Teaching Quartile Location-Using Sample Size Divisibility

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Quartiles are descriptive measures of location that can be introduced to students as early as primary school and are taught at the tertiary education level across the world. To successfully locate the quartiles of a univariate data set, basic counting and arithmetic are required. However, particularly in the tertiary-level statistics course, there is often confusion among students when textbooks and technology explain quartile location using complex computational algorithms (i.e., interpolation and cumulative distribution functions). We can eliminate such confusion by providing a table of closed-form solutions for locating the three quartiles.

Review of Quartiles and Their Meanings

Quartiles are the boundaries that divide a univariate data set into four (almost) equal subsets. These measures of location are instrumental in teaching topics such as interquartile range, boxplots, and Tukey's outlier detection method.

There exists a plethora of mathematically valid methods to estimate the quartiles, which run the gamut in difficulty from basic to rigorous. Computer software (i.e., Excel, SPSS, SAS, R, and STATA) is often incorporated, particularly at the tertiary level, for quick calculation. However, these technological resources are many times complicated to understand, because they give estimates based on highly sophisticated algorithms. Some textbooks are also at fault for basing their description of quartile location on these methods. This may leave students confused, especially in their first statistics class.

First Quartile

The first quartile, sometimes referred to as Q_1, is a descriptive measure that indicates the 25th percentile of a univariate data set. This location separates the bottom 25% of the data set from the top 75% of the data set.

Second Quartile

The second quartile, sometimes referred to as Q_2, is a descriptive measure that indicates the 50th percentile of a univariate data set. This location separates the bottom 50% of the data set from the top 50% of the data set. The Q_2 is also referred to as the median.

Third Quartile

The third quartile, sometimes referred to as Q_3, is a descriptive measure that indicates the 75th percentile of a univariate data set. This location separates the bottom 75% of the data set from the top 25% of the data set.

Locating Quartiles Based on the Median

The most fundamental approach to locating the quartiles is to use the median (or Q_2) as the separator of the "lower half" and the "upper half" of the ordered data set. After Q_2 has been identified, Q_1 can be defined as the "median of the lower half" and Q_3 can be defined as the "median of the upper half." However, ambiguity occurs when deciding whether to include the Q_2 when locating Q_1 and Q_3 from the lower and upper partitions, respectfully. This is an important distinction, as inclusion or exclusion of the median will give different locations and values for Q_1

and Q_3. Ultimately, this decision is left to the instructors' discretion and should be maintained consistently throughout the course.

Using this method of partitioning the data, we have observed students typically make errors in locating the quartiles when the data set is large (say, greater than 25 data points). This is not surprising. As the number of data points increases, students are more likely to make mistakes. Also, this method is time consuming, since three separate medians must be found when partitioning the data. In order to combat this problem, can a method for teaching quartile location be implemented that is mathematically principled, easy to use, and provides a check of work?

Table Method Based on Sample Size Divisibility

This method can be broken down into the following steps. First, the data must be arranged in increasing order $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$, where $x_{(i)}$ indicates the ith ordered data value. Next, find the remainder when the sample size, n, is divided by four. Finally, apply the formulas in Table 1 or Table 2, which give the location of the quartiles based on the divisibility of n. Use Table 1 when the median is excluded, and use Table 2 when the median is included.

Table 1. Quartile Locations Based on Divisibility of Sample Size n, While Excluding the Median

Divisibility of *n*	**First Quartile**	**Second Quartile**	**Third Quartile**
$n/4$ with 0 remainder	$\frac{1}{2}(x_{(\frac{n}{4})} + x_{(\frac{n}{4}+1)})$	$\frac{1}{2}(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)})$	$\frac{1}{2}(x_{(\frac{3n}{4})} + x_{(\frac{3n}{4}+1)})$
$n/4$ with 1 remainder	$\frac{1}{2}(x_{(\frac{n-1}{4})} + x_{(\frac{n-1}{4}+1)})$	$x_{(\frac{n+1}{2})}$	$\frac{1}{2}(x_{(\frac{3n+1}{4})} + x_{(\frac{3n+1}{4}+1)})$
$n/4$ with 2 remainder	$x_{(\frac{n-2}{4}+1)}$	$\frac{1}{2}(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)})$	$x_{(\frac{3n+2}{4})}$
$n/4$ with 3 remainder	$x_{(\frac{n-3}{4}+1)}$	$x_{(\frac{n+1}{2})}$	$x_{(\frac{3n+3}{4})}$

