Investigation 12

Chances of Getting the Flu?
Simulations

Overview

This investigation develops a probability distribution through the design and use of a simulation. It follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are formulate a statistical question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level B activity.

This activity is based on a simulation problem from *The Art and Techniques of Simulation*, published by Dale Seymour and the American Statistical Association. (This module is part of the Quantitative Literacy Series. Though out of print, the book is available through book resale sites and on Amazon.com.)

Instructional Plan

Brief Overview

» Read and discuss the scenario about the spread of flu in an apartment building.

» Formulate the statistical/probabilistic question: "What is an estimate for the probability that all six people who live in an apartment building will get the flu?"

» Demonstrate the steps to conduct a simulation to answer the probabilistic question.

» Have students conduct the simulation using a die or technology and report their results.

» Collect class data in a table, convert the results to relative frequencies and a probability distribution.

» Use the probability distribution to answer the statistical/probabilistic question.

Hand out Student Worksheet 12.1 Flu Epidemic. Direct your students to read the first paragraph in the scenario.

Scenario

Did you get a flu vaccine last year? If so, did you still get the flu?

Infectious diseases (or diseases that are often caused by a bacteria or virus) are extensively researched in the medical field. These diseases result in colds, seasonal flu, and major epidemics that affect large numbers of people or animals in some cases.

In the fall of 1918, a flu pandemic erupted and became one of the greatest loss of lives the world had ever seen. By many accounts, the flu claimed between 2.5% and 5% of the global population. At that time, there was no flu vaccine, no antiviral drugs, and no antibiotics to help lessen the number of patients who got the flu or aid in the recovery from the flu.

As a result of this pandemic, countries began to put a greater emphasis on the study of patterns, causes, and effects of diseases.

Medical researchers are actively involved in understanding what causes the disease, how it is spread, how long it lasts, and other data related to the health of patients.

Source: www.smithsonianmag.com/history/how-1918-flu-pandemic-revolutionized-public-health-180965025

Learning Goals

- Design and carry out a simulation to estimate the probability of a random event.
- Develop a non-uniform probability distribution based on a simulation.

Mathematical Practices Through a Statistical Lens

MP5. Use appropriate tools strategically.

Statistically proficient students are able to use technological tools to carry out simulations for exploring and deepening their understanding of statistical and probabilistic concepts.

Materials

Student worksheets are available at *www.statisticsteacher.org/statistics-teacher-publications/focus*.

- Large foam die
- Die for each pair of students
- Technology like the TI-84 graphing calculator with ProbSim app or a similar rolling die application such as *www.random.org/dice*
- Student Worksheet 12.1 Flu Epidemic
- Student Worksheet 12.2 Simulation Steps
- Exit Ticket

Estimated Time

One 50-minute class period

Pre-Knowledge

Students should already be able to find the probability of simple events.

Students understand the probability of an event E is equal to:

$$P(E) = \frac{\text{Number of trials favorable to E}}{\text{Total number of trials in the experiment}}$$

Discuss with students the flu scenario and ask what type of precautions they can take to avoid getting the flu.

Ask your students to read the flu example.

Flu Example

Consider the following simple example of an infectious disease, like a cold or flu, and how it spreads throughout a small apartment building.

Suppose a strain of the flu has a one-day infection period (i.e., a person with the flu can only infect another person for one day and, after that day, the person can't spread the flu and is immune—that is, once you get the flu, you can't get this strain of flu again). This strain of flu is potent; if a person comes into contact with someone with the flu, that person will get the flu for certain.

Six people live in a small apartment building. One person catches this very infectious strain of flu and randomly encounters one of the other tenants during the infection period, and this second tenant gets this strain of flu. This second tenant infected with the flu visits a third tenant at random during the next day, and this third tenant gets the flu. The process continues with a newly infected person randomly visiting someone who hasn't had the flu or visiting an immune person and the strain of flu dies out. If an infected person visits an immune person, then the spread of the flu will end, as the flu in this example has only a one-day infection period.

Ask your students to summarize how this strain of flu spreads.

