Teaching Notes

Lesson 12
Completing the Recursive Model with the Foundation Layer

Overview:

This lesson completes the development of the Recursive Model. Counts for each of the other age groups in 2020 and beyond were derived in Lesson 11 by multiplying the appropriate population factors and the counts of the connected age groups. The foundation count, or the count of the 0 – 4 years old age group in 2020, has a major impact on a country's future. The previous lesson provided an estimate of the total of the 2020 population except for the people who were born or moved into the country within the ages of 0 – 4 years old after the start of 2015. The "hole" (or blank cell) left by the unknown estimate of the 0 – 4 years old is filled in this lesson by making a second important assumption, namely, that the proportion of the total population who were 0 – 4 years old at the start of 2015 will be the same proportion at the start of 2020. This assumption is continued to the conclusion of this model, or 2050.

The domino effect is again visible. Filling the hole of the foundation age group in 2020 is then passed to the next iteration of estimating counts. In the same way, the next iteration for 2025, and then the other years highlighted in the handout, are derived. The handout that students complete has color-coded the cells representing the characters in Kristin's story. Students are encouraged to think about how the projected estimates (or the numbers) will impact these characters in the future. Throughout this module, reflecting on the impact of "crunching the numbers" on the characters was intentional.

This lesson further pushes students' thinking to levels 3 and 4 of the **Modeling Continuum**. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 2,3	Problems: 4, 5, 6	Problems: 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	Problems: 17, 18, 19, 20, 21

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent, recursion. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 12 for each student. This lesson also requires 1 additional handout for students to complete the problems, **Handout 6: *United States 2010 – 2050 (Student Edition)*.** Provide an electronic or printed copy of the handout.

Launch:

Start this lesson by discussing the data story, **Kristin's Story – Chapter 7**. This chapter highlights new characters who are important in the problems. Estimating the count of children who are born or under the age of 5 is derived differently in the recursive model. As mentioned in previous lessons, the young people in that category are not connected to a previous age group. They are the people who will be listed in the next population summary as 0 – 4 years old, but at the present, they are not counted. Incorporating this group of people into the model poses an interesting challenge and is the goal of this lesson. Who are these children, and how do they impact the future shape of the country?

Implementation Ideas:

After students read the data story, direct them to complete the problems individually or in small groups. Summaries or highlights of several problems are provided to help determine if students are understanding the material. Provide opportunities for students to verbally summarize their responses or answers of the problems. This lesson addresses an important feature of the recursive model that will impact the age group estimates for several decades.

Consider organizing a bulletin board highlighting the images of people who are considered the decision-making leaders, the entertainment or sports leaders, or the general cultural leaders of the country. Also consider including in the display advertisements designed to sell clothes, furniture, cars, food, etc. (Collect either printed ads or images from online advertisements.) Ask students to identify the target age groups of these ads. Would people 65 years old or older be interested in the ads? Would people in their 20's be interested in the items or services these ads are trying to sell? The dominate age groups (or the age groups with the greatest count of people) are a driving force of a country's economy and culture. Often the entertainment industry primarily targets the dominant age groups. If an age group is targeted that is not the dominant age group, then the challenge is even more critical that the targeted age groups are motivated by the ads. Continue these discussions throughout this lesson and the lessons that follow.

Lesson 12 – Problems

1. Adeline was born in 2012. What is her age at the start of 2015?
   She would be turning 3 years old in 2015.

2. In what age group would Adeline be counted in the 2015 histogram?
   She would be counted in the 0 – 4 years old age group of 2015.

3. Estimate the number of young people counted in that age group.
   Based on the histogram, there were approximately 20 million people in that age group.

4. Based on Kristin's story, what do you know about Mathew's young sister and Dominic's younger brother at the start of 2015?
   They were born after 2015 and before 2020. As a result, they will not be counted on a population graph until 2020.

