Name: ______________________________ Date: ______________

Investigation 1.1: Formulating a Statistical Question

Assessment

Level A

A third-grader's favorite sport was soccer. She asked all the students in her room, "Who likes to watch a soccer game?" Explain why this is a statistical question.

Name: ______________________ Date: ______________

Investigation 1.1: Formulating a Statistical Question

Assessment

Level B

A group of seventh-grade students asked the question, “What’s the fastest animal in the world?”

1. Explain why this is not a statistical question.

2. Rewrite the question so it is a statistical question.

Name: ______________________ Date: ______________

Investigation 1.1: Formulating a Statistical Question

For each question, indicate whether it is a statistical question. If yes, specify the population, measurement, and expected variation. If not, explain why and rewrite the question so that it is a statistical question.

Table 1.1.1: Level A Questions

Question	Statistical Question (Y or N)	Explain Your Answer	Question	Statistical Question (Y or N)	Explain Your Answer
What colors are the shoes worn by the teachers in our school?			How many languages does my friend speak?		
What are the shapes of all the buttons on the clothes worn by the students in this class?			How far can I jump?		
How many times does the word "bridge" appear in the rhyme "London Bridge Is Falling Down"?			Does my best friend like McDonald's Happy Meals?		
How many pockets do I have?			Is my last name the longest name in class?		
What is my fifth-grade sister's favorite animal at the zoo?			What is the favorite lunch of third-graders in our school?		

Name: ______________________________ Date: ______________

Investigation 1.1: Formulating a Statistical Question

For each question, indicate whether it is a statistical question. If yes, specify the population, measurement, and expected variation. If not, explain why and rewrite the question so that it is a statistical question.

Table 1.1.2: Level B Questions

Question	Statistical Question (Y or No)	Explain Your Answer	Question	Statistical Question (Y or No)	Explain Your Answer
Can I roll my tongue?			Who was the oldest U.S. president when inaugurated?		
How do the lengths of the first names of students in class compare to the lengths of their last names?			Are students in our class who are 4′5″ or taller able to jump higher than students who are shorter than 4′5″?		
Am I going to win a prize at the school carnival?			Which brand of pizza has the most pepperoni?		
What is the longest-lasting brand of AA batteries?			Do plants grow better under colored lights?		
A teacher asks her class, "What is your shoe size?"			Is it easier to remember a set of objects or a list of words?		
Which brand of bubble gum holds its flavor the longest?					

Name: ______________________________ Date: ______________

Investigation 2.1: What Colors Are Our Shoes?

Assessment

1. Suppose the color of shoes for a class of 20 students was as follows. W stands for white, B for black, and G for green.

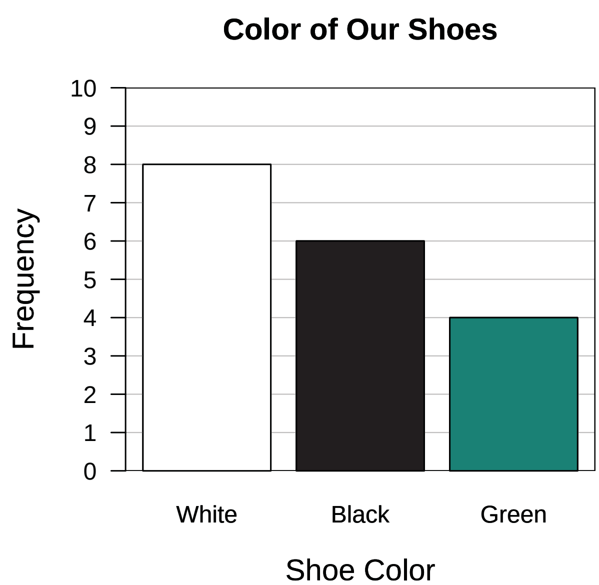
G G B W W W B G W B B B W W W G B G W W

Construct two data displays for these data. Choose one of your data displays and write a letter to the president of a shoe company describing what your chosen display tells you about the color of shoes for that class. Include in your letter to the president why you chose a certain data display.

Example of 'Interpret the Results'

Dear Shoe Company President,

You asked us to tell you something about the shoes we wear. We decided to collect data and analyze the color of our shoes. We counted the number of shoes of each color and represented our findings in the bar chart below. We chose a bar chart because we thought the heights of the bars showed the comparison of the colors the best.



There were only three colors of shoes in our class: black, white, and green. There were eight of us who had white shoes, six who had black shoes, and four who had green. There were more white shoes than black or green, so we would recommend you concentrate on making white shoes. Actually, we want to continue our study and see if our classmates across the hall agree with our distribution of colors. We will let you know.

Also, we want to help you by looking at something other than color, like the type of shoes we wear. Many of us wear an athletic shoe, but there are other types, too.

We hope our data analysis helps you.
Thank you for asking us.

Mrs. Franklin's Class

Investigation 2.1: What Colors Are Our Shoes?

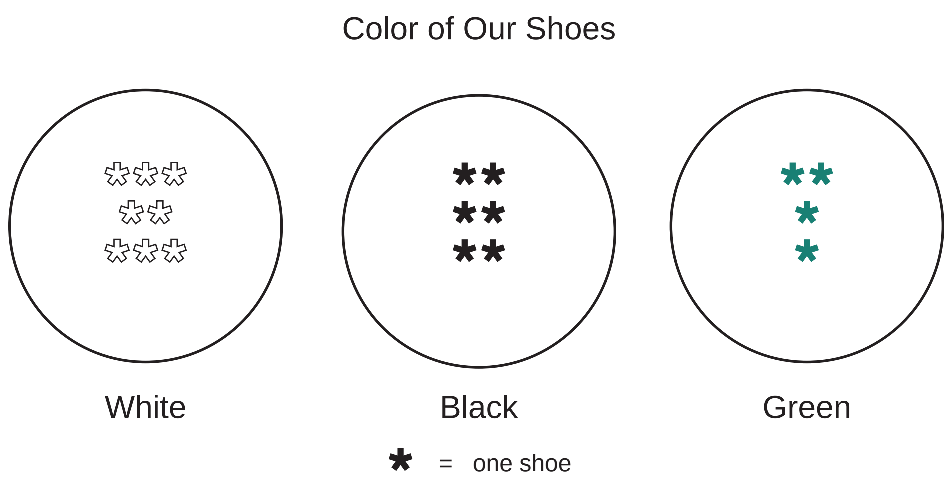


Figure 2.1.1 Venn diagram of children's shoes

Investigation 2.1: What Colors Are Our Shoes?