What is the least number of tenants who could get the flu?

Answer: *Two tenants, The first tenant gets the flu and visits a second tenant, who then goes back and visits the first tenant.*

What is the highest number of tenants who could get the flu?

Answer: *All six tenants*

Formulate a Statistical Question

Discuss with your students that one way to investigate an estimate of the number of people who would get the flu in this apartment building is to design and conduct a simulation. A simulation is a procedure developed for answering questions about real problems by running experiments that resemble the real-life situation. Instead of finding a large number of apartment buildings with six apartments and one person with the flu, a simulation could be designed to provide outcomes of the number of people who get the flu.

Ask students to consider the statistical/probabilistic question: "What is an estimate for the probability that all six people who live in an apartment building will get the flu?"

Collect Appropriate Data

To help your students understand the scenario, conduct a simulation involving them.

- Select six students and have them come to the front of the room. These six students represent the people living in the apartment building. Number each student from 1 to 6.
- *Day 1:* Roll the large foam die to determine Patient Zero, who will have the flu first. For example, if a 3 is rolled, then Person 3 has the flu. Have Person 3 roll the die, and then have Person 3 visit the person whose number is rolled. For example, a 4 is rolled. Remember this flu is potent; if a person is "visited," they will get the flu. Now two people have gotten the flu—persons 3 and 4. If Person 3 rolled a 3, then Person 3 would roll again since a person can't visit him/herself.

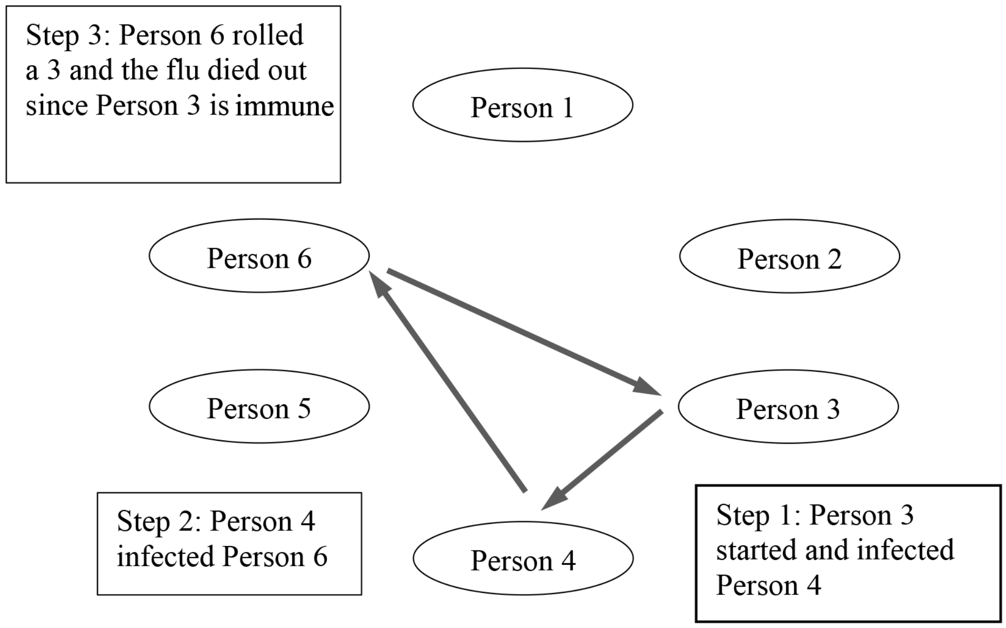

Figure 12.1

- » *Day 2:* Person 3 is now immune (once you have had the flu, you can't get it again and you are no longer contagious) and Person 4, who now has the flu rolls a die and visits (infects) the person whose number was selected. For example, Person 6. Three people (3, 4, and 6) now have had the flu, unless Person 4 was to roll a 3. In that case, the flu would die out since the infected person visited a person who already had the flu. If the person rolls his/her own number, have the person roll again since a person can't visit him/herself.
- » *Day 3:* Person 3 and Person 4 are immune. Person 6, who now has the flu, rolls a die and visits a person. Continue until a person visits someone who has already had the flu (i.e., immune) or someone who has not been infected. If the person rolls his or her own number, have the person roll again since a person can't visit him or herself.