5. Most population graphs prepared by the United States Census Bureau use the 5-year age groups represented in the above histogram. There are people not counted in the 2015 histograms who are part of the population from 2015 to 2019. Describe these people.
   The people who are not counted in the 2015 population would be people born during 2015 to 2019 or people under 5 years old during that time who moved into the country. They would be in the age group 0 – 4 of the 2020 graph as that graph counts the people who are 0 – 4 years old at the start of 2020.

6. Assume a histogram of the actual count of people was prepared in 2015 by the United States Census Bureau. If a histogram is prepared every 5 years by the Census Bureau of the actual count, when will the people described in problem 5 be represented on a United States population graph?
   The people described in question 5 would be counted in the 0 – 4 years old age group of 2020.

There were several blanks cells in **Handout 5** that could not be completed as there was no estimate of the projected number of people born in the 0 – 4 age group after 2015. **Handout 6** begins to fill in these blank cells. This ***new*** group of people, or the ***Foundation Layer***, is the first domino that impacts a country's shape looking forward. A country's shape can drastically change from events that impact the count of people born in a 5-year period.

7. If the actual count of people in the 0 – 4 age group in 2020 turns out to be larger than what people expected, what might explain the larger number of people?

   One explanation is a higher birth rate. If more births occurred during 2015 to 2019, then the count would be higher. Another explanation is possibly more people moved into the country who were 0 – 4 years old at the start of 2020. Possibly more immigrants, or possibly more adoptions from outside of the United States occurred during that period of time.

8. In a similar way, if the actual count of people in the 0 – 4 age group in 2020 is less than what people expected, what might explain the smaller number of people?

   Just the opposite would explain a lower count. A lower birth rate resulting in fewer births. It is also possible that fewer people of that age were moving into the country.

9. **Handout 6** indicates that the total number of people in the United States in 2010 was 309.35 million people. It also indicates there were 20.19 million people who were estimated to be 0 – 4 years old. What is the proportion of the population in 2010 who were counted in the 0 – 4 age group? Express this proportion to the nearest thousandth.

   $\frac{20.19}{309.35}$ *approximately equals 0.065*

10. Based on the above proportion, what is the percent of the people in the United States who were counted in the 0 – 4 age group at the start of 2010?

    The above proportion indicates that approximately 6.5% of the population in 2010 were 0 – 4 years old.

11. In the same way, use **Handout 6** to derive the proportion of the United States population who were counted in the 0 – 4 age group at the start of 2015.

    Handout 6 *indicates there were 19.91 million people in 2015 who were 0 – 4 years old. The total population of the United States in 2015 was 320.91 million. Therefore, the proportion of people who were counted in the 0 – 4 age group at the start of 2015 is:* $\frac{19.91}{320.91}$*, this proportion is approximately equal to 0.062.*

12. What is the percent of the United States population who were counted in the 0 – 4 age group at the start of 2015?

    The percent of the population who were counted in the 0 – 4 age group is approximately 6.2%.

13. What is the difference in the count of people who were 0 – 4 years old in 2010 to the count of people who were 0 – 4 years old in 2015? What might be an explanation for this difference in the count of people born in the 5 years before 2010 and the count of people born in the 5 years before 2015?

The approximate difference in the count is 20.19 million people – 19.91 million people or approximately 0.28 million people. This difference represents approximately 280,000 people. The difference is a result of fewer births (lower birth rate) from 2010 to 2014, or fewer children moving into the country who were 0 – 4 years old prior to 2015.

14. An estimate of the count of people in 2020 who were 5 years old or older to 100+ years was 311.21 million people. Explain how this estimate was determined using **Handout 6**. Is 311.21 million people the total population of the country in 2020? Explain your answer.

This estimate was determined by adding the counts of people in all of the age groups from 5 – 9 years old to 100+ years old. The estimates for each age group were determined first by multiplying the 2015 counts by the population factors for the connected age groups. This sum is not the total population in 2020 as it is missing the count of people 0 – 4 years old.