Table 2. Quartile Locations Based on Divisibility of Sample Size n, While Including the Median

Divisibility of *n*	**First Quartile**	**Second Quartile**	**Third Quartile**
$n/4$ with 0 remainder	$x_{(\frac{n}{4}+1)}$	$\frac{1}{2}(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)})$	$x_{(\frac{3n}{4})}$
$n/4$ with 1 remainder	$x_{(\frac{n-1}{4}+1)}$	$x_{(\frac{n+1}{2})}$	$x_{(\frac{3n+1}{4})}$
$n/4$ with 2 remainder	$\frac{1}{2}(x_{(\frac{n+2}{4})} + x_{(\frac{n+2}{4}+1)})$	$\frac{1}{2}(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)})$	$\frac{1}{2}(x_{(\frac{3n-2}{4})} + x_{(\frac{3n-2}{4}+1)})$
$n/4$ with 3 remainder	$\frac{1}{2}(x_{(\frac{n+1}{4})} + x_{(\frac{n+1}{4}+1)})$	$x_{(\frac{n+1}{2})}$	$\frac{1}{2}(x_{(\frac{3n-1}{4})} + x_{(\frac{3n-1}{4}+1)})$

Conveniently, a quick check exists for this method. All that is needed is to count from the subscript of Q_3 up to $x_{(n)}$. If the index matches the sample size value n, then the student can be more confident they have provided the correct answer. If not, they must have gone wrong by accidentally excluding or over-counting data points. Next, we present the derivations for the quartile locations using median-exclusion. The derivations for median-inclusion can be done in a similar way. Ideally, these derivations should be included in a lesson plan, as they help motivate the understanding of quartiles. In the following, assume the data is ordered $x_{(1)}, x_{(2)},\dots, x_{(n)}$.

<u>Case $n/4$ with 0 remainder:</u>
The median is calculated by $\frac{1}{2}(x_{(n/2)} + x_{(n/2+1)})$.
Since n is even and $x_{(1)}, x_{(2)},\dots, x_{(n/2)}$ indicates the "bottom half" of the data, let $k = n/2$.
Here, k is even, so $Q_1=\frac{1}{2}(x_{(k/2)}+x_{(k/2+1)})=\frac{1}{2}(x_{((n/2)/2)}+x_{((n/2)/2+1)})=\frac{1}{2}(x_{(n/4)}+ x_{(n/4+1)})$.
Next, to find Q_3, simply add $n/2$ to the indices in Q_1.
Doing so gives the following: $Q_3 = \frac{1}{2}(x_{(n/4+ n/2)} + x_{((n/4+ n/2+1))}) =\frac{1}{2}(x_{(3n/4)} + x_{(3n/4+1)})$
Finally, count from the location of Q_3 to $x_{(n)}$ to verify all n data points were used.

<u>Case $n/4$ with 1 remainder:</u>
The median is calculated by $x_{((n+1)/2)}$.
Since n is odd and $x_{(1)}, x_{(2)},\dots, x_{((n-1)/2)}$ indicates the "bottom half" of the data, let $k=(n–1)/2$.
Here, k is even, so $Q_1=\frac{1}{2}(x_{(k/2)}+x_{(k/2+1)})=\frac{1}{2}(x_{((n-1)/2/2}+x_{((n-1)/2/2+1)}) =\frac{1}{2}(x_{(n/4-1/4)}+ x_{(n/4+3/4)})$.
Next, to find Q_3, simply add $(n+1)/2$ to the indices in Q_1.
$Q_3 = \frac{1}{2}(x_{(n/4-1/4 +(n+1)/2)} + x_{(n/4+3/4 + (n+1)/2)}) = \frac{1}{2}(x_{(3n/4+1/4)} + x_{(3n/4+5/4)})$
Finally, count from the location of Q_3 to $x_{(n)}$ to verify all n data points were used.