Make a note of the number of people who got the flu.

Note: Students could also draw six circles—one for each person in the apartment building—and draw lines connecting the circles to show how the flu spreads as the simulation progresses.

Figure 12.1 illustrates the example above showing one trial in which three people were infected before the flu died out.

Emphasize that the goal is to design and conduct a simulation to find an estimate for the probability that all six people living in an apartment building will get the flu.

Share (consider posting) the steps to designing and conducting a simulation. Student Worksheet 12.2 Simulation Steps lists the steps.

Steps

1. State the problem or statistical/probabilistic question.
2. Define the simple events that form the basis of the simulation.
3. State any underlying conditions that need to be made so the answer to the probabilistic question can be determined.
4. Decide on a model that will be used to match the probabilities. Describe how random numbers will be assigned to match the probabilities described in the problem. Determine what constitutes a trial and what will be recorded.
5. Conduct the first trial.
6. Record the results of the trial.
7. Continue to run trials. Run a large number of trials. Remember to report the result of each trial.
8. Summarize the results of the trials and draw conclusions.

Go through the steps for this simulation using a die or large foam die.

1. State the problem (probabilistic question) so the objective of the simulation is clear. *What is an estimate for the probability all six people living in an apartment building will get the flu?*
2. Define the simple events that form the basis of the simulation. *Infected person randomly visits another person in the apartment building. If a person is randomly visited, they will get the flu, unless they have already had the flu.*
3. State any underlying conditions that need to be made so the answer to the probabilistic question can be determined.

Table 12.1

Trial Number	Who Was Infected (# on Each Roll	Number of People Infected
1	3,4,2,5,3	4
2	6,6,2,6	2
3		

Conditions: Visits are done randomly. Only one person can become infected at a time. Person can infect others for only one day.

4. Decide on a model that will be used to match the probabilities. Describe how random numbers will be assigned to match the probabilities described in the problem. Determine what constitutes a trial and what will be recorded. *Number the people from 1 to 6. Roll a die to simulate the visit by the infected person. (Persons can't visit themselves.) A trial is rolling the die until the flu dies out—a person with the flu visits someone who is immune (already had the flu). The number of people infected will be recorded.*
5. Define and conduct the first trial. *The first roll of the die determines which person was the first person to get the flu. Continue to roll the die until whoever is the current infected person visits an immune person (someone who has already had the flu). That is, roll until a number (other than the infected person's) is repeated. The trial is then over.*
6. Record the results of the trial. *Record the trial number, the results of each roll, and the number of people infected in a table, as shown in Table 12.1.*
7. Continue to run several more trials. Remember to record the result of each trial. *Repeat steps 5 and 6 a large number of times (at least 50 for the class). Give each pair of*

Table 12.2

Number of People Infected	Frequency
2	
3	
4	
5	
6	
Total	

Table 12.3

Number of People Infected	Frequency
2	17
3	33
4	22
5	8
6	2
Total	82

students a die and have them conduct at least five trials and collect the class results in a table.

Explain that an accurate estimate for a probability requires that a large number of trials be conducted (at least 50 for the whole class). Divide the students into groups of two. One person rolls the die and the other records the outcomes in a chart. Ask each group of students to conduct at least five trials.

After the groups have completed at least five trials, collect each group's results in Table 12.2. You are collecting the number of people infected for each trial.

Sample results from a class of 9th graders are shown in Table 12.3.

Option: Demonstrate how to use technology (e.g., ProbSim app on TI-84 Graphing calculator or other rolling die simulator) to collect a large number of trial results.

Analyze the Data

After the simulation has been run for a large number of trials and the results collected in a table, ask the students to answer questions 1 to 4.

