Teaching Notes
Lesson 4
The Center and Spread of a Country's Shape

Overview:

Describing and summarizing data are the primary goals of this lesson. Lessons 1, 2, and 3 introduced students to population data by focusing on the shape of the population pyramid graphs or the histograms. This lesson examines measures of center, specifically the median and mean ages, and the spread of the data as additional tools in understanding the population distributions of the United States, Kenya, and Japan. The centers and the spread work together in providing important summaries of the data. The spread developed in this lesson is a generalization of the Interquartile Range (or IQR) for grouped data and provides a description of how spread out the data is from the median. Lessons in Unit 2 will look back at the population distributions and use the centers and spread to summarize changes in each country's population distribution. Lessons in Unit 3 will analyze projected or future population data sets of each of the countries and will also summarize the changes by discussing the centers and the spread.

The topics addressed in this lesson might be familiar to students based on their previous work involving statistics. A different twist to these familiar statistical topics, however, is that the median age, the spread, and the mean age are derived for grouped data. A list of all of the data of the population is obviously not known and too massive, therefore, a summary of the data by age groups is provided. Deriving centers and spread based on working with grouped data might be new to most students.

A major challenge in completing this lesson is estimating the number of 50-minute class periods needed to complete the entire lesson. If students are familiar with these topics, anticipate 2 class periods. If these topics are relatively new to students, anticipate 3 to 4 class periods. Assigning students to work in small groups (especially when students are completing the tables) is again suggested and may decrease the time needed to complete the lesson. This lesson addresses student's analysis at the higher levels of the **Modeling Continuum**.

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 5, 6,	Problems: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 17, 18, 19	Problems: 14, 15, 16, 20, 21, 22, 21, 24, 25	Problems: 12

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, mean of a data set, median of a data set, and spread of a data set.
See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy, either electronic or printed, of a complete Lesson 4 for each student. This lesson does not require any additional handouts although it is also longer than most lessons. Consider providing students printed copies of the selected tables you direct them to complete. Students can answer several of the remaining problems using an electronic copy of the lesson.

Launch:

Begin this lesson by directing students to study the 1980 population histogram of the United States. After a minute or two of examining the graph, ask students questions similar to the following:

- What age group had the greatest count of people?
- What was the total population of the country in 1980?
- What description of the shape of the country would you estimate for this graph?

Discuss with students that often a population is described by a single value, generally identified as a center of the population. For example, ask students what age they might use to describe a typical person from the United States. Is 80 years old a good description? Why or why not? Is 20 years old a good description of a typical person? And again, why or why not? The focus of this lesson is to derive centers that are often used to describe a collection of data, along with a value called a spread that is also necessary to summarize the data.

After highlighting several of these type of questions, direct students to complete the problems. Monitor their responses. Provide opportunities to discuss with students their answers to the problems. Also provide opportunities to discuss problems as a class, especially if they are not initially clear to students.

Implementation Ideas:

Work through the steps outlined in the lesson to find the mean and median ages for a person in the United States. At the end of the problem set, students are asked to find the mean and median ages for Kenya and Japan. If students have access to scientific or graphing calculators, then organize students in small groups to complete the problems. Assign one group to find the mean and median ages for Kenya and the other group to find the mean and median ages for Japan. The process to derive each of these centers is organized for students in the tables that are part of the lesson.

Another possibility is to move the calculations to a spreadsheet or to a graphing calculator. The list options of calculators similar to the TI-84 allow for multiple calculations to be completed by the calculator. Similarly, multiple calculations can be completed with a spreadsheet. After

students complete the calculations (either individually or in small groups), discuss as whole group the estimates of the mean and median ages for each country as directed in the problems.

