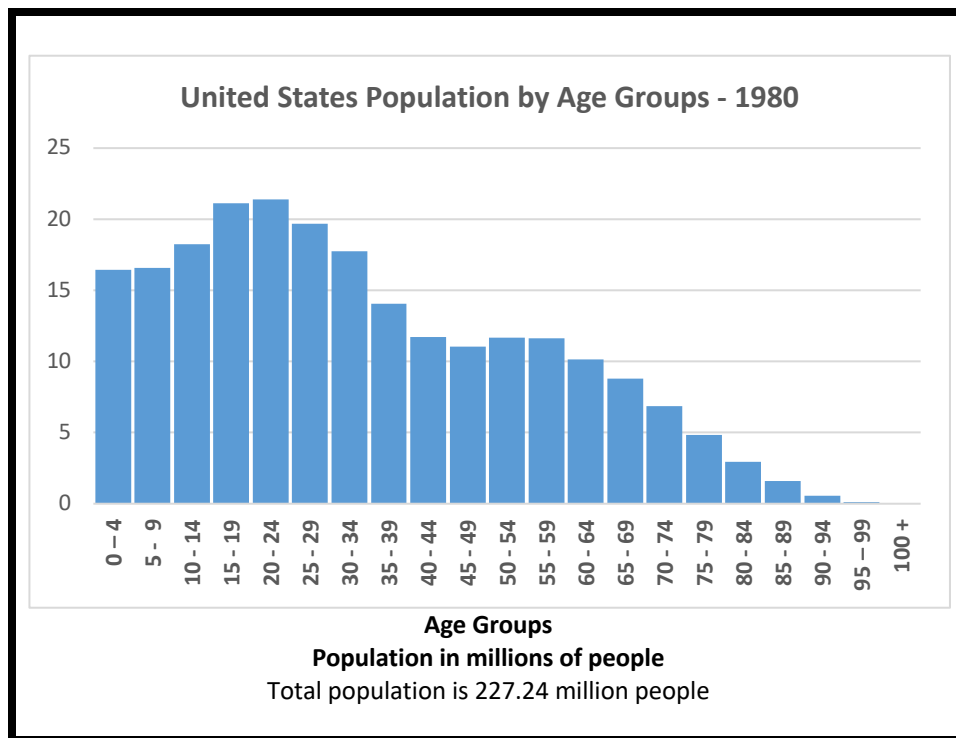


## Lesson 4

### The Center and Spread of a Country's Shape

Kristin was 36 years old at the start of 2015. Her birthday was in February of 1979. The first histogram that counted Kristin was the 1980 population histogram. She was counted in the 0 – 4 years old age group of that histogram.