<u>Case $n/4$ with 2 remainder:</u>
The median is calculated by $\frac{1}{2}(x_{(n/2)} + x_{(n/2+1)})$.
Here, n is even and $x_{(1)}, x_{(2)},\dots, x_{(n/2)}$ indicates the "bottom half" of the data; let $k = n/2$.
Here, k is odd, so $Q_1 = x_{((k+1)/2)} = x_{((n/2+1)/2)} = x_{(n/4+1/2)}$.
Next, to find Q_3, simply add $n/2$ to the indices in Q_1.
$Q_3 = x_{(n/4+ n/2+1/2)} = x_{(3n/4+1/2)}$
Finally, count from the location of Q_3 to $x_{(n)}$ to verify all n data points were used.

<u>Case $n/4$ with 3 remainder:</u>
The median is calculated by $x_{((n+1)/2)}$.
Here, n is odd and $x_{(1)}, x_{(2)},\dots, x_{((n-1)/2)}$ indicates the "bottom half" of the data; let $k = (n–1)/2$.
Here, k is odd, so $Q_1 = x_{((k+1)/2)} = x_{(((n-1)/2+1)/2)} = x_{(n/4+1/4)}$.
Next, to find Q_3, simply add $(n+1)/2$ to the indices in Q_1.
$Q_3 = x_{(n/4+1/4)} = x_{(n/4+1/4 +(n+1)/2)} = x_{(3n/4+3/4)}$
Finally, count from the location of Q_3 to $x_{(n)}$ to verify all n data points were used.

Advantages of Using the Table-Based Method
Using this novel method, we claim students can locate quartiles quicker and more accurately than finding the medians of the upper half and lower half of the ordered data. Using the table method, the data only needs to be counted n times, specifically counting from $x_{(1)}$ up to $x_{(n)}$. Whereas using the method that finds Q_2 and then the medians of the upper half and lower half, the data needs to counted $2n$ times. The reasoning is that three separate medians must be found. The data would need to be counted n times to get Q_2, then $n/2$ times to find Q_1, and finally another $n/2$ times to find Q_3. Combining the terms n+n/2+n/2 gives $2n$, so it can be strongly

argued it takes about twice as long as the table-based method. Therefore, the larger the data set, the more time-efficient it is to use this table-based method. Moreover, it comes with a simple check as previously stated. Using this table-based method, the students could either use a calculator or their basic arithmetic skills to locate the positions of the quartiles. In fact, students could use the aid of a calculator for the entirety of the problem if they felt more comfortable.

Illustrative Examples

Twelve hypothetical sample size cases are considered below. There are three cases for each of the four possible remainder values when *n* is divided by four. We use arithmetic calculations and visual illustrations to show how the formulas in Table 1 and Table 2 give exact locations for the three quartiles as the method of separating the data set into two partitions by Q_2 and then finding Q_1 and Q_3 from each partition. Using median-exclusion, Table 3 gives the location of the three quartiles using the appropriate arithmetic formulas. These locations are found by plugging in the sample size value of *n* into the formulas given in Table 1. Table 4 shows a visual location of the three quartiles by first locating Q_2 and then finding the median of the bottom half and top half of the data set using median-exclusion. The locations of the three quartiles are shown using upward pointing arrows. Table 5 gives the location of the quartiles while including the median using the appropriate arithmetic formulas from Table 2. Table 6 shows the visual location of the three quartiles by first locating Q_2 and then finding the median of the bottom half and top half of the data set using median-inclusion.

Table 3. Location of Quartiles Based on Sample Size Divisibility, While Excluding the Median