Figure 2.1.2 Picture graph of children's shoes

Investigation 2.1: What Colors Are Our Shoes?

Table 2.1.1 Tally Chart/Frequency Table of Children's Shoes

Color	Tally	Frequency
White	𝍸 𝍷𝍷𝍷	8
Black	𝍸 𝍷	6
Green	𝍷𝍷𝍷𝍷	4

Investigation 2.1: What Colors Are Our Shoes?

Figure 2.1.3 Bar graph of children's shoes

Name: ______________________________ Date: ______________

Investigation 2.2: What Shapes Are Our Buttons?

Assessment

A group of students recorded the type of buttons they had on their clothes. Table 2.2.3 shows the tallies of the type of buttons.

Table 2.2.3 Tally Chart of Button Shapes

Shape	Tally
Triangle	|||
Round	~~||||~~ ||
Square	~~||||~~ ~~||||~~ |
Other	||

1. Which button shape is the most common?

2. How many more square buttons are there than triangle buttons?

3. Make a bar graph of the different button shapes.

4. Write a report that indicates how the bar graph for this group of students' button shapes differs from your bar graph.

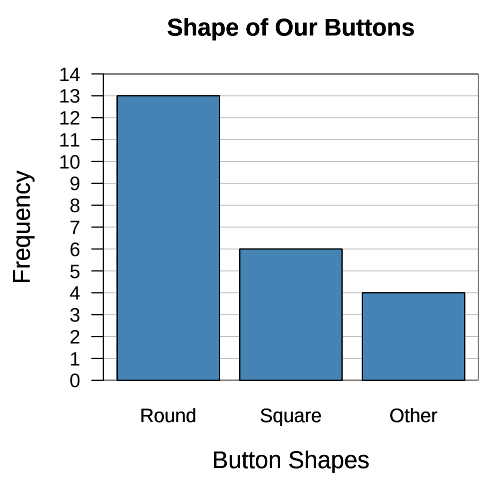
Investigation 2.2: What Shapes Are Our Buttons?

Example of 'Interpret the Results'

Our class read the story "A Lost Button," in which Toad loses a button that can be described by five attributes: size, color, thickness, shape, and number of holes. His button was big, white, thick, round, and had four holes. We decided to ask a statistical question for ourselves, "What shape are our buttons?" It turned out that not all of us had buttons, and some of us had more than one shape. We counted all of them. We put a tally mark for each and then counted them as frequencies in the following table.

Shape	Tally	Count/Frequency
Round	𝍸𝍸|||	13
Square	𝍸|	6
Other	||||	4

Then, to see the results better, we drew the graph below, called a bar graph.



From the bar graph, it is easy to see that there were more round buttons than the other shapes. Round is called the mode shape for our data. Also, we noticed that 13 of the buttons were like the shape of the one Toad lost—round. In addition to our own buttons, we are going to ask our parents and grandparents what shape of buttons they usually wear. Their mode might be different than ours. Doing this activity was a lot of fun. Another analysis we want to do is to look at one of the other attributes, such as number of holes, to see if we match Toad's four holes.

Investigation 2.2: What Shapes Are Our Buttons?

Table 2.2.1 Tally Chart of Button Shapes

Shape	Tally
Round	
Square	
Other	

Investigation 2.2: What Shapes Are Our Buttons?

Table 2.2.2 Count/Frequency Table of Button Shapes

Shape	Tally	Count/Frequency
Round	𝍸𝍸𝍷𝍷𝍷	13
Square	𝍸𝍷	6
Other	𝍷𝍷𝍷𝍷	4

Investigation 2.2: What Shapes Are Our Buttons?

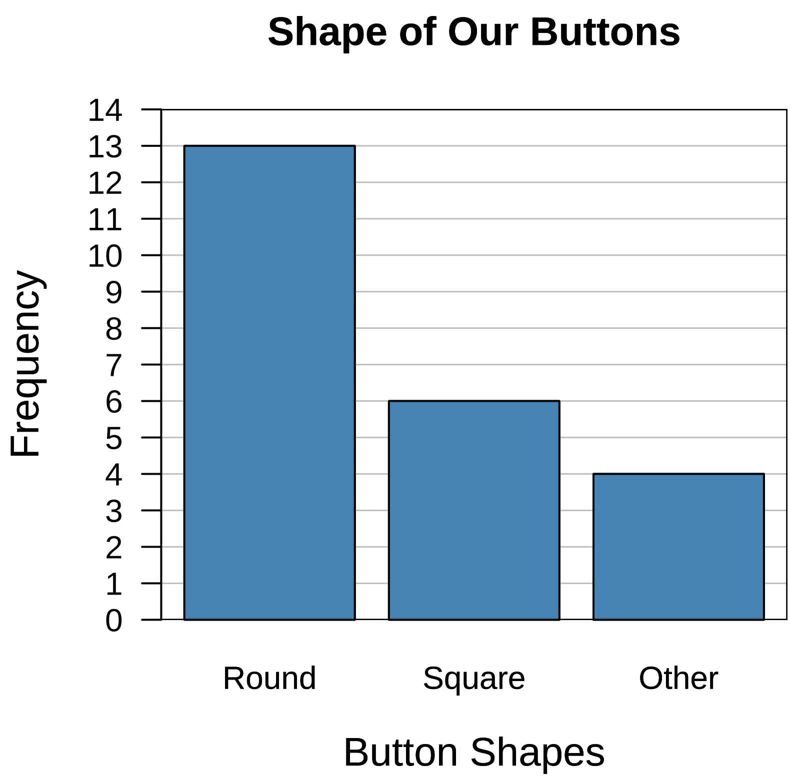


Figure 2.2.1 Bar graph of button shapes

Name: ______________________________ Date: ______________

Investigation 2.3: Is London Bridge Falling Down?

Assessment

1. Use the nursery rhyme "Jack Be Nimble" and complete the tally chart and frequency table.

Jack, be nimble,
Jack, be quick,
Jack, jump over the candlestick.
Jack, be nimble,
Jack, be quick,
Jack, jump over the candlestick!