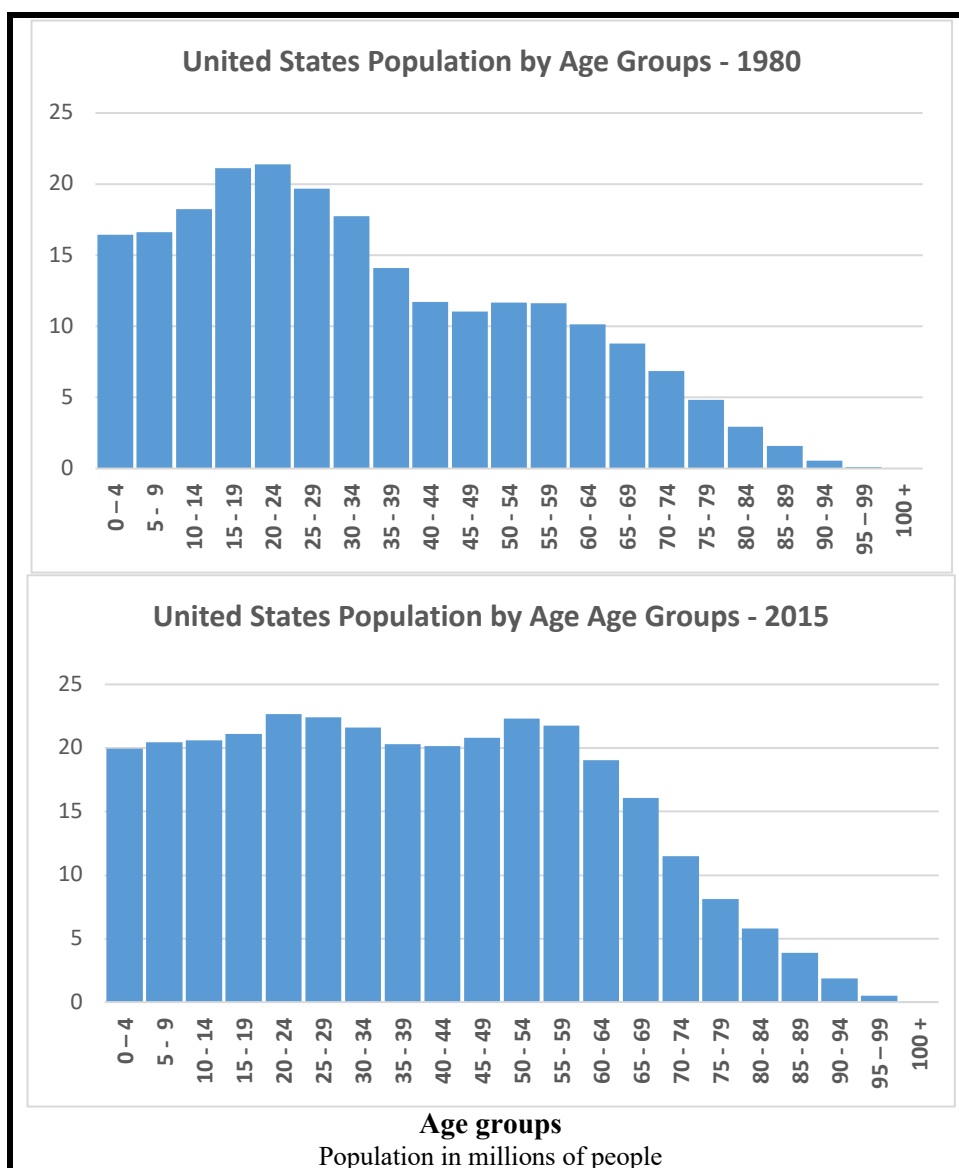
A 1980 report prepared by the United States Census Bureau indicated a typical person in the United States was 34 years old, with a little more than 50% of the country's population between the ages of 15 and 54 years old. Describing the age of a typical person does not mean that most of the people are 34 years old. Identify in the 1980 population histogram the age group in which a person 34 years old would be counted. Note that the count of people who were 15 to 19 years old, 20 to 24 years old, and 25 – 29 years old in 1980 had more people than the 30 – 34 years old age group. Why was 34 years old used to identify a typical person in the United States?

The problems in this lesson derive two summaries of the distribution of ages, namely the median age and the mean age. Although other summaries could be included in this list, the

mean age and the median age are often used to describe a “typical” age of a person in a country. This lesson will also explain that there is more to the story of a typical age than just the mean or median age. Equally important in understanding a country’s population distribution is the **spread** of the ages. The problems in this lesson will also derive the spread of the ages based on an estimate of the median age.

## Lesson 4 – Problems

To begin this lesson, compare the 1980 population histogram to the 2015 population histogram:



1. Answer the following based on the 1980 and the 2015 population histograms.
  - a. Do you think the population increased from 1980 to 2015? Explain your answer.
  - b. What two age groups recorded the greatest count of people in 1980? What two age groups recorded the greatest count of people in 2015?
  - c. Identify the age group where a person 34 years old would be counted on the 1980 population histogram. Also identify the age group where a person 34 years old would be counted on the 2015 population histogram. Do you think the typical age of a person in 2015 is also 34 years old? If not, do you think the typical age will be older or younger? Explain your answer. Several of the problems in this lesson will develop an answer to these questions more precisely.