Student responses or descriptions

Lesson 4 – Problems

1. Answer the following based on the 1980 and the 2015 population histograms.
   a. Do you think the population increased from 1980 to 2015? Explain your answer.
   Anticipate that students will indicate the population increased based on the increased area of the 2015 histogram. Students might quickly add up the values of the 2015 histogram and determine that they exceeded the total value included on 1980 histogram of 227.24 million people.
   b. What two age groups recorded the greatest count of people in 1980? What two age groups recorded the greatest count of people in 2015?
   The greatest counts in 1980 were the 20 – 24 years old age group (approximately 21 million people and the 15 – 19 years old age group (also approximately 21 million people). The greatest counts in 2015 are more challenging as there are several age groups that have similar counts for the first and second greatest count of people. The 20 – 24 years old age groups had the greatest count. The 25 – 29 years old age group had the second greatest count. Accept 50 – 54 years old, 55 – 59 years old, and 30 – 34 years old age groups as estimates of the second greatest count. Each of these age counts appear to have similar heights on the 2015 histogram. A table of the age group counts is needed to distinguish the counts.
   c. Identify the age group where a person 34 years old would be counted on the 1980 population histogram. Also identify the age group where a person 34 years old would be counted on the 2015 population histogram. Do you think the typical age of a person in 2015 is also 34 years old? If not, do you think the typical age will be older or younger? Explain your answer. Several of the problems in this lesson will develop an answer to these questions more precisely.

   The counts of the older age groups have increased. As a result, an estimate of a typical age would identify an age that is older in 2015 than in 1980.

Finding the Median Age of the United States Population in 2015

2. Are the ages of the people summarized in the histograms from 1980 and 2015 in order from youngest to oldest? Explain your answer.
   The ages are essentially in order by age groups. They could be considered a summary of the population in order.

3. Is it possible to determine how many people in the United States were exactly 34 years old? Explain your answer.

   *Yes, **if** it were possible obtain the ages of everyone in the United States. The task of finding everyone and counting everyone, however, is not likely. From that standpoint, the answer would be no.*

4. Answer the following questions based on the histogram of the 2015 population provided in this lesson.
   a. Do you think a person who is 20 years old would be an estimate of the median age? Why or why not
      No, as most of the area of the histogram is above (or older) than this estimate. Therefore, there would be more than 50% above this age.
   b. Do you think 40 years would be an estimate of the median age? Why or why not?
      This estimate is difficult to determine based on the area of the histogram. Visually, the areas above and below 40 years appear similar, so this age could be an estimate of the median age.
   c. Do you think 80 years would be an estimate of the median age? Why or why not?
      No, as most of the area of the histogram is below (or younger) than this estimate. Therefore, there would be more than 50% below this age.

Discuss with students as a class the following table. Point out the column headings and the values completed in the table. Particularly discuss the **Cumulative count of people** and how the values in that column are calculated.

United States - 2015

Age group	**Count** of people in each age group (in millions of people to the nearest hundredth)	**Cumulative count** of people (in millions of people to the nearest hundredth)	Proportion of cumulative count of people to the total population of the country (to the near thousandth)	Proportion as a percent (to the nearest tenth of a percent)
0 – 4	19.91	19.91	$\frac{19.91}{320.91} = 0.062$	6.2%
5 - 9	20.48	40.39	$\frac{40.39}{320.91} = 0.126$	12.6%
10 - 14	20.61	61.00	$\frac{61.00}{320.91} = 0.190$	19.0%
15 - 19	21.09	82.09	$\frac{82.09}{320.91} = 0.256$	25.6%
20 - 24	22.69	104.78	$\frac{104.78}{320.91} = 0.327$	32.7%
25 - 29	22.40	*127.18*	$\frac{127.18}{320.91} = 0.396$	*39.6%*
30 - 34	21.62	*148.80*	$\frac{148.80}{320.91} = 0.464$	46.4%
35 - 39	20.31	169.11	$\frac{169.11}{320.91} = 0.527$	52.7%
40 - 44	20.16	*189.27*	$\frac{189.27}{320.91} = 0.590$	*59.0%*