Word	Tally	Frequency

2. Use the tally chart and frequency table to make a bar graph of the word count for the rhyme "Jack Be Nimble."

Name: ______________________________ Date: ______________

Investigation 2.3: Is London Bridge Falling Down?

3. Use the tally chart and frequency table or the graph to answer the following questions:

   a. How many words are actually in the rhyme? How can you use the table to find the answer?

   b. How many different (distinct) words are in the rhyme? How can you use the table to answer the question?

   c. Which word or words appear most often? How many times?

   d. Which word or words appear least often? How many times?

   e. Which words appear more than three times?

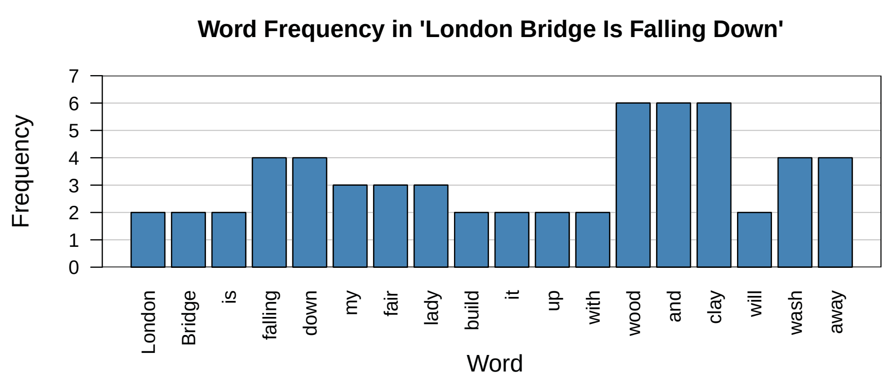
   f. How many more times does the word "Jack" appear than the word "jump"?

Example of 'Interpret the Results'

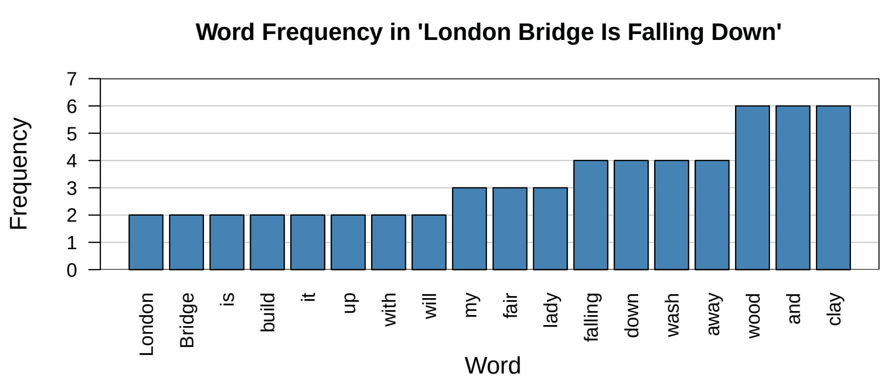
In our history class, we were studying the origins of various literary pieces including nursery rhymes. Some of us wondered what we could do with these rhymes in our mathematics class. Since we have been studying frequency tables and bar graphs there, we thought about doing a statistical analysis of a nursery rhyme. Our teacher suggested "London Bridge Is Falling Down." The statistical question we came up with was "How often do the words in the London Bridge nursery rhyme appear?" We made a tally chart listing all the words and then put a tally beside each word as we sang the rhyme slowly. Then, we counted the number of tallies and made a frequency table as follows:

London	Bridge	is	falling	down	my	fair	lady	build
2	2	2	4	4	3	3	3	2
it	up	with	wood	and	clay	will	wash	away
2	2	2	6	6	6	2	4	4

Sometimes, it's easier to make conclusions by looking at a picture, so we made a bar graph. Our teacher said that was okay as long as we kept the right vertical spacing for the counts. Here it is:



She suggested it might be even easier to discuss our question if we put the data in order.



Investigation 2.3: Is London Bridge Falling Down?

Our teacher was right, because it is clear that the words that occur most often—six times each—are "wood," "and," and "clay." All of them are modes. We also see that the mode words occurred four more times each than did the eight words that only occurred twice each (6 – 2). Now, we are wondering if there are any nursery rhymes that have unique words, since it looks like nursery rhymes like to repeat words. That will be one of our next data analysis studies.

'London Bridge Is Falling Down'

London Bridge is falling down,
Falling down, falling down,
London Bridge is falling down,
My fair Lady.

Build it up with wood and clay,
Wood and clay, wood and clay,
Build it up with wood and clay,
My fair Lady.

Wood and clay will wash away,
Wash away, wash away,
Wood and clay will wash away,
My fair Lady.

Investigation 2.3: Is London Bridge Falling Down?

Table 2.3.1 Tally Chart for Words in 'London Bridge Is Falling Down'

<table>
<tr><th>Word</th><th>Tally</th><th>Count/ Frequency</th><th>Word</th><th>Tally</th><th>Count/ Frequency</th></tr>
<tr><td>London</td><td>||</td><td>2</td><td>It</td><td>||</td><td>2</td></tr>
<tr><td>Bridge</td><td>||</td><td>2</td><td>Up</td><td>||</td><td>2</td></tr>
<tr><td>Is</td><td>||</td><td>2</td><td>With</td><td>||</td><td>2</td></tr>
<tr><td>Falling</td><td>||||</td><td>4</td><td>Wood</td><td>𝍸|</td><td>6</td></tr>
<tr><td>Down</td><td>||||</td><td>4</td><td>And</td><td>𝍸|</td><td>6</td></tr>
<tr><td>My</td><td>|||</td><td>3</td><td>Clay</td><td>𝍸|</td><td>6</td></tr>
<tr><td>Fair</td><td>|||</td><td>3</td><td>Will</td><td>||</td><td>2</td></tr>
<tr><td>Lady</td><td>|||</td><td>3</td><td>Wash</td><td>||||</td><td>4</td></tr>
<tr><td>Build</td><td>||</td><td>2</td><td>Away</td><td>||||</td><td>4</td></tr>
</table>

Name: ______________________________ Date: ______________

Investigation 2.3: Is London Bridge Falling Down?