15. Consider the following equation:

$$\frac{x}{x+311.21} = 0.062$$

If the above equation is used to determine an estimate of the count of people 0 – 4 years old in 2020, answer the following:

a. What does 0.062 represent in this equation? What does 311.21 represent?

The proportion of the 2015 population who were approximately 0 – 4 years old is 0.062. Assume the same proportion of the population in 2020 is approximately 0 – 4 years old. 311.21 million people represents the sum of the population who are estimated to be 5 – 9 years old to 100+ years old in 2020.

b. What does x represent in this equation?

x represents the count of people 0 – 4 years old at the start of 2020 in millions of people.

c. Solve for x to the nearest hundredth.

x = 0.062(x + 311.21) => x = 0.062x + 19.30 => x - 0.062x = 19.30

- ⇨ *0.938x = 19.30*
- ⇨ *x = 19.30/0.938 which is approximately 20.57 million people.*

d. If the count of people in the 5 – 9 years old age group to the 100+ age group is greater than 311.21 millions of people, what happens to the value of x if the population factor stays the same? What happens to the count of the total population?

The value of x also increases. Either use the equation to point this out to students or consider changing the value of 311.21 to a larger value but keeping the foundation factor at 0.062. Direct students to solve for x based on the increased value. The total population also increases as a result of an increase in x.

16. Your solution to the above equation is an estimate of the 0 - 4 years old age group in the 2020 population. Place this value in the appropriate cell of **Handout 6**. What is your estimate of the total population of the United States in 2020? Indicate in the space below how you derived the value of the total population. Also enter this value in the appropriate cell of **Handout 6**.

*Students place the value 20.57 million to the 0 – 4 years old age group for 2020. They also add this value to the 311.21 million people estimating the total population at 331.78 million people. Verify correct entry using the Teacher Edition of **Handout 6.***

17. Given your estimate of the 0 - 4 age group for 2020, estimate the number of people in the 5 – 9 age group for the 2025 population group by multiplying the count derived for the 0 – 4 age group by the Population Factor for the connecting age groups of 0 – 4 to 5 – 9 years old. Indicate in the space below how you derived this value. Also enter this value in the appropriate cell of **Handout 6**.

*The estimate of the number of people in the 5 – 9 years old age group for 2025 is 20.57 x 1.014. This product is approximately 20.87 million people. Place this estimate in the proper cell of **Handout 6**. (Estimates can differ from this result by +/- 0.01 due to round off procedures.)*

18. Revise the equation in problem 15 to estimate the count of people in the 0 – 4 age group for 2025 population using the same foundation factor of 0.062 for 2015 and 2020. Place your estimate in the proper cell of **Handout 6**. Why was it necessary to derive the estimate for the 5 – 9 age group before you derived the above estimate for the 0 – 4 age group?

Students use the same equation that was set-up in problem 15. After estimating the count of the 5 – 9 years old age group in the previous problem, they calculate the sum of the population for age groups 5 – 9 years old to 100+ years old. That sum is approximately 320.67 millions of people. Using the equation, students solve for x or the population of the 0 – 4 years old in 2025:

$\frac{x}{x+320.67} = 0.062$ *=> x = 0.062(x + 320.67) => x = 0.062x + 19.88*

⇨ *0.938x = 19.88 => x = 19.88/0.938 which is approximately 21.19 or 21.20*

Enter 21.20 in the blank cell for the count of 0 – 4 years old in 2025.

19. Derive estimates for the remaining blank cells in **Handout 6**. (Remember your estimates can differ from some of the listed estimates by + 0.01 or - 0.01.)