45 - 49	20.80	210.07	$\frac{210.07}{320.91} = 0.655$	65.5%
50 - 54	22.29	*232.36*	$\frac{232.36}{320.91} = 0.724$	72.4%
55 - 59	21.77	254.15	$\frac{254.13}{320.91} = 0.792$	79.2%
60 - 64	19.04	273.17	$\frac{273.17}{320.91} = 0.851$	85.1%
65 - 69	16.05	*289.22*	$\frac{289.22}{320.91} = 0.901$	*90.1%*
70 - 74	11.48	300.70	$\frac{300.70}{320.91} = 0.937$	93.7%
75 - 79	8.12	*308.82*	$\frac{308.82}{320.91} = 0.962$	*96.2%*
80 - 84	5.80	314.62	$\frac{314.62}{320.91} = 0.980$	98.0%
85 - 89	3.86	*318.48*	$\frac{318.48}{320.91} = 0.992$	*99.2%*
90 - 94	1.85	320.33	$\frac{320.33}{320.91} = 0.998$	99.8%
95 – 99	0.50	*320.83*	$\frac{320.83}{320.91} = 0.999$	99.9%
100+	0.08	**320.91**	$\frac{320.91}{320.91} = 1.000$	100.0%
Total	**320.91**			

5. What does each of the columns of the above table summarize? What does the cumulative count summarize? Answer each of the following questions based on the table to indicate your understanding of each of the columns.
   a. How many people are younger than 5 years old?
      19.91 million people
   b. What is the percent of people younger than 5 years old?
      6.2%
   c. How many people are younger than 10 years old?
      40.39 million people
   d. What is the percent of people younger than 10 years old?
      12.6%
   e. What is the percent of people younger than 20 years old?
      25.6%

f. How many people are younger than 50 years old?

210.07 million people

g. What is the percent of people younger than 50 years old?

65.5%

6. There are several blank cells in the table. For each blank cell, complete the expected calculations.

See the above completed table.

7. What is the first age group that captures at least 50% of the cumulative population?

The 35 – 39 years captures 52.7% of the cumulative population.

8. Estimate the age group in which the median age is located.

The estimate of the median age would be within the age group of 35 - 39 years age. This age group captures at least 50% of the population.

9. Is it possible to identify the exact age within the age group identified in problem 8 that would be the median age (or the age where at most 50% of the population would be less than this age)? Explain your answer.

As we do not know the specific ages (just a grouping of the ages as indicated by age group), we cannot find an exact age.

10. Using the above description of spread and the completed table, answer the following:
    a. What is the first age group that captures 75% of the cumulative population?
    The age group 55 – 59 years captures 79.2% of the cumulative population. It is the first age group that has at least 75% of the cumulative population within that age group.
    b. What is the first age group that captures 25% of the cumulative population?
    The age group 15 – 19 years captures 25.6%. It is the first age group that has at least 25% of the cumulative population within that age group.
    c. Calculate the spread of the population based on the difference in ages of the two age groups in 10(a) and 10(b) using the oldest age from the age group that captures 75% of the cumulative population and the youngest age from the age group that captures 25% of the cumulative population. Approximately what percent of the total population is captured between the two ages used to calculate the spread?
    At least 50% of the cumulative population is captured between 15 – 19 years old and 55-59 years old. The difference of 59 years and 15 years is 44 years. This difference is an estimate of the spread of the population. The actual spread based on the goal of finding where 50% of the population is less than 44 years.

d. Why is the last cell in the column representing the proportion as a percent equal to 100.0%?
The last cell is the percent based on the proportion of the entire count of people or the entire population. Therefore, the resulting proportion is 1.00 and the percent is 100%.

11. Describe the typical person in 2015. How does this person differ from the 1980 typical person?
The median age is estimated to be between 35 – 39 years old, which is slightly higher than the median age reported in 1980 of 34 years old. The spread of approximately 44 years is slightly higher than the reported spread in 1980 which was estimated at 39 years (the difference of 54 years and 15 years). The greater value of the spread, however, is based on the age groups which are overestimating where 50% of the cumulative population would be counted. As a result, the estimate of 44 years is greater than the actual spread; therefore, the spread for 2015 is similar to the spread reported for 1980.