“Typical” generally starts by describing a **center** of a data set. The first center derived in this lesson is the *median age*. The second center derived is the *mean age*. What these centers summarize about the population is explored in the following problems.

### Finding the Median Age of the United States Population in 2015

An important summary of the population is the median age. The exact median age requires access to everyone’s age in the country. Is that possible? Of course not. Imagine the number of ages this would represent (over 300 million). Also, at any moment in time, there is a person who leaves the country, a person who dies, a person who enters the country, and a person who is born. The process outlined in this lesson is an estimate of the median age based on the count of people in each of the age groups represented in the 2015 histogram.

What is the median age? If it were possible to collect the ages of all the people in the country, the first step would be to put those ages in order from 0 years old to 100+ years old (or from the youngest to the oldest person). **The median age is the age that marks where approximately one-half of the population, or 50%, is below this age and approximately one-half, or 50% of the population, is above this age.** A more precise definition of the median age is based on data compiled from every person’s age in a population of over 300 million people. For this lesson, an estimate of the median age is derived using the counts of people in each of the age groups. The result will be an estimate of the median age and not the exact median age.

2. Are the ages of the people summarized in the histograms from 1980 and 2015 in order from youngest to oldest? Explain your answer.
3. Is it possible to estimate how many people in the United States were exactly 34 years old? Explain your answer.

4. Answer the following questions based on the histogram of the 2015 population provided in this lesson.
  - a. Do you think a person who is 20 years old would be an estimate of the median age? Why or why not?
  - b. Do you think 40 years would be an estimate of the median age? Why or why not?
  - c. Do you think 80 years would be an estimate of the median age? Why or why not?

The table that follows is used to derive an estimate of the median age, or identify the age where approximately 50% of the population is greater than this age and 50% of the population is less than this age. For the purposes of this lesson, the 2015 population is provided in millions of people to the nearest hundredth. Given the size of the population (over 300 million people), approximations listed in the table are sufficient for deriving an estimate of the median age.

Carefully examine the column headings of this table. In particular, note the column labeled **Cumulative count** of people. The first value of the **Cumulative count** of people is the count of the 0 – 4 years old age group. The next value of the cumulative count represents the count of people 0 – 9 years old. This cumulative count is the sum of the 0 – 4 years old and the 5 – 9 years old age groups. In the same way, the next value of the cumulative count represents the count of people 0 – 14 years old. It is the sum of the previous cumulative count (or the 0 – 9 years old age groups) and the 10 – 14 years old age group. This process continues until the entire population of the country is summarized in the cumulative count.

Study the following partially completed table designed to estimate the population median age:

**United States - 2015**

<b>Age group</b>	<b>Count of people in each age group (in millions of people to the nearest hundredth)</b>	<b>Cumulative count of people (in millions of people to the nearest hundredth)</b>	<b>Proportion of cumulative count of people to the total population of the country (to the near thousandth)</b>	<b>Proportion as a percent (to the nearest tenth of a percent)</b>
<b>0 - 4</b>	19.91	19.91	$\frac{19.91}{320.91} = 0.062$	6.2%
<b>5 - 9</b>	20.48	40.39	$\frac{40.39}{320.91} = 0.126$	12.6%
<b>10 - 14</b>	20.61	61.00	$\frac{61.00}{320.91} = 0.190$	19.0%
<b>15 - 19</b>	21.09	82.09	$\frac{82.09}{320.91} = 0.256$	25.6%
<b>20 - 24</b>	22.69	104.78	$\frac{104.78}{320.91} = 0.327$	32.7%
<b>25 - 29</b>	22.40		$\frac{127.18}{320.91} = 0.396$	
<b>30 - 34</b>	21.62			46.4%
<b>35 - 39</b>	20.31	169.11	$\frac{169.11}{320.91} = 0.527$	52.7%
<b>40 - 44</b>	20.16		$\frac{189.27}{320.91} = 0.590$	
<b>45 - 49</b>	20.80	210.07	$\frac{210.07}{320.91} = 0.655$	65.5%
<b>50 - 54</b>	22.29		$\frac{232.36}{320.91} = 0.724$	72.4%
<b>55 - 59</b>	21.77	254.13		79.2%
<b>60 - 64</b>	19.04	273.17	$\frac{273.17}{320.91} = 0.851$	85.1%