Word	Tally	Count/ Frequency	Word	Tally	Count/ Frequency
London			It		
Bridge			Up		
Is			With		
Falling			Wood		
Down			And		
My			Clay		
Fair			Will		
Lady			Wash		
Build			Away		

Investigation 2.3: Is London Bridge Falling Down?

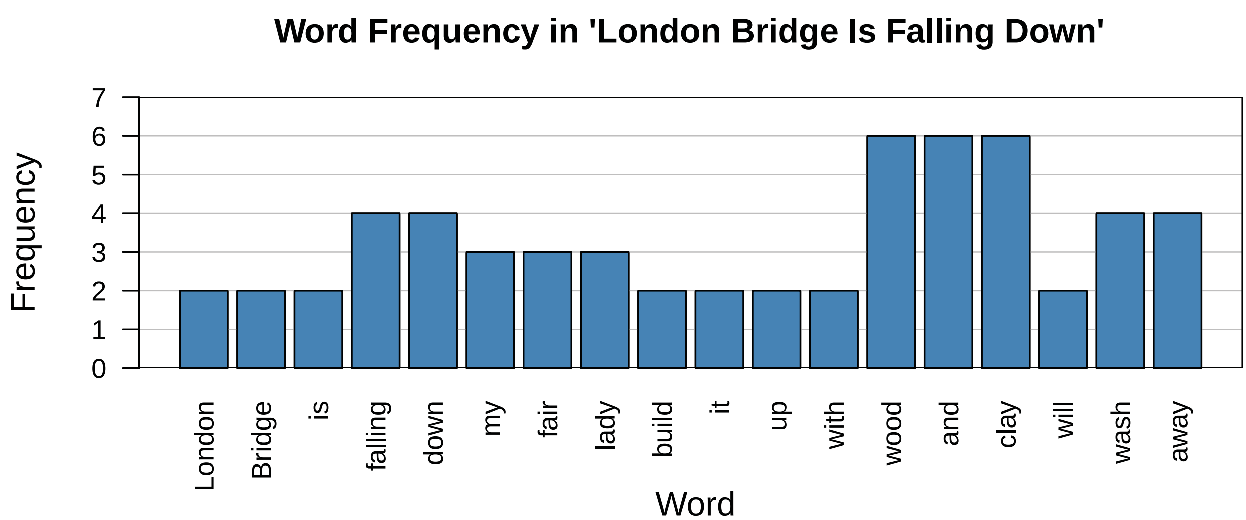


Figure 2.3.1 Bar graph of the word frequency

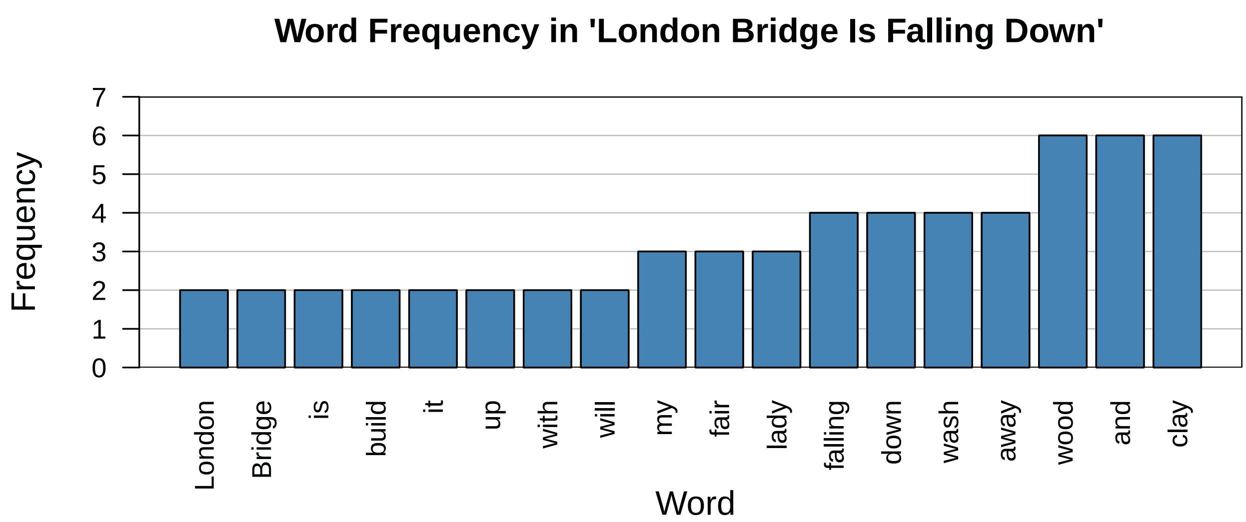


Figure 2.3.2 Bar graph of the word frequency ordered

London Bridge Rap

London Bridge is falling down,
Whatcha gonna do when you go to town?
I say, London Bridge is falling down.
Hold on there, pretty lady.

Gonna build the bridge up with bricks and clay
Gotta get across, can't take all day!
Build up that bridge with bricks and clay.
Wait right there, pretty lady.
Dangerous to cross right now,
Can't 'llow no one to be goin' down.
Take the key, can't cross right now,
Chill out now, pretty lady.

Name: ______________________________ Date: ______________

Investigation 2.4: How Can We Sort Our Junk?

Data Collection Sheet

Object (Individuals)				
Attribute (Variable)				
Possible Values				

Name: ______________________________ Date: ______________

Investigation 2.4: How Can We Sort Our Junk?

Assessment

Give each of your students the following pattern blocks: 6 Yellow Hexagons, 4 Green Triangles, 3 Red Trapezoids, 2 Blue Parallelograms, and 2 Tan Parallelograms. Instruct your students to sort the shapes and complete the following questions:

1. How did you decide to sort the shapes?

2. Draw a pictograph of how you sorted the shapes.

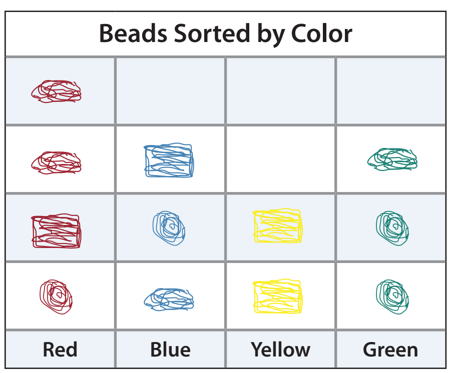
3. Which category had the most shapes?