Sample Size	Remainder	Q_1	Q_2	Q_3
$n = 8$	0	$\frac{1}{2}(x_{(2)} + x_{(3)})$	$\frac{1}{2}(x_{(4)} + x_{(5)})$	$\frac{1}{2}(x_{(6)} + x_{(7)})$
$n = 9$	1	$\frac{1}{2}(x_{(2)} + x_{(3)})$	$x_{(5)}$	$\frac{1}{2}(x_{(7)} + x_{(8)})$
$n = 10$	2	$x_{(3)}$	$\frac{1}{2}(x_{(5)} + x_{(6)})$	$x_{(8)}$
$n = 11$	3	$x_{(3)}$	$x_{(6)}$	$x_{(9)}$
$n = 20$	0	$\frac{1}{2}(x_{(5)} + x_{(6)})$	$\frac{1}{2}(x_{(10)} + x_{(11)})$	$\frac{1}{2}(x_{(15)} + x_{(16)})$
$n = 21$	1	$\frac{1}{2}(x_{(5)} + x_{(6)})$	$x_{(11)}$	$\frac{1}{2}(x_{(16)} + x_{(17)})$
$n = 22$	2	$x_{(6)}$	$\frac{1}{2}(x_{(11)} + x_{(12)})$	$x_{(17)}$
$n = 23$	3	$x_{(6)}$	$x_{(12)}$	$x_{(18)}$
$n = 32$	0	$\frac{1}{2}(x_{(8)} + x_{(9)})$	$\frac{1}{2}(x_{(16)} + x_{(17)})$	$\frac{1}{2}(x_{(24)} + x_{(25)})$
$n = 33$	1	$\frac{1}{2}(x_{(8)} + x_{(9)})$	$x_{(17)}$	$\frac{1}{2}(x_{(25)} + x_{(26)})$
$n = 34$	2	$x_{(9)}$	$\frac{1}{2}(x_{(17)} + x_{(18)})$	$x_{(26)}$
$n = 35$	3	$x_{(9)}$	$x_{(18)}$	$x_{(27)}$

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Table 4. Location of Quartiles Based in an Ordered Data Set, While Excluding the Median

Sample Size	Remainder	Locations of the quartiles in an ordered data set

$n = 8$	0
$n = 9$	1
$n = 10$	2
$n = 11$	3
$n = 20$	0
$n = 21$	1
$n = 22$	2
$n = 23$	3
$n = 32$	0
$n = 33$	1
$n = 34$	2
$n = 35$	3

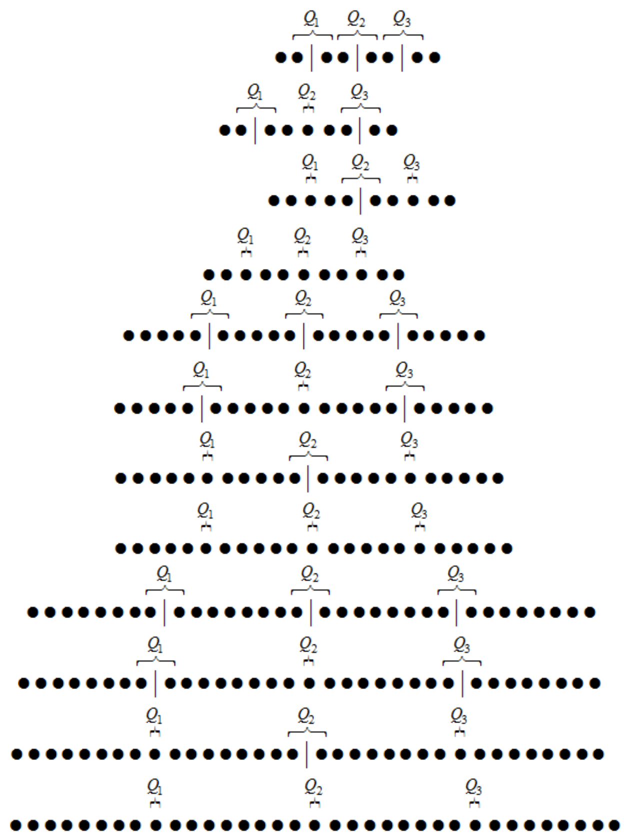

Table 5. Location of Quartiles Based on Sample Size Divisibility, While Including the Median