<b>65 - 69</b>	16.05		$\frac{289.22}{320.91} = 0.901$	
<b>70 - 74</b>	11.48	300.70		93.7%
<b>75 - 79</b>	8.12			
<b>80 - 84</b>	5.80	314.62		98.0%
<b>85 - 89</b>	3.86		$\frac{318.48}{320.91} = 0.992$	
<b>90 - 94</b>	1.85	320.33		99.8%
<b>95 - 99</b>	0.50		$\frac{320.83}{320.91} = 0.999$	99.9%
<b>100+</b>	0.08	<b>320.91</b>	$\frac{320.91}{320.91} = 1.000$	100.0%
<b>Total</b>	<b>320.91</b>			

5. What does each of the columns of the above table summarize? What does the cumulative count summarize? Answer each of the following questions based on the table to indicate your understanding of each of the columns.
- How many people are younger than 5 years old?
  - What is the percent of people younger than 5 years old?
  - How many people are younger than 10 years old?
  - What is the percent of people younger than 10 years old?
  - What is the percent of people younger than 20 years old?
  - How many people are younger than 50 years old?
  - What is the percent of people younger than 50 years old?

6. There are several blank cells in the table. For each blank cell, complete the expected calculations.
7. What is the first age group that captures at least 50% of the cumulative population?
8. Estimate the **age group** in which the median age is located.
9. Is it possible to identify the exact age within the age group identified in problem 8 that would be the median age (or the age where at most 50% of the population would be less than this age)? Explain your answer.

In addition to the center or median age of the population, the **spread** of the population based on the above table also provides an important description of the ages in the United States. There are several summaries that are used to describe spread. For this lesson, spread is identified by two important age groups. The first age group captures 75% of the cumulative population. The second age group captures 25% of the cumulative population. Between these two age groups at least 50% of the ages of every person in the country are captured. Note that the age group capturing the median age is within these two age groups. **The difference of the age groups that captures 75% of the population and the age group that captures 25% of the population summarizes a spread.** This difference is derived by subtracting the youngest age of the age group capturing 25% of the population from the oldest age of the age group capturing 75% of the population.

10. Using the above description of spread and the completed table, answer the following:
  - a. What is the first age group that captures 75% of the cumulative population?
  - b. What is the first age group that captures 25% of the cumulative population?
  - c. Calculate the spread of the population based on the difference in ages of the two age groups in 10(a) and 10(b) using the oldest age from the age group that captures 75% of the cumulative population and the youngest age from the age group that captures 25% of the cumulative population. Approximately what percent of the total population is captured between the two ages used to calculate the spread?
  - d. Why is the last cell in the column representing the proportion as a percent equal to 100.0%?

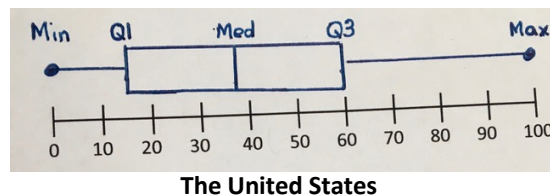
11. Describe the typical person in 2015 based on the median age and the spread. How does this person differ from the 1980 typical person?

A helpful visual of the median age and spread of a distribution is a **box plot**. For this lesson, the box plot is a modified box plot as the data are grouped in intervals of 5-years. A more precise box plot is based on using the ages of the entire population.

The box plot for the United States is defined by a **5-number summary**.

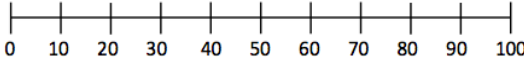
- The first number of the summary is called the minimum (or **Min**) and represents the youngest age in the data set. For the United States population that age would be 0 years old.
- The second number of the summary is called **quartile 1** (or **Q1**) and is the youngest age of the age group capturing 25% of the population.
- The next number is the median age (or **Med**). For this lesson, this age is an age within the age group that captures 50% of the population.
- The next number is **quartile 3** (**Q3**). For this lesson, Q3 is the oldest age of the age group capturing 75% of the population.
- Finally, the last number is the maximum age (or **Max**). For this lesson, the value of 100 years will be used (although we know there actually are individuals older than 100 years old).