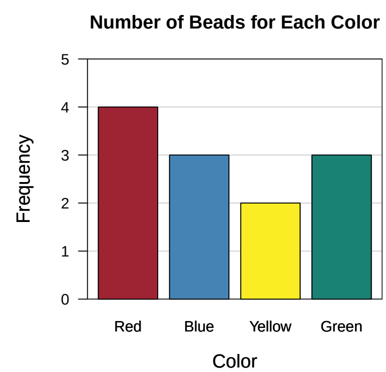
4. Which category had the fewest shapes?

Example of 'Interpret the Results'

Our teacher asked us if we collected junk. Most of us said we do. He then asked if we had any sort of preference for one characteristic of our junk over another, like color, size, or design. We never really thought about it in that way, so he brought in a bag of "junk" he had collected from doing class activities over a long time. Our problem was to investigate the question, "How can we sort his bag of junk?" We decided to just look at the 12 beads in the bag and sorted the actual beads on two pieces of grid paper, one with regard to color and the other with regard to shape.

We chose color just because we like color, but we chose shape because we have been looking at different shapes in our geometry class. Then, we drew a pictograph to show graphs of how we sorted the beads. We used graph paper to keep our rows the same so the same number of beads—regardless of their shape—took up the same amount of vertical space. Another group in class drew a bar graph. They didn't have to make sure the same number of beads for different colors took up the same amount of space because the rectangles do that automatically. So maybe it's easier to draw a bar graph, but we liked the pictographs because we can see the beads.

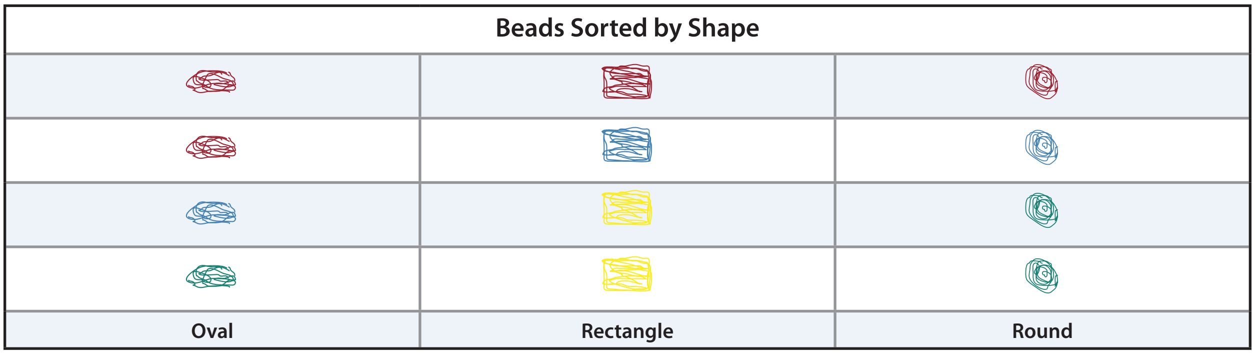




We can see from either graph that the mode color is red, but if we wanted to compare red to non-red, then the mode would be non-red, since there would be eight of them compared to the four red.

Investigation 2.4: How Can We Sort Our Junk?

When we sorted by shape, the graphs looked different from the color graphs. The shape graphs only have three columns because there are three shapes: oval, rectangle, and round. Also, the heights were all equal because there were four beads in each shape category. That means there was the same number of each shape.



Investigation 2.4: How Can We Sort Our Junk?

Object (Individuals)	Beads			
Attribute (Variable)	Color			
Possible Values	Red	Blue	Yellow	Green

Figure 2.4.1 Data collection sheet for attribute of color

Investigation 2.4: How Can We Sort Our Junk?

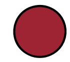
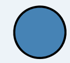
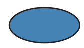
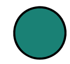
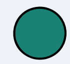
Object (Individuals)	Beads		
Attribute (Variable)	Shape		
			
			
			
			
Possible Values	Oval	Rectangle	Round

Figure 2.4.2 Data collection sheet for attribute of shape

Investigation 2.4: How Can We Sort Our Junk?

