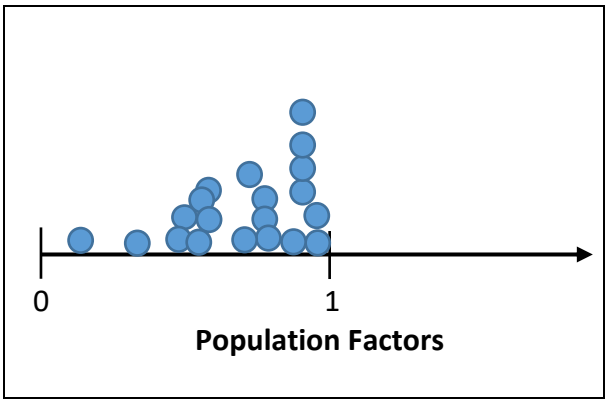
17. Is it possible for the population factor of connecting age groups be equal to 0? Explain your answer.
Yes. If everyone after the start of the year in an age group died or moved out of the country, and no one moved into the country during the 5-year period, the count in the connecting age group would be 0.

18. Changes in the count of people over a 5-year period of connected age groups are explained by **birth**, **death**, **immigration**, and **emigration** (people leaving a country). Use the value of the population factor to Identify what changed the counts in the following Connected Age groups from 2010 to 2015. Identify the most dominant explanation for the changes in the connected age groups in the last column. (The first connecting age group is completed for you.)

Connected Age groups from 2010 to 2015	Population Factor for the connected age groups	What could explain the changes in the count of people in the connected age groups?	What was the dominant explanation of the change in the connected age groups?
0 – 4 to 5 – 9	1.014	Immigration, emigration, deaths	Immigration
20 – 24 to 25 – 29	*1.032*	*Immigration, emigration, deaths*	*immigration*
40 – 44 to 45 – 49	*0.995*	*Immigration, emigration, deaths*	*deaths*
85 – 89 to 90 – 94	*0.509*	*Immigration, emigration, deaths*	*deaths*
95 to 99 to 100+	*0.205*	*Immigration, emigration, deaths*	*deaths*

19. Summarize what a population factor indicates about the connected age groups.
Population factors provide an explanation for the change in counts of connected age groups. If a population factor is less than 1, then the dominate explanation for the change is death or people moving out of the country or both. People could still move into the country, but the count is less than the count of people dying and moving out of the country. If a population factor is greater than 1, then the dominate explanation for the change is immigration. People could still move out of the country or die, but the counts are less than the count of people moving into the country. A population factor equal to 1 indicates that the counts that decrease connected age groups by death and leaving the country equal the count that increases the age group by immigration.

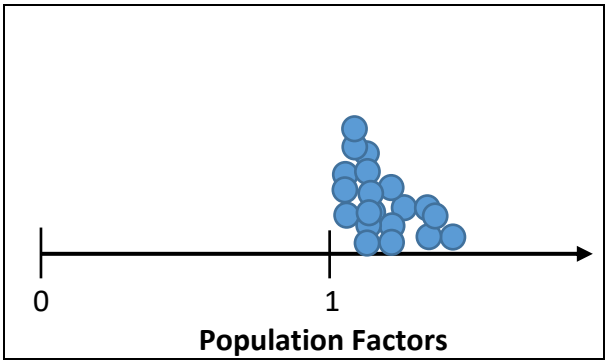
20. Consider the following dot plot of the population factors of a fictitious country:



a. What is the dominant explanation of change in the connecting age groups for a country represented by the above dot plot?
The dominant explanation of change is death or people leaving the country.

b. Do you think it is possible for a country with the above population factors to have an increase in its total population during a 5-year period? Explain.
Yes. Population factors do not provide a summary of births or immigration in the 0 – 4 years old age group during the previous 5 years. There is no connecting age group to the 0 – 4 years old age group. As a result, a large count of births or possibly a large count of children who immigrated into the country could still result in a total population increase.

21. Consider the following dot plot of the population factors for another fictitious country::



a. What is the dominant explanation of change in the connecting age groups for the above dot plot?
The dominant explanation of change is immigration

b. Do you think it is possible for a country with the above population factors to have a decrease in its total population during a 5-year period? Explain.
The above dot plot would suggest a country with all population factors greater than 1 would increase its population. In most cases, this summary is accurate. However, if the country had an unusually large count of older people (for example 90 - 94 years old, 95 – 99 years old, 100+ years old) and an unusually small count of younger people in 2015, the changes in 5 years could decrease. Over a longer period of time, the counts increase. (After students have an understanding of the recursive models as organized in the EXCEL files, you may want to return to this question and allow students to play around with the 2015 counts to see if they can configure a 2015 population that results in a declining population in 2020.)

How can the population factors be used to estimate future counts, or population projections? Applying the above population factors to the actual count of people in 2015 will start the recursive model.

Assessment ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of this lesson.

The city of Awesome had 1000 people who were 0 – 4 years old in 2015. The population factor for the connecting layer of 0 – 4 years old to 5 – 9 years old is 1.05.

a. What is an estimate of the count of people for the 5 – 9 years old in 2020?

b. Explain the reasons behind the change in population for 0 – 4 years old to 5 – 9 years old.

c. The population of 85 – 89 years old in 2010 was 200 people. The population of people who were 90 – 94 years old in 2015 is 300 people. What is the population factor for these connecting layers?

d. What do you know about the changes in the connecting layers in (c)? What reasonable explanation might explain the changes?

Comments on the Assessment Task:

The questions expect students to recognize that a population factor of 1.05 indicates an anticipated increase of 5% of the population in the connecting layer of 5 – 9 years old for 2020, or a 5% increase in the count of 1000 people. An estimate of 5 – 9 years old in 2020 would be 1050 people. The primary reason for the change would be more people moving into the country during the 5-year interval.

The population factor for (c) would be 1.5 as there is a 50% increase in the population during the 5-year period. An explanation might be related to a health issue or a lifestyle issue. For example, this country may have a particularly attractive climate for older people, resulting in older people moving into the country.

Additional Assessment Ideas:

Students' explanations to questions 19 – 21 provide a key insight to their understanding of population factors. What a population factor indicates about a connecting age group is a main component of the recursive model that will be continued in the following lessons.

Do not necessarily expect students to correctly explain whether or not a country will increase or decrease in population over time as that is related to an understanding of another feature of this recursive model, the foundation factor. If students' answers do not address the count of the 0 – 4 years old age group in the future, revisit the questions about when Adeline was counted in this lesson to remind them that this age group impacts the total population differently than the connected age groups. The Foundation Factors will be developed in the next several lessons and will complete the design of the recursive model. Questions 19 – 21 are designed to have students think about all of the age groups, and their possible impact on the country's population.

Consider assigning an **Exit Summary** for this lesson.