Putting these 5 numbers derived in this lesson forms the following box plot:





12. Sketch a histogram of a country with the same median age as the United States in 2015 but with a spread that is one-half of the value derived for the United States. Also sketch an approximate box plot of this country using the following grid:

<p>Sketch a possible histogram:</p>	<p>Sketch a possible box plot:</p> 
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### Finding the Mean Age of the United States Population in 2015

The mean age of the United States population in 1980 was reported to be 33.6 years. Similar to a median age, the mean age is also considered a center. Recall that a mean of a set of data is the arithmetic average of the data. The mean age is derived by dividing the sum of the ages of all people in the United States by the total count of people. The mean is also identified as the “balance point” of a set of data (or ages) plotted on a number line. Is it possible to estimate the mean age using the counts in the age groups provided by the Census Bureau rather than all ages of the population? To answer this question, an estimate of the mean age of the United States in 2015 will be derived from the grouped data and then compared to the 1980 mean age. The following template will be used to derive an estimate of the mean age. Study the following column headings used to organize your calculation of this estimate.

### United States - 2015

Age group	Mid-interval Age of Age group	Count of people (in millions of people to the nearest hundredth)	Sum of the ages in each age group: (Estimated in millions of years)
0 – 4	2	19.91	$2 \times 19.91 = 39.82$
5 - 9	7	20.48	$7 \times 20.48 = 143.36$
10 - 14	12	20.61	$12 \times 20.61 = 247.32$
15 - 19	17	21.09	$17 \times 21.09 = 358.53$
20 - 24		22.69	
25 - 29	27	22.40	$27 \times 22.40 = 604.8$
30 - 34	32	21.62	$32 \times 21.62 = 691.84$
35 - 39	37	20.31	
40 - 44	42	20.16	$42 \times 20.16 = 846.72$
45 - 49		20.80	$47 \times 20.80 = 977.6$
50 - 54	52	22.29	$52 \times 22.29 = 1159.08$
55 - 59	57	21.77	$57 \times 21.77 = 1240.89$
60 - 64	62	19.04	
65 - 69		16.05	
70 - 74		11.48	$72 \times 11.48 = 826.56$
75 - 79	77	8.12	$77 \times 8.12 = 625.24$
80 - 84	82	5.80	$82 \times 5.80 = 475.6$
85 - 89	87	3.86	
90 – 94		1.85	
95 – 99	97	0.50	$97 \times 0.50 = 48.5$
100+	102	0.08	$102 \times 0.08 = 8.16$
	<b>Total</b>	<b>320.91</b>	<b>12306.52</b>

13. Explain how the mid-interval ages were determined for each age group.

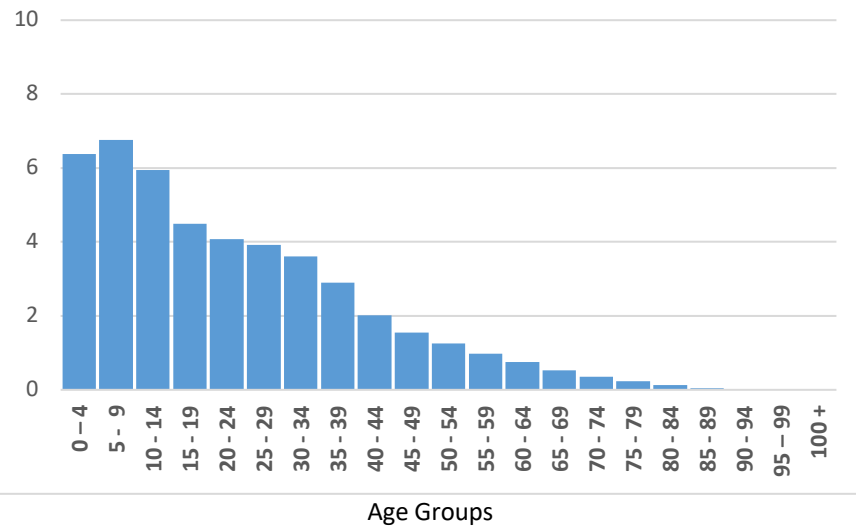
14. Do you think all of the 19.91 million people who were 0 – 4 years old are 2 years old? Why might, however, the mid-interval age of 2 years be a reasonable estimate of the age of each of the children in the 0 – 4 years old age group?