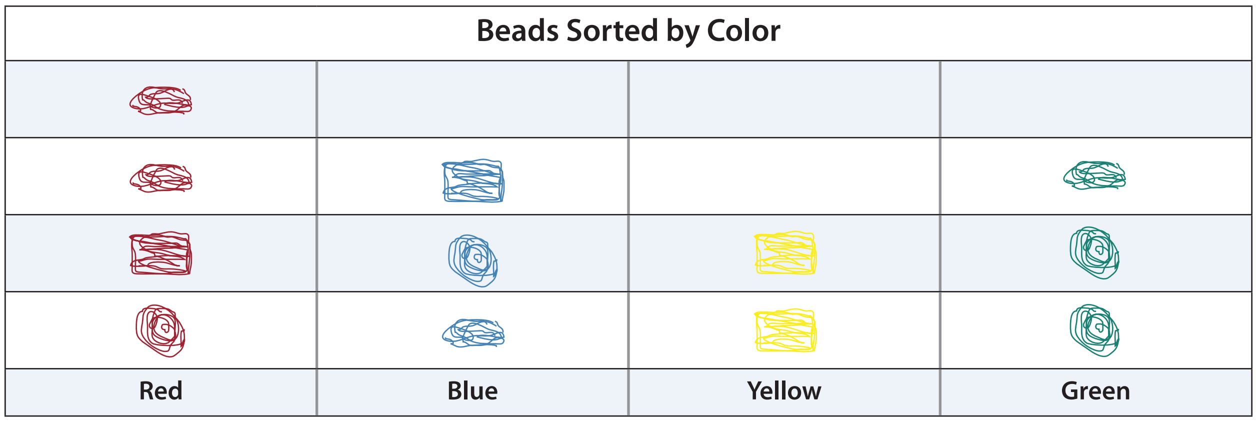


Figure 2.4.3 Pictograph of the beads sorted by color

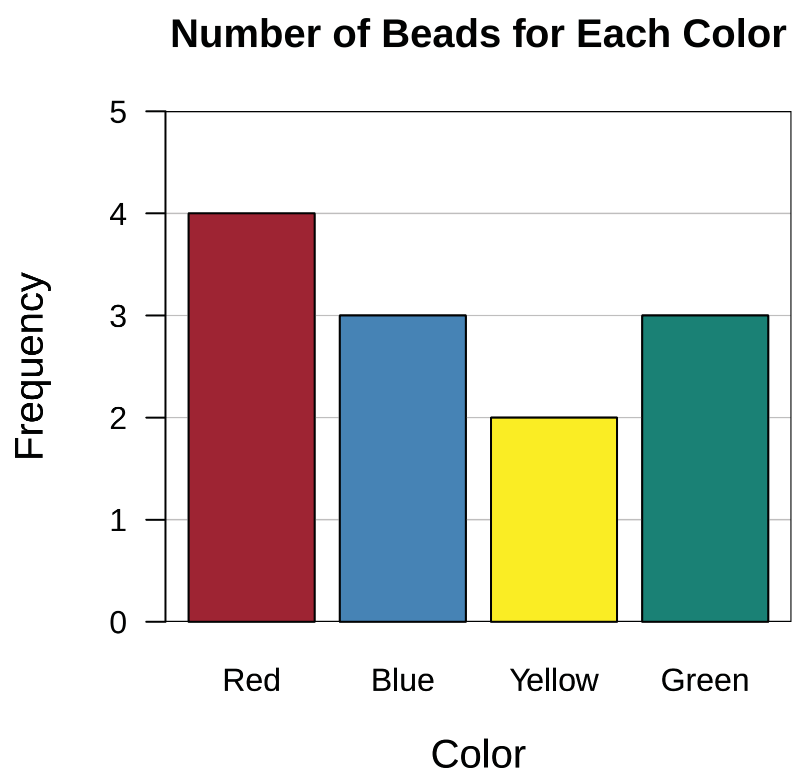


Figure 2.4.4 Bar graph of the beads sorted by color

Name: ______________________________ Date: ______________

Investigation 3.1: How Many Pockets?

Assessment

A group of students counted the number of pockets in their clothes and drew the two graphs shown below.

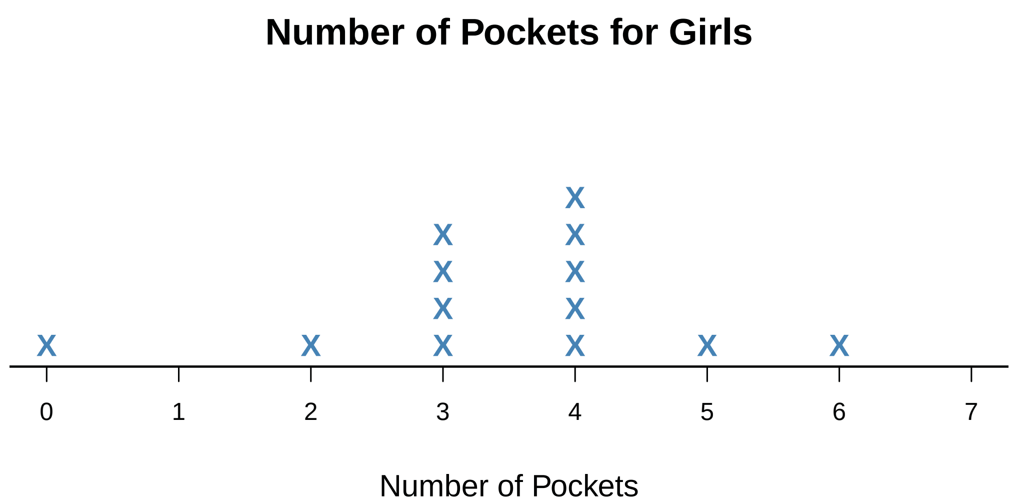


Figure 3.1.4 Dotplot of number of pockets for girls

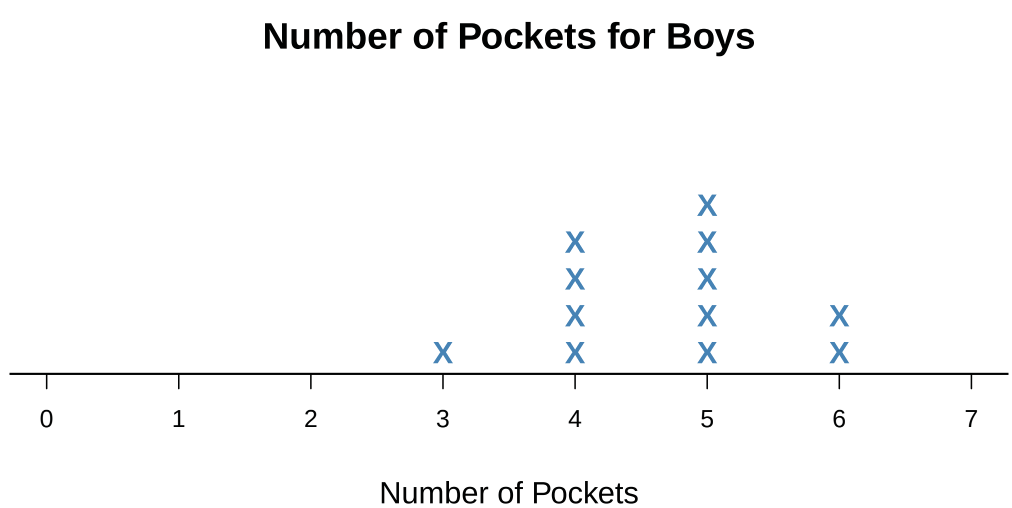


Figure 3.1.5 Dotplot of number of pockets for boys

Name: ______________________ Date: ______________

Investigation 3.1: How Many Pockets?

1. What is the minimum number of pockets for the girls?

2. What is the maximum number of pockets for the boys?

3. What is the mode number of pockets for the girls?

4. How many more girls had four pockets than boys who had four pockets?

5. Who has more pockets, boys or girls? Use words, numbers, and graphs to explain your answer.

Example of 'Interpret the Results'

After we listened to the story *A Pocket for Corduroy*, we became curious about the number of pockets we have on our clothes and whether girls or boys have more. So, on a green sticky note, each boy in our class wrote the number of pockets he had and the girls did the same thing, except their sticky notes were yellow. Then, we put them all in a dotplot. When we tried to answer our statistical question about whether girls or boys had more pockets, it was a little hard to do with all the green and yellow sticky notes together. So, to make it easier to answer our question, we drew two dotplots with the same scale, one for the boys and one for the girls.