Sample Size	Remainder	Q_1	Q_2	Q_3
$n = 8$	0	$x_{(3)}$	$\frac{1}{2}(x_{(4)} + x_{(5)})$	$x_{(6)}$
$n = 9$	1	$x_{(3)}$	$x_{(5)}$	$x_{(6)}$
$n = 10$	2	$\frac{1}{2}(x_{(3)} + x_{(4)})$	$\frac{1}{2}(x_{(5)} + x_{(6)})$	$\frac{1}{2}(x_{(7)} + x_{(8)})$
$n = 11$	3	$\frac{1}{2}(x_{(3)} + x_{(4)})$	$x_{(6)}$	$\frac{1}{2}(x_{(8)} + x_{(9)})$
$n = 20$	0	$x_{(6)}$	$\frac{1}{2}(x_{(10)} + x_{(11)})$	$x_{(15)}$
$n = 21$	1	$x_{(6)}$	$x_{(11)}$	$x_{(15)}$
$n = 22$	2	$\frac{1}{2}(x_{(6)} + x_{(7)})$	$\frac{1}{2}(x_{(11)} + x_{(12)})$	$\frac{1}{2}(x_{(16)} + x_{(17)})$
$n = 23$	3	$\frac{1}{2}(x_{(6)} + x_{(7)})$	$x_{(12)}$	$\frac{1}{2}(x_{(17)} + x_{(18)})$
$n = 32$	0	$x_{(9)}$	$\frac{1}{2}(x_{(16)} + x_{(17)})$	$x_{(24)}$
$n = 33$	1	$x_{(9)}$	$x_{(17)}$	$x_{(24)}$
$n = 34$	2	$\frac{1}{2}(x_{(9)} + x_{(10)})$	$\frac{1}{2}(x_{(17)} + x_{(18)})$	$\frac{1}{2}(x_{(25)} + x_{(26)})$
$n = 35$	3	$\frac{1}{2}(x_{(9)} + x_{(10)})$	$x_{(18)}$	$\frac{1}{2}(x_{(26)} + x_{(27)})$

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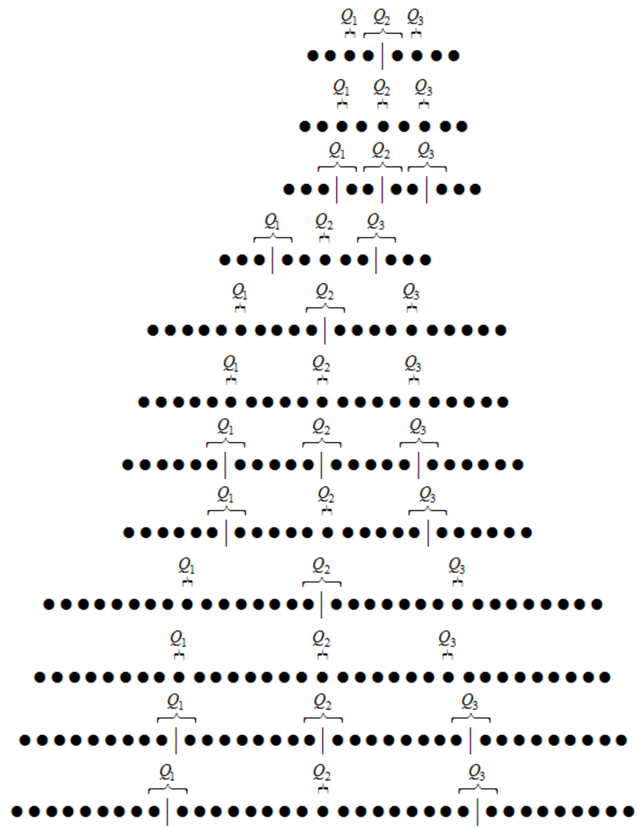
Table 6. Location of Quartiles Based in an Ordered Data Set, While Including the Median

Sample Size	Remainder	Locations of the quartiles in an ordered data set
$n = 8$	0	
$n = 9$	1	
$n = 10$	2	
$n = 11$	3	
$n = 20$	0	
$n = 21$	1	
$n = 22$	2	
$n = 23$	3	
$n = 32$	0	
$n = 33$	1	
$n = 34$	2	
$n = 35$	3	



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Example 1

In the first example, we look at the age (in years) of the 45 United States presidents at inauguration. When 45 is divided by four, it leaves a remainder of one, so the formulas in the second row of Tables 1 and 2 were used to find the quartiles.