12. Sketch a histogram of a country with the same median age as the United States in 2015 but with a spread that is one-half of the value derived for the United States. Also sketch an approximate box plot of this country using the following grid:

A sketch of the population histogram would be "pushed" more closely toward the estimate of the median age. The age group where at least 75% of the cumulative population is captured would probably be younger than the age group for the United States. Similarly, the age group where at least 25% of the cumulative population is captured would probably be older than the age group for the United States. It is likely there would be a noticeable build-up of the population around the age group containing the median age.

Finding the Mean Age of the United States Population in 2015

Topics for Discussion:

• Mean as a Balance Point

Point out to students that the mean is an important summary of a population. Although often summarized as a simple "average" of data, it is a value that also indicates a **balance point** of the data. If students do not understand the significance of the mean as a balance point, take a few minutes to share an example of this description. Tape coins to a light-weight ruler. Possibly start off by taping a stack of 5 pennies at the end of a 12-inch ruler (position equals 12 inches) and 10 pennies taped together at the other end of the ruler (position equals 0 inches). Place a pencil under the ruler and try to identify the position that balances the ruler. The ruler should approximately balance at the position of 4 inches. Indicate to students that 4 is the mean or: (10 x 0 + 5 x 12)/15. A frequent clarification that might be needed is that the mean for this example is derived by dividing the sum of the positions for ***each*** of the 15 pennies (ten 0's and five 12's) by the total number of pennies (15 pennies).

In addition, point out that the distance of the mean to the position 12 is 8 inches. The sum of the distances from the mean of 4 inches to the 5 pennies taped at the position 12 inches is 5 x 8 inches or 40 inches to the left of the mean. The distance of the mean to the position 0 is 4 inches. The sum of the distances from the mean of 4 inches to the 10 pennies taped at the position 0 is 10 x 4 inches or 40 inches to the right of the mean. The mean is a special summary in which the sum of the distances to the right and the sum of the distances to the left are equal. This is what makes the mean a balance point. (A resource for additional examples and lessons can be found on the ASA website and the **Data-Driven Module Exploring Centers**, or: https://www.amstat.org/asa/files/pdfs/ddmseries/ExploringCenters--TeachersEdition.pdf)

• Mid-Intervals as an Estimate

The table uses the mid-interval age of an age group as an estimate of all of the ages in an age group. This estimate is then used to calculate the sum of all of the ages in that age group. This technique will likely not result in an exact value of the sum of the ages, but given that the exact values are not known, it is considered a reasonable estimate. If further discussion about why this might result in a reasonable estimate, consider providing a specific example. For example, if you have 5 people with the ages of 1, 1, 2, 3, and 4, the sum of the ages is 11 years. If you use the mid-interval value of 0 – 4 years old, or 2 years as estimate of each person's age, the sum of the ages would be 10 years. Although not exact, it is close. There could, however, be examples in which the estimate is not close. For example, what if the ages of the 5 people were 0, 0, 0, 0, 1. The sum of the five ages is 1 which is considerably less than 10 years. The main point to discuss with students is that with a large count of people in each age group, the mid-interval age is a reasonable estimate that is larger than the ages that are smaller and smaller than the ages that are larger, and therefore, it should be a good estimate of an age for deriving the sum of the ages in that age group.