By comparing the two dotplots, we concluded that, overall, boys have more pockets. We had to be careful because the number of boys and the number of girls was not the same. There were 12 boys and 13 girls. Still, 11 out of the 12 boys had four or more pockets; that's way over half of the boys. But 7 of the 13 girls, about half of them, had four or more pockets, so we concluded from our analysis that boys have more pockets than girls. We're going to continue this activity by asking our parents and grandparents how many pockets they usually have on their everyday clothes.

Investigation 3.1: How Many Pockets?

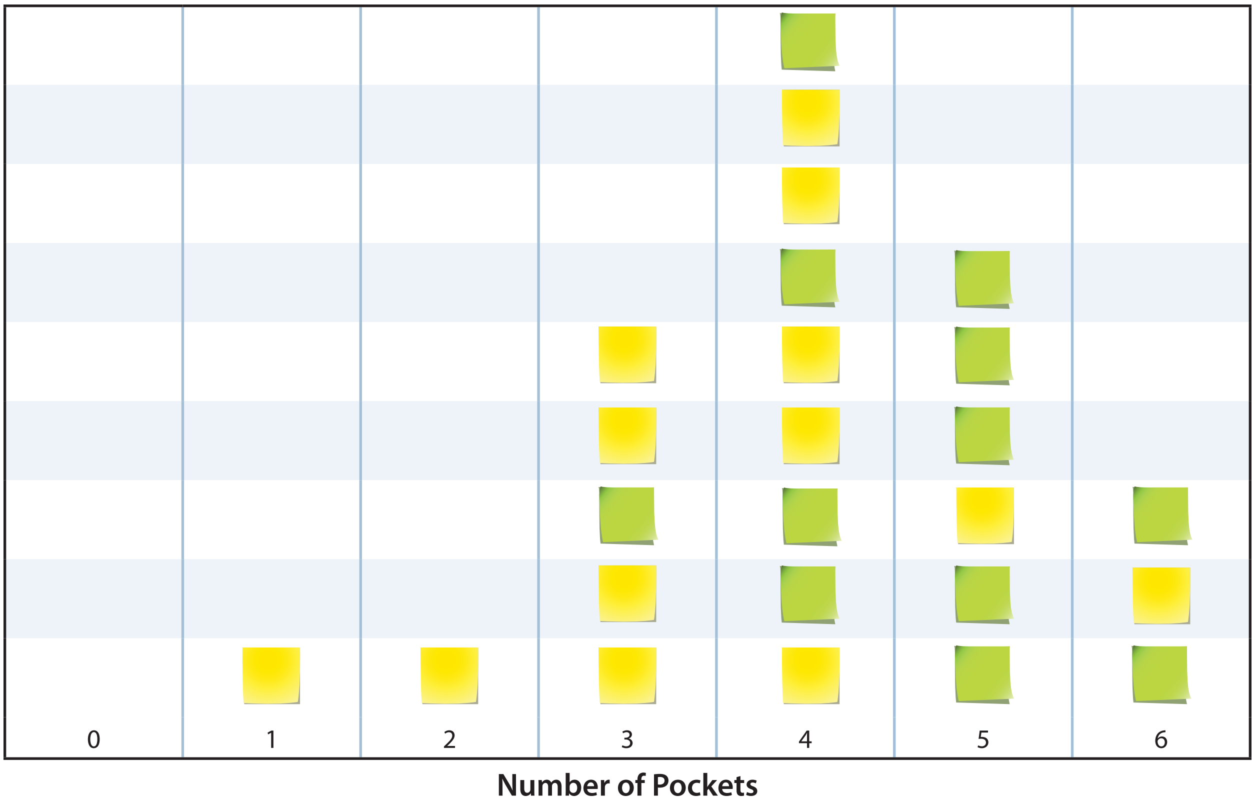


Figure 3.1.1 Dotplot of number of pockets

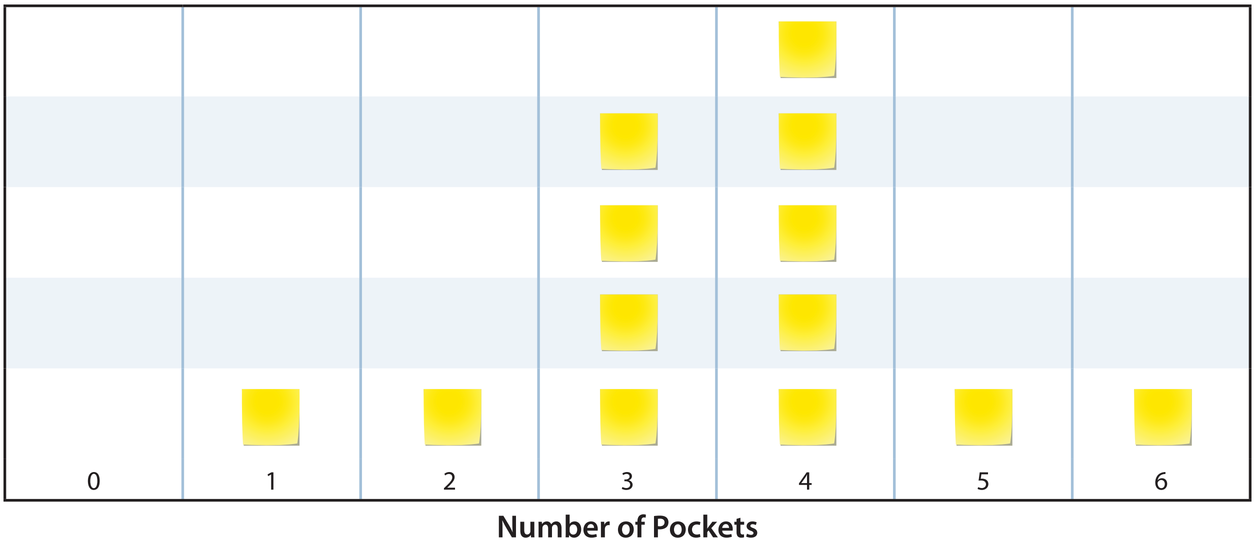


Figure 3.1.2 Dotplot of number of pockets for girls

Figure 3.1.3 Dotplot of number of pockets for boys

Name: ______________________________ Date: ______________

Investigation 3.2: Who Has the Longest First Name?