The sorted data is below:

42 43 46 46 47 47 48 49 49 50 51 51 51 51 51 52 52 54 54 54 54 54 55 55 55 55 56 56 56 57 57 57 57 58 60 61 61 61 62 64 64 65 68 69 70

Quartile locations while excluding the median

$$Q_1 = \frac{1}{2}(x_{(\frac{n-1}{4})} + x_{(\frac{n-1}{4}+1)}) = \frac{1}{2}(x_{(\frac{45-1}{4})} + x_{(\frac{45-1}{4}+1)}) = \frac{1}{2}(x_{(11)} + x_{(12)}) = \frac{1}{2}(51+51) = 51$$

$$Q_2 = x_{(\frac{n+1}{2})} = x_{(\frac{45+1}{2})} = x_{(23)} = 55$$

$$Q_3 = \frac{1}{2}(x_{(\frac{3n+1}{4})} + x_{(\frac{3n+1}{4}+1)}) = \frac{1}{2}(x_{(\frac{3\cdot 45+1}{4})} + x_{(\frac{3\cdot 45+1}{4}+1)}) = \frac{1}{2}(x_{(34)} + x_{(35)}) = \frac{1}{2}(58+60) = 59$$

$$42\ 43\ 46\ 46\ 47\ 47\ 48\ 49\ 49\ 50\ \overbrace{51\ 51}^{Q_1}\ 51\ 51\ 51\ 52\ 52\ 54\ 54\ 54\ 54\ 54\ \overbrace{55}^{Q_2}\ 55\ 55\ 55\ 55\ 56\ 56\ 56\ 57\ 57\ 57\ 57\ \overbrace{58\ 60}^{Q_3}\ 61\ 61\ 61\ 62\ 64\ 64\ 65\ 68\ 69\ 70$$

Quartile locations while including the median

$$Q_1 = x_{(\frac{n-1}{4}+1)} = x_{(\frac{45-1}{4}+1)} = x_{(12)} = 51$$

$$Q_2 = x_{(\frac{n+1}{2})} = x_{(\frac{45+1}{2})} = x_{(23)} = 55$$

$$Q_3 = x_{(\frac{3n+1}{4})} = x_{(\frac{3\cdot 45+1}{4})} = x_{(34)} = 58$$

$$42\ 43\ 46\ 46\ 47\ 47\ 48\ 49\ 49\ 50\ 51\ \overbrace{51}^{Q_1}\ 51\ 51\ 51\ 52\ 52\ 54\ 54\ 54\ 54\ 54\ \overbrace{55}^{Q_2}\ 55\ 55\ 55\ 55\ 56\ 56\ 56\ 57\ 57\ 57\ 57\ \overbrace{58}^{Q_3}\ 60\ 61\ 61\ 61\ 62\ 64\ 64\ 65\ 68\ 69\ 70$$

Example 2

Next, we collected data on the average goals scored per game during the past 20 FIFA World Cup tournaments. When the sample size of 20 is divided by four, it gives a remainder of zero, so the formulas in the first row of Tables 1 and 2 were utilized to find the quartiles.

The sorted data is below:
2.21 2.27 2.30 2.52 2.54 2.55 2.67 2.67 2.68 2.71 2.78 2.78 2.81 2.97 3.60 3.89 4.00 4.12 4.67 5.38

Quartile locations while excluding the median

$$Q_1 = \frac{1}{2}(x_{(\frac{n}{4})} + x_{(\frac{n}{4}+1)}) = \frac{1}{2}(x_{(\frac{20}{4})} + x_{(\frac{20}{4}+1)}) = \frac{1}{2}(x_{(5)} + x_{(6)}) = \frac{1}{2}(2.54 + 2.55) = 2.545$$

$$Q_2 = \frac{1}{2}(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}) = \frac{1}{2}(x_{(\frac{20}{2})} + x_{(\frac{20}{2}+1)}) = \frac{1}{2}(x_{(10)} + x_{(11)}) = \frac{1}{2}(2.71 + 2.78) = 2.745$$

$$Q_3 = \frac{1}{2}(x_{(\frac{3n}{4})} + x_{(\frac{3n}{4}+1)}) = \frac{1}{2}(x_{(\frac{3\cdot 20}{4})} + x_{(\frac{3\cdot 20}{4}+1)}) = \frac{1}{2}(x_{(15)} + x_{(16)}) = \frac{1}{2}(3.60 + 3.89) = 3.745$$

$$2.21\ 2.27\ 2.30\ 2.52\ \overbrace{2.54\ 2.55}^{Q_1}\ 2.67\ 2.67\ 2.68\ \overbrace{2.71\ 2.78}^{Q_2}\ 2.78\ 2.81\ 2.97\ \overbrace{3.60\ 3.89}^{Q_3}\ 4.00\ 4.12\ 4.67\ 5.38$$