United States – 2015
Results for finding the mean age of the United States

Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	19.91	2 X 19.91 = 39.82
5 - 9	7	20.48	7 X 20.48 = 143.36
10 - 14	12	20.61	12 X 20.61 = 247.32
15 - 19	17	21.09	17 X 21.09 = 358.53
20 - 24	*22*	22.69	*22 X 22.69 = 499.18*
25 - 29	27	22.40	27 X 22.40 = 604.8
30 - 34	32	21.62	32 X 21.62 = 691.84
35 - 39	37	20.31	*37 X 20.31 = 751.47*
40 - 44	42	20.16	42 X 20.16 = 846.72
45 - 49	*47*	20.80	47 X 20.80 = 977.6
50 - 54	52	22.29	52 X 22.29 = 1159.08
55 - 59	57	21.77	57 X 21.77 = 1240.89
60 - 64	62	19.04	*62 X 19.04 = 1180.48*
65 - 69	*67*	16.05	*67 X 16.05 = 1075.35*
70 - 74	*72*	11.48	72 X 11.48 = 826.56
75 - 79	77	8.12	77 X 8.12 = 625.24
80 - 84	82	5.80	82 X 5.80 = 475.6
85 - 89	87	3.86	*87 X 3.86 = 335.82*
90 – 94	*92*	1.85	*92 X 1.85 = 170.2*
95 – 99	97	0.50	97 X 0.50 = 48.5
100+	102	0.08	102 X 0.08 = 8.16
	Total	**320.91**	**12306.52**

13. Explain how the mid-interval ages were determined for each age group.

    The mid-interval ages for each age group can be found by adding the endpoints and dividing by 2. Also, the mid-interval age for an age group of 5 years is the age within the group that has two ages above it and two ages below it.

14. Do you think all of the 19.91 million people who were 0 – 4 years old are 2 years old? Why might, however, the mid-interval age of 2 years be a reasonable estimate of the age of each of the children in the 0 – 4 years old age group?

    Clearly all of the people within the age group are not 2 years old, however, it is a good estimate of the age of each person within that interval as there are likely a similar count of people above 2 years as there are people below 2 years.

15. To determine the mean age, the sum of the ages in each age group is needed. Consider the age group 0 – 4 years old. If 2 years is a good estimate of the age of all of the people in that age group, what does the product of 2 and 19.91 represent?

    As indicated, 2 years is a good estimate of the age of each person within that age group. As a result, to find the sum of all of the ages in that age group, the product of 2 and the count of people in that age group, or 19.91, is a good estimate.

16. In the same way, what does the product of 7 and 20.48 represent?

    The age of 7 years is a good estimate of the age of each person within the age group of 5 – 9 years old. As a result, the sum of all of the ages in that age group could be estimated by the product of 7 and the count of people within the age group.

17. Analyze what is missing in each of the blank cells of the above table. For each blank cell, complete the expected calculations.

    See the completed cells in the table.

18. The last column of the above table represents an estimate of the sum of the ages for each age group. To determine the mean, the sum of all of the ages for 320.91 million people is needed. Based on the estimates recorded for each age group, 12306.52 million is the approximate sum of all of the age groups in that column. Describe the last step needed to calculate an estimate of the mean age of a person in the United States.
    The last step needed is to divide the sum of the all of the ages by the total population of the country, or divide 12306.52 million years by 321.91 million people.

19. What is an estimate of the mean age of a person in the United States?
    Divide the sum of the ages (12306.53 million) by the number of people or 320.91 million. The mean age is approximately 38.35 years old. This estimate is similar to what the Census Bureau estimated as the mean age for 2015.

20. Do you think the estimated mean age is a good description of a typical person in 2015? Explain why or why not.
    Mean age is 38.35 years. This estimate is similar to what the Census Bureau estimates for 2015 and could be used to describe a typical person.

21. The estimate of the mean age in 2015 is greater than the estimate for 1980. What does this indicate about the change in the population during this time?

    The mean age is greater as the count of older people in the 2015 population is greater. Compare the 1980 and 2015 histograms and observe the higher bars (or greater counts) for ages 40 and older.

Tables are provided for calculating the median and mean values for Kenya and Japan. The tables are designed so that the calculations of the mean and median ages are completed in the same way as the calculations for mean and median ages were done for the United States. Consider organizing this part of the lesson in small groups, with each group and each member of the group completing a portion of the calculations. Also consider assigning some groups to derive the median age, the spread, and the mean age for Kenya, and assigning different groups to derive the median age, the spread, and the mean age for Japan.