15. To determine the mean age, the sum of the ages in each age group is needed. Consider the age group 0 – 4 years old. If 2 years is a good estimate of the age of all of the people in that age group, what does the product of 2 and 19.91 represent?

16. In the same way, what does the product of 7 and 20.48 represent?
17. Analyze what is missing in each of the blank cells of the above table. For each blank cell, complete the expected calculations.
18. The last column of the above table represents an estimate of the sum of the ages for each age group. To determine the mean, the sum of all of the ages for 320.91 million people is needed. Based on the estimates recorded for each age group, 12306.52 million is the approximate sum of all of the age groups in that column. Describe the last step needed to calculate an estimate of the mean age of a person in the United States.
19. What is an estimate of the mean age of a person in the United States?
20. Do you think the estimated mean age is a good description of a typical person in 2015? Explain why or why not.
21. The estimate of the mean age in 2015 is greater than the estimate for 1980. What does this indicate about the change in the population during this time?

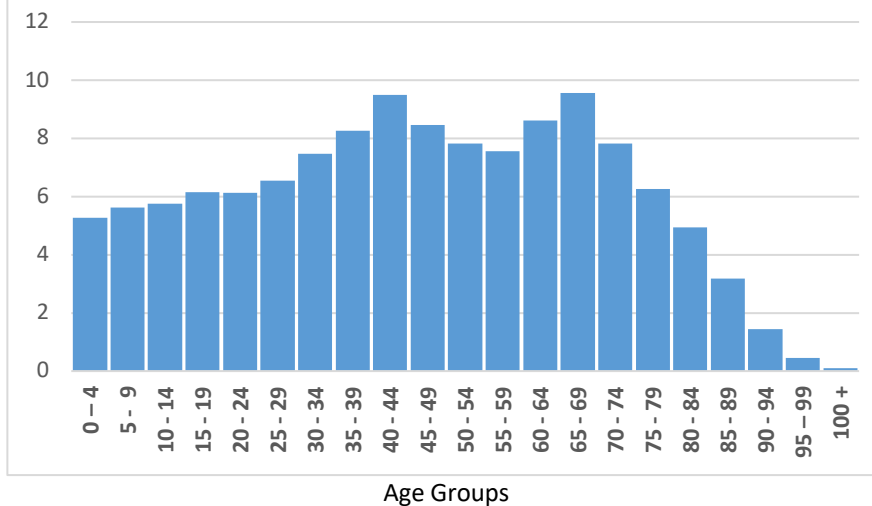
The estimates of the mean age and the median age are similar for the United States but not exactly the same. This similarity is not always the case, however, for other countries. Use the following templates prepared to derive an estimate of the median ages and spread and the mean ages for Kenya and Japan in 2015. One of these two countries will have a noticeable difference in the estimates of the median age and the mean age. What does it indicate about the population if the two centers are different?