Quartile locations while including the median

$$Q_1 = x_{(\frac{n}{4}+1)} = x_{(\frac{20}{4}+1)} = x_{(6)} = 2.55$$

$$Q_2 = \frac{1}{2}(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}) = \frac{1}{2}(x_{(\frac{20}{2})} + x_{(\frac{20}{2}+1)}) = \frac{1}{2}(x_{(10)} + x_{(11)}) = \frac{1}{2}(2.71 + 2.78) = 2.745$$

$$Q_3 = x_{(\frac{3n}{4})} = x_{(\frac{3\cdot 20}{4})} = x_{(15)} = 3.60$$

$$2.21\ 2.27\ 2.30\ 2.52\ 2.54\ \overbrace{2.55}^{Q_1}\ 2.67\ 2.67\ 2.68\ \overbrace{2.71\ 2.78}^{Q_2}\ 2.78\ 2.81\ 2.97\ \overbrace{3.60}^{Q_3}\ 3.89\ 4.00\ 4.12\ 4.67\ 5.38$$

Example 3

Finally, the winning times (in seconds) of the last 15 Kentucky Derby horse races were analyzed. When the sample size of 15 is divided by four, it leaves a remainder of three, so the formulas in the fourth row of Tables 1 and 2 were utilized to find the quartiles.

The sorted data is here:
121.31 121.36 121.82 121.83 122.04 122.17 122.66 122.75 122.89 123.02 123.59 123.66 124.06 124.20 124.45

<u>Quartile locations while excluding the median</u>

$$Q_1 = x_{(\frac{n-3}{4}+1)} = x_{(\frac{15-3}{4}+1)} = x_{(4)} = 121.83$$

$$Q_2 = x_{(\frac{n+1}{2})} = x_{(\frac{15+1}{2})} = x_{(8)} = 122.75$$

$$Q_3 = x_{(\frac{3n+3}{4})} = x_{(\frac{3\cdot 15+3}{4}+1)} = x_{(12)} = 123.66$$

$$121.31\ 121.36\ 121.82\ \overbrace{121.83}^{Q_1}\ 122.04\ 122.17\ 122.66\ \overbrace{122.75}^{Q_2}\ 122.89\ 123.02\ 123.59\ \overbrace{123.66}^{Q_3}\ 124.06\ 124.20\ 124.45$$

<u>Quartile locations while including the median</u>

$$Q_1 = \frac{1}{2}(x_{(\frac{n+1}{4})} + x_{(\frac{n+1}{4}+1)}) = \frac{1}{2}(x_{(\frac{15+1}{4})} + x_{(\frac{15+1}{4}+1)}) = \frac{1}{2}(x_{(4)} + x_{(5)}) = \frac{1}{2}(121.83 + 122.04) = 121.935$$

$$Q_2 = x_{(\frac{n+1}{2})} = x_{(\frac{15+1}{2})} = x_{(8)} = 122.75$$

$$Q_3 = \frac{1}{2}(x_{(\frac{3n-1}{4})} + x_{(\frac{3n-1}{4}+1)}) = \frac{1}{2}(x_{(\frac{3\cdot 15-1}{4})} + x_{(\frac{3\cdot 15-1}{4}+1)}) = \frac{1}{2}(x_{(11)} + x_{(12)}) = \frac{1}{2}(123.59 + 123.66) = 123.625$$

$$121.31\ 121.36\ 121.82\ \overbrace{121.83\ 122.04}^{Q_1}\ 122.17\ 122.66\ \overbrace{122.75}^{Q_2}\ 122.89\ 123.02\ \overbrace{123.59\ 123.66}^{Q_3}\ 124.06\ 124.20\ 124.45$$