Kenya 2015

Results for finding the median age of Kenya:

Age group	Count of people (in millions of people)	**Cumulative count** of people (in millions)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	6.38	*6.38*	*0.138*	*13.8%*
5 - 9	6.76	*13.14*	*0.286*	*28.6%*
10 - 14	5.95	*19.09*	*0.416*	*41.6%*
15 - 19	4.49	*23.58*	*0.513*	*51.3%*
20 - 24	4.08	*27.66*	*0.602*	*60.2%*
25 - 29	3.92	*31.58*	*0.686*	*68.6%*
30 - 34	3.60	*35.18*	*0.766*	*76.6%*
35 - 39	2.89	*38.07*	*0.829*	*83.9%*
40 - 44	2.01	*40.08*	*0.873*	*87.3%*
45 - 49	1.55	*41.63*	*0.906*	*90.6%*
50 - 54	1.25	*42.88*	*0.934*	*93.4%*
55 - 59	0.98	*43.86*	*0.955*	*95.5%*
60 - 64	0.75	*44.61*	*0.971*	*97.1%*
65 - 69	0.53	*45.14*	*0.983*	*98.3%*
70 - 74	0.36	*45.50*	*0.991*	*99.1%*
75 - 79	0.23	*45.73*	*0.996*	*99.6%*
80 - 84	0.12	*45.85*	*0.998*	*99.8%*
85 - 89	0.05	*45.90*	*0.999*	*99.9%*
90 - 94	0.01	*45.91*	*1.000*	*100.0%*
95 – 99	0.01	*45.92*	*1.000*	*100.0%*
100+	0.01	*45.93*	*1.000*	*100.0%*

22. Derive an estimate of the median age group for Kenya and a description of the spread based on the definition of spread in this lesson.

The median age would be within the age group of 15 – 19 years as it captures 50% of the cumulative population. The age group 5 - 9 captures 25% of the cumulative population and the age group 30 – 34 captures 75% of the cumulative population. The spread as defined in this lesson would be the difference of 34 years and 5 years, or approximately 29 years. The median age and spread is less than the United States indicating a younger population.

Kenya – 2015
Results for finding the mean age of Kenya:

Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	6.38	*12.76*
5 - 9	7	6.76	*47.32*
10 - 14	12	5.95	*71.40*
15 - 19	17	4.49	*76.33*
20 - 24	22	4.08	*89.76*
25 - 29	27	3.92	*105.84*
30 - 34	32	3.60	*115.20*
35 - 39	37	2.89	*106.93*
40 - 44	42	2.01	*84.42*
45 - 49	47	1.55	*72.85*
50 - 54	52	1.25	*65.00*
55 - 59	57	0.98	*55.86*
60 - 64	62	0.75	*46.5*
65 - 69	67	0.53	*35.51*
70 - 74	72	0.36	*25.92*
75 - 79	77	0.23	*17.71*
80 - 84	82	0.12	*9.84*
85 - 89	87	0.05	*4.35*
90 – 94	92	0.01	*0.92*
95 – 99	97	0.01	*0.97*
100+	102	0.01	*1.02*
	Total	45.93	*1046.41*

23. Derive an estimate of the mean age of Kenya. Compare the estimated mean age to the median age. Are they similar? Explain.

The mean age is 1046.91 million years divided by 45.93 million people, or approximately 22.78 years. The mean age is pulled by the older age groups resulting in a higher estimate of the age of a typical person than the median age. Remind students that an estimate of the mean age is more impacted by the older age groups. The difference between the estimates for the mean age of 22.78 years and median ages for Kenya is large.