### Kenya Population by Age Groups - 2015



Population in millions of people  
Total Population is 45.93 millions of people

### Japan Population by Age Groups - 2015



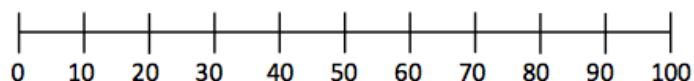
Population in millions of people  
Total Population is 45.93 millions of people

## Kenya 2015

### Template for finding the estimate of the median age:

Age group	Count of people (in millions of people to the nearest hundredth)	Cumulative count of people (in millions of people to the nearest hundredth)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	6.38			
5 - 9	6.76			
10 - 14	5.95			
15 - 19	4.49			
20 - 24	4.08			
25 - 29	3.92			
30 - 34	3.60			
35 - 39	2.89			
40 - 44	2.01			
45 - 49	1.55			
50 - 54	1.25			
55 - 59	0.98			
60 - 64	0.75			
65 - 69	0.53			
70 - 74	0.36			
75 - 79	0.23			
80 - 84	0.12			
85 - 89	0.05			
90 - 94	0.01			
95 – 99	0.01			
100+	0.01			
Total	45.93			

22. Derive an estimate of the median age group for Kenya and a description of the spread based on the definition of spread in this lesson. After you have estimated the median age and the spread, sketch a modified box plot using the following grid:



## Kenya – 2015

Template for finding the estimate of the mean age:

Age group	Mid-interval Age of Age group	Count of people (in millions of people to the nearest hundredth)	Sum of all ages in age group: (Estimated in millions of years)
0 – 4	2	6.38	
5 - 9	7	6.76	
10 - 14	12	5.95	
15 - 19	17	4.49	
20 - 24	22	4.08	
25 - 29	27	3.92	
30 - 34	32	3.6	
35 - 39	37	2.89	
40 - 44	42	2.01	
45 - 49	47	1.55	
50 - 54	52	1.25	
55 - 59	57	0.98	
60 - 64	62	0.75	
65 - 69	67	0.53	
70 - 74	72	0.36	
75 - 79	77	0.23	
80 - 84	82	0.12	
85 - 89	87	0.05	
90 – 94	92	0.01	
95 – 99	97	0.01	
100+	102	0.01	
	<b>Total</b>	45.93	

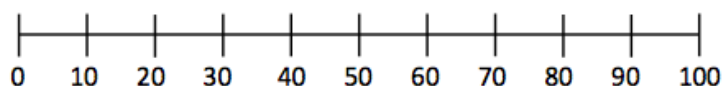
23. Derive an estimate of the mean age of Kenya. Compare the estimated mean age to the median age. Are they similar? Explain.

## Japan 2015

Template for finding the estimate of the median age:

Age group	Count of people (in millions of people to the nearest hundredth)	Cumulative count of people (in millions to the nearest hundredth)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	5.27			
5 - 9	5.61			
10 - 14	5.75			
15 - 19	6.15			
20 - 24	6.13			
25 - 29	6.54			
30 - 34	7.47			
35 - 39	8.27			
40 - 44	9.50			
45 - 49	8.46			
50 - 54	7.82			
55 - 59	7.57			
60 - 64	8.62			
65 - 69	9.57			
70 - 74	7.82			
75 - 79	6.26			
80 - 84	4.95			
85 - 89	3.17			
90 - 94	1.45			
95 – 99	0.44			
100+	0.09			
Total	126.91			

24. Derive an estimate of the median age group for Japan and a description of the spread based on the definition of spread in this lesson. After you have estimated the median age and the spread, sketch a modified box plot using the following grid:



# Japan – 2015

Template for finding the estimate of the mean age:

Age group	Mid-interval Age of Age group	Count of people (in millions of people to the nearest hundredth)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	5.27	
5 - 9	7	5.61	
10 - 14	12	5.75	
15 - 19	17	6.15	
20 - 24	22	6.13	
25 - 29	27	6.54	
30 - 34	32	7.47	
35 - 39	37	8.27	
40 - 44	42	9.50	
45 - 49	47	8.46	
50 - 54	52	7.82	
55 - 59	57	7.57	
60 - 64	62	8.62	
65 - 69	67	9.57	
70 - 74	72	7.82	
75 - 79	77	6.26	
80 - 84	82	4.95	
85 - 89	87	3.17	
90 – 94	92	1.45	
95 – 99	97	0.44	
100+	102	0.09	
	<b>Total</b>	<b>126.91</b>	

25. Derive an estimate of the mean age of Japan. Compare Japan’s estimate of the mean age to the median age. Are they similar? Explain.