*Verify correct entries for the blank cells using the Teacher Edition of **Handout 6.***

20. After all blank cells have been filled, Identify the age group with the greatest number of people for each of the following:

Year	Age group with the greatest number of people	Percent of the country within age group (nearest tenth of a percent)
2010	*45 – 49 age group*	$\frac{22.64}{309.35}$ *or 0.073 = 7.3%*
2015	*20 – 24 age group*	$\frac{22.69}{320.91}$ *or 0.071 = 7.1%*
2020	*25 – 29 age group*	$\frac{23.42}{331.78}$ *or 0.071 = 7.1%*
2025	*30 – 34 age group*	$\frac{23.94}{341.87}$ *or 0.070 = 7.0%*
2030	*35 – 39 age group*	$\frac{24.23}{350.87}$ *or 0.069 = 6.9%*
2035	*40 – 44 age group*	$\frac{24.33}{358.55}$ or 0.068 or 6.8%
2040	*45 – 49 age group*	$\frac{24.20}{364.94}$ *or 0.066 or 6.6%*
2045	*50 – 54 age group*	$\frac{23.82}{370.45}$ *or 0.064 or 6.4%*
2050	*25 – 29 age group*	$\frac{23.69}{375.67}$ *= 0.063 or 6.3%*

Note: After students complete the above table, ask them what they observed about the count and percent of the age group that has the greatest number of people. In general, the count slightly declines over each 5 years (there are a few exceptions), and the percent of the country within the dominant age group is also declining. This general pattern is also addressed in the next lesson.

21. Why might a person who was 22 years old at the start of 2015 be highlighted in an online commercial over a person who was 42 years old at the start of 2015?
A person in 2015 who was 22 years old belonged to the age group that had the largest count of people (with approximately 7.1% of the population). A person 42 years old belonged to an age group that had less people. In addition, there were also higher counts of people older than a person 42 years old. (Notice the slight dip of the age groups around 42 years old in the histogram.) The higher counts of people influence the target audience of a commercial, as well as what movies or other form of entertainment are produced, what form of transportation is established, and what type of housing options are created including the size of a house or apartment, or preferences for renting or owning a house. A country's economy and cultural are linked to those age groups who make up the greater percent of the country's population.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of this lesson.

The country of Awesome recorded that there were 1,000 people 0 – 4 years old at the start of 2015. Awesome also recorded a total population of 25,000 people at the start of 2015.

a. What is the value of the foundation factor for Awesome at the beginning of 2015?

b. Adeline lives in Awesome and turned 3 years old in 2015. Was she one of the 1,000 people counted in the 0 – 4 years old age group? Explain.

c. If the total population in Awesome at the start of 2020 is 30,000 people, and if the foundation factor did not change, estimate the count of 0 – 4 years old in 2020?

d. Adeline's friend Dominic was born in 2015. Is Dominic one of the 1,000 people counted in the 0 – 4 years old age group? Explain.

Comments on the Assessment Task:

With 1000 people in the 0 – 4 years old age group, there would be a 0.04 or 4% foundation factor for 2015. Students derive the foundation factor by setting up the proportion of 1000/25000.

Adeline would be counted in the 0 – 4 years old age group as she was 2 years old at the start of 2015.

An estimate of the 0 – 4 years old age group in 2020 would be 1,200 people, or 4% of 30,000 people.

Finally, Dominic would not be counted in the 2015 summary of the population as he was not born at the start of 2015.

Additional Assessment Summary:

This lesson highlights the second type of factor used to estimate future counts of the population, namely the ***foundation factor***. Discuss with students why the term "foundation factor" is a good description for this important value. Also, ask students why this factor is important in projecting future counts. The previous description of falling dominos could be used again to explain the impact of the 0 – 4 years old age group. As the projection for the 0 – 4 years old age group was determined in one 5-year period, all of the other age groups dependent on that estimate can be estimated in the next 5 -year periods.

Summarize with students the completed **Handout 6**. Highlight with them any patterns they observe in the handout, such as the summary mentioned in question 20. For each year except 2050, the age group with the most people kept getting older by 5 years from the previous 5-year age group. Yet, the percent in the age group with the most people slightly declined. Many other patterns can be identified that show how the population ages. Consider asking students to describe how the country might change for a person 22 years old in 2015.

Consider directing students to complete an **Exit Summary** (or Handout 13) for this lesson.