Japan 2015

Results for finding the median age of Japan:

Age group	Count of people (in millions of people)	**Cumulative count** of people (in millions)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	5.27	*5.27*	*0.042*	*4.2%*
5 - 9	5.61	*10.88*	*0.086*	*8.6%*
10 - 14	5.75	*16.63*	*0.131*	*13.1%*
15 - 19	6.15	*22.78*	*0.179*	*17.9%*
20 - 24	6.13	*28.91*	*0.228*	*22.8%*
25 - 29	6.54	*35.45*	*0.279*	*27.9%*
30 - 34	7.47	*42.92*	*0.338*	*33.8%*
35 - 39	8.27	*51.19*	*0.403*	*40.3%*
40 - 44	9.50	*60.69*	*0.478*	*47.8%*
45 - 49	8.46	*69.15*	*0.545*	*54.5%*
50 - 54	7.82	*76.97*	*0.606*	*60.6%*
55 - 59	7.57	*88.54*	*0.698*	*69.8%*
60 - 64	8.62	*98.46*	*0.776*	*77.6%*
65 - 69	9.57	*102.73*	*0.809*	*80.9%*
70 - 74	7.82	*110.55*	*0.871*	*87.1%*
75 - 79	6.26	*116.81*	*0.920*	*92.0%*
80 - 84	4.95	*121.76*	*0.959*	*95.9%*
85 - 89	3.17	*124.93*	*0.984*	*98.4%*
90 - 94	1.45	*126.38*	*0.996*	*99.6%*
95 – 99	0.44	*126.82*	*0.999*	*99.9%*
100+	0.09	*126.91*	*1.000*	*100.0%*
Total	**126.91**			

24. Derive an estimate of the median age group for Japan and a description of the spread based on the definition of spread in this lesson.

The median age would be within the age group of 45- 49 years as it captures 50%of the cumulative population. The age group 25 -29 captures 25% of the cumulative population and the age group 60 – 64 captures 75% of the cumulative population. The spread as defined in this lesson would be the difference of 64 years and 25 years, or approximately 39 years. The median age is greater than the estimate of the median range for the United States, with a similar spread. This estimate indicates that a typical person in Japan is older than the typical person in the United States.

Japan – 2015
Results for finding the mean age of Japan:

Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	5.27	*10.54*
5 - 9	7	5.61	*39.27*
10 - 14	12	5.75	*69.00*
15 - 19	17	6.15	*104.55*
20 - 24	22	6.13	*134.86*
25 - 29	27	6.54	*176.58*
30 - 34	32	7.47	*239.04*
35 - 39	37	8.27	*305.99*
40 - 44	42	9.50	*399.00*
45 - 49	47	8.46	*397.62*
50 - 54	52	7.82	*406.64*
55 - 59	57	7.57	*431.49*
60 - 64	62	8.62	*534.44*
65 - 69	67	9.57	*641.19*
70 - 74	72	7.82	*563.04*
75 - 79	77	6.26	*482.02*
80 - 84	82	4.95	*405.90*
85 - 89	87	3.17	*275.79*
90 – 94	92	1.45	*133.40*
95 – 99	97	0.44	*42.68*
100+	102	0.09	*9.18*
	Total	**126.91**	*5802.22*

25. Derive an estimate of the mean age of Japan. Compare Japan's estimate of the mean age to the median age. Are they similar? Explain.

    The mean age is 5802.22 millions of years divided by 126.91 millions of people. This is approximately equal to 45.72 years old. This is also within the age group of the median age.