1. Fill in Table 12.2 using the class simulation results.
2. Construct a dot plot of the class simulation results.

Possible answer: *Sample results from a 9th grade class in Figure 12.2*

3. What is the most likely number of people living in the apartment building who will get the flu?

Possible answer: *Three people*

4. Add a column to Table 12.3. Label the column Relative Frequency. Complete the relative frequency column in Table 12.4.

Answer: *Based on the example*

Explain that Table 12.4 gives estimates for the relative frequency of various successes (the number of persons who become infected). The relative frequencies for the different number of successes can be thought of as the probability of the number of successes. This table describes a *probability distribution*.

Let X = Number of people infected and P(X) = the probability of x people being infected.

5. What is an estimate for the probability that all six people living in an apartment building will get the flu?

Answer (based on the example in Table 12.5): *0.024, or 2.4%*

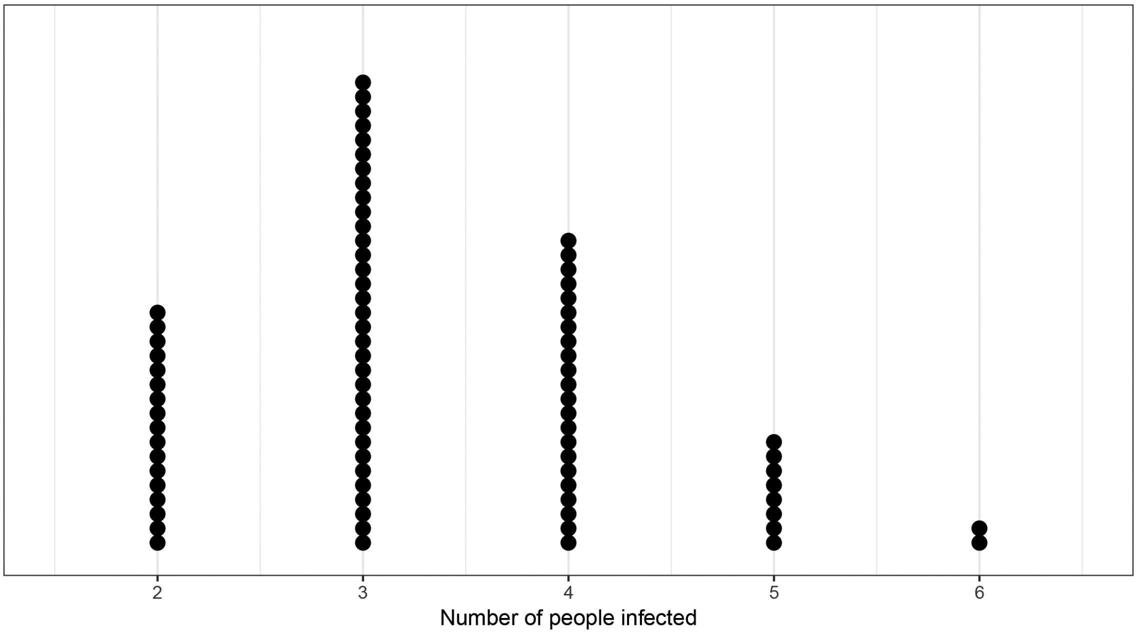

Figure 12.2: Dot plot of the class simulation results

Interpret the Results in the Context of the Original Question

Ask students to answer this question based on the simulation model they designed and conducted.

6. How did you model the spread of the flu in the apartment building? And how did you use this model to find an estimate for the probability that all six people living in the apartment building will get the flu?

Possible answer: *We modeled the spread of the flu by using a six-sided die. Each side of the die represented one person in the apartment building. We rolled the die and recorded the person who got the flu. We continued until a person visited someone with the flu, which caused the flu to die out. We recorded the number of people who got the flu and repeated the simulation a large number of times. After many trials, we were able to estimate the probability of all six people getting the flu as 2.4%*

Summary

To help summarize this simulation, ask your students the following questions:

7. What model could be used if there were eight people in the apartment building?

Table 12.4

Number of People Infected	Frequency	Relative Frequency
2	17	17/82 = 0.207
3	33	33/82 = 0.402
4	22	22/82 = 0.268
5	8	8/82 = 0.098
6	2	2/82 = 0.024
Total	82	1.0

Table 12.5

X	P(X)
2	17/82 = 0.207
3	33/82 = 0.402
4	22/82 = 0.268
5	8/82 = 0.098
6	2/82 = 0.024
Total	1.0

Possible answers: *An eight-sided die, randomly selecting numbers from 1 to 8 from a hat or bag, random number generator on computer or calculator*

8. How do you think the probability of all eight people in an apartment building getting the flu compares with the probability of all six people getting the flu?

Answer: *The probability of eight would be smaller than the probability of six getting the flu.*

Additional Ideas

1. Design and conduct a simulation for the following problem: The chance of contracting strep throat when encountering an infected person is estimated as 0.15. Suppose the four children of a family encounter an infected person. Conduct a simulation to estimate the probability of at least two of the children getting strep throat. State the conditions needed to simulate the problem.
2. A high-school algebra teacher has eight keys, but she never recalls which one fits her office door lock. She tries one key at a time, each time choosing one of the keys at random from her pocket. (All the keys look the same but she does not put a key back in her pocket once she has tried that key.) Conduct a simulation to estimate the probability it will take more than four tries to find the right key.

Exit Ticket

Your math teacher owns 10 ties and randomly chooses a tie to wear to work each school day (not much fashion sense). You notice he sometimes wears the same tie more than once during the week. You wonder if this is likely to happen often, so you decide you would like to find an estimate for the probability he wears the same tie more than once in a five-day workweek. To find this estimate, you design and conduct a simulation.

1. Describe the simple event.

Answer: *Randomly choosing a tie.*

2. Describe a model that would be appropriate to use for the simple event.

Answer: *Number the ties from 1 to 10*

Exit Ticket Cont.

3. Describe a trial and what you would record for each trial.

Answer: *Randomly choose five numbers between 1 and 10. The numbers could be chosen using a 10-sided die, randomly selecting numbers from 1 to 10 from a hat or bag or using the randint(1,10,5) function of the TI-84 graphing calculator or another rolling die simulator.*

Record whether or not a number is repeated.

4. Using the results below, what is an estimate for the probability he wears the same tie more than once in a five-day workweek?

Answer: *An estimate for the probability that he wears the same tie more than once in a five-day workweek is 17/28 = 0.61.*

Table 12.6: Results for the Simulation

Trial Number	Wears Same Tie More Than Once (Y/N)	Trial Number	Wears Same Tie More Than Once (Y/N)
1	Y	15	N
2	N	16	Y
3	N	17	Y
4	N	18	Y
5	N	19	Y
6	Y	20	Y
7	Y	21	Y
8	N	22	N
9	N	23	Y
10	Y	24	N
11	Y	25	Y
12	Y	26	N
13	Y	27	N
14	Y	28	Y

Further Explorations and Extension

Investigating the flu probability problem further.

The simulation gave an estimate for the probability of all six getting the flu. Using formal probability rules, find the exact probability of 2, 3, 4, 5, and 6 people getting the flu. Compare these answers to the simulated probability distribution developed in this lesson.

The probabilities can be calculated in the following manner:

Let X = the number of people infected

P(X=2) = 5/5 * 1/5 = 1/5 = 0.2

P(X=3) = 5/5 * 4/5 * 2/5 = 40/125 = 0.32

P(X=4) = 5/5 * 4/5* 3/5* 3/5 = 180/625 = 0.288

P(X=5) = 5/5 * 4/5* 3/5* 2/5 *4/5 = 480/3125= 0.1536

P(X=6) = 5/5 * 4/5 * 3/5* 2/5 * 1/5 = 120/3125 = 0.0384

Explanation for P(X=3)

5/5 = the probability Person 1 picks another person

4/5 = the probability Person 2 picks another person other than Person 1

2/5 = the probability the third person picks Person 1 or Person 2, which stops the flu.

Answer: *Table 12.7 Probability Distribution*

Table 12.7

X	P(X)
2	0.2
3	0.32
4	0.288
5	0.1536
6	0.0384
Total	1.0