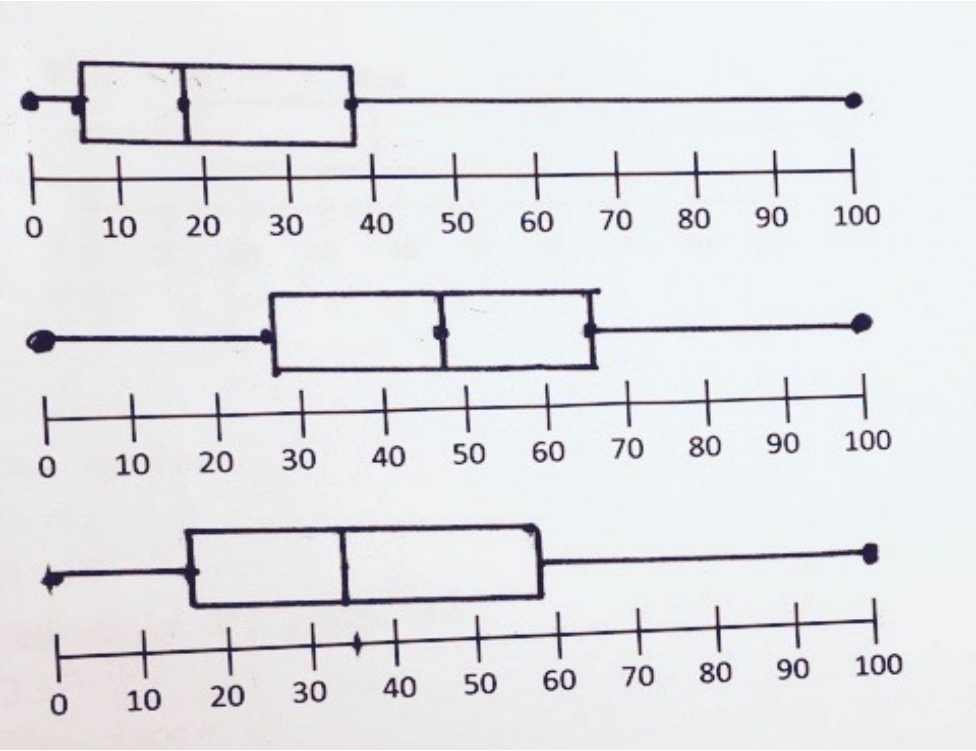
Consider discussing the following with students: The estimated mean and median ages for a typical person in the United States are similar ages. This observation is also true for a typical person in Japan. The median for Kenya, however, is within the age group 15 – 19 years old, and the mean age was estimated to be 22.78 years old. The difference in the estimated values of median and mean for Kenya is one of the larger differences of median and mean ages for countries of the world. Recall that the mean is a balance point (a type of fulcrum). Although there are not many people in the older age groups in Kenya, the older people result in a "pull" on the location of the mean age. The mean and median ages calculated in this lesson for each of the countries are very similar to what is reported by the United States Census Bureau. The Census Bureau emphasizes the median age in its reports as the median is considered a more appropriate estimate for skewed distribution.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to evaluate a student's understanding of the lesson.

The following 3 box plots were sketched by a student completing this lesson. One box plot represents the country of Kenya, one box plot represents the country of Japan, and one box plot represents the country of the United States. Unfortunately, the student forgot to write a label of the country for each box plot. Study the box plots and identify the correct country that is represented by each box plot.



Comments on the Assessment Task:
The top box plot is Kenya. The second box plot is Japan and the bottom box plot is the United States.

Students connect their understanding of the 5-number summaries to the box plots . The box plots also provide visuals that highlight the differences of each country's population distribution and their different shapes. Consider aligning the box plots to the population histograms studied in Lessons 1, 2, and 3.

Highlight a few of the summaries that a comparison of the box plots point out. For example, discuss how more than 50% of the population in Japan is older than 25% of the population in Kenya. Also discuss that Q1 for the United States is approximately equal to the median age of Kenya.

Additional Assessment Ideas:
This lesson has multiple questions and problems that serve as formative assessments. The last set of problems (problems 22 to 25) provide opportunities to assess a student's understanding of centers and spread. Students use the guided work of estimating the centers and spread for the United States to derive centers and spread for Kenya and Japan. These problems also extend a student's thinking to levels 3 and 4 of the **Modeling Continuum**. Consider conducting a whole class discussion on what role students' think the centers and spread play in analyzing what are the challenges a country might face. Are the challenges faced by the 3 countries studied in this module similar or different? Do the data and the estimates of center and spread suggest the challenges faced by these countries?