Assessment

With the help of his family and friends, Jose collected data regarding the lengths of first names of his family and friends. Table 3.2.1 shows the data Jose collected.

Table 3.2.1 Length of First Name

	Family and Friends First Names	Number of Letters in First Name
1	Hector	6
2	Amada	5
3	Che	3
4	Ricardo	6
5	Camila	6
6	Roberto	7
7	Carlos	6
8	Raymundo	8
9	Gabriela	8
10	Diego	5
11	Tia	3

1. Make a dotplot of the length of the first names of Jose's family and friends.

Name: ______________________________ Date: ______________

Investigation 3.2: Who Has the Longest First Name?

2. Find the value of each of the following:

   Maximum value:

   Minimum value:

   Mode:

   Median:

   Range:

3. Write a summary of what you observed about the length of the first names of Jose's family and friends. Your summary should include reference to the dotplot and the measures of center and spread that you found.

Example of 'Interpret the Results'

On the first day of school, our teacher had us play games to learn each other's names. After that, she was showing us some statistics by having us study the lengths of our first names. The question was, "How do the lengths of first names vary in our class?"
We used sticky notes with our names and the number of letters in our names written on them. After putting all of the sticky notes on the board all messed up, we organized them into a dotplot with the number of letters on the horizontal axis. But, we didn't do the graph the right way the first time, because we didn't keep the columns nice and straight and in line with the other columns. We had to remember to keep the rows in line, also. When we corrected that, we saw that there were more of us whose first names had six letters than any other number. It was the highest in the dotplot. That's called the mode number of letters.

We also calculated the middle number of letters by lining up from fewest number of letters to most and then having low and high sit down until we got to one person left. That number is called the median. It's the middle number of letters, 6 (Alicia), with 12 of us below Alicia and 12 of us above Alicia.

The day after we did that analysis, we got a new student in class, Seraphinia. When we added her, she had the longest name. The range of letters was 9 – 3 = 6 before Seraphinia, but 10 – 3 = 7 letters with her. The mode stayed at 6 because it was still the highest. To find the median, we sat down like before, but now there were two middles, Alicia and Connor. They both have 6 letters in their names, so the median is still 6.

We want to continue doing this study by looking at names from different countries to see if their number of letters differs from ours. We think that maybe Chinese names are shorter.

Investigation 3.2: Who Has the Longest First Name?

Figure 3.2.1 Example of sticky notes with names and lengths

Investigation 3.2: Who Has the Longest First Name?

Figure 3.2.2 Example of sticky notes organized by number of letters

Investigation 3.2: Who Has the Longest First Name?

Figure 3.2.3 Dotplot of length of first names

Name: ______________________________ Date: ______________

Investigation 3.3: How Expensive Is Your Name?

Assessment

1. Jose collected first names from his family and friends. Table 3.3.1 shows the first names and their lengths. Determine the amount of money it would cost for each family member to have their first name sewn on a T-shirt with a cost of 4 cents per letter and complete the last column in Table 3.3.1.

Table 3.3.1 Cost to Sew on First Name

	Family and Friends First Name	Number of Letters in First Name	Cost to Sew on First Name
1	Hector	6	
2	Amada	5	
3	Che	3	
4	Ricardo	6	
5	Camila	6	
6	Roberto	7	
7	Carlos	6	
8	Raymundo	8	
9	Gabriela	8	
10	Diego	5	
11	Tia	3	

2. Find the mean, median, and mode of the number of letters.

	Number of Letters	Cost of Letters
Mean		
Median		
Mode		

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Investigation 3.3: How Expensive Is Your Name?

3. Use words, numbers, and graphs to explain how the mean, median, and mode of the number of letters compare to the mean, median, and mode of the cost of the letters. A dotplot of each data set should be included with the location of the mean, median, and mode labeled on the graph.

Investigation 3.3: How Expensive Is Your Name?

Example of 'Interpret the Results'

In a previous activity, we analyzed the number of letters in first names by drawing a dotplot that looked like this for our class.

Our student council voted to have T-shirts made with our first names on them and asked our class to do a statistical study on the cost. So, we made a statistical question of "How expensive is it to sew first names onto T-shirts?" We asked our domestic arts teacher what it would cost to do the sewing. She said most embroidery businesses would charge around 5 cents per letter. With that estimate, we first determined the data set for the cost of sewing first names based on our data set for the number

Investigation 3.3: How Expensive Is Your Name?

of letters in our first names. For example, Janice has six letters in her name, so her cost would be five times six, or 30 cents. We did that for all 25 students in our class and produced the following dotplot:

We decided to compare our two dotplots. The first thing is that the scales are different. The number of letters scale is 3, 4, 5, 6, 7, 8, 9. The cost of sewing scale is 15, 20, 25, 30, 35, 40, 45. It is five times the letters scale. This makes sense since each letter costs 5 cents.

From our last study, we found the mode (most often) number of letters was 6 and the median (middle) number of letters was also 6. So it makes sense that the mode and median for the cost data set should be five times as much (i.e., 30 cents). Our teacher had us learn another measure of center called the mean. It is a fair share, or equal number, for everyone.

Investigation 3.3: How Expensive Is Your Name?

What we did was work in groups of four, and working with cubes that represented the number of letters in our names, put them all together in a pile and then handed them out to each other. If we had done that for our whole class according to our dotplot, all the letters in our first names would have totaled 148. Handing them out to all 25 of us gave each of us 5 cubes with 23 cubes left over. To hand out the 23 evenly, each of us would get 23/25 of a cube, so the mean (fair share) center is 5 23/25 letters for the whole data set.

After we found that the mean number of letters was 5 23/25, we found the mean cost of the letters by multiplying 5 23/25 by 5 (the cost of each letter). The answer was a mean cost of 29 3/5 cents, which was the cost each one of us would have if we all had the same cost.

The last thing we did was to compare the shapes of the two dotplots. We saw that they were the same except the cost one was more spread out. Just like we find measures of center (mode, median, mean), our teacher said we also will learn how to measure how spread out a data set is. We can't wait to find out.

Investigation 3.3: How Expensive Is Your Name?

Figure 3.3.1 Dotplot of length of first names

Investigation 3.3: How Expensive Is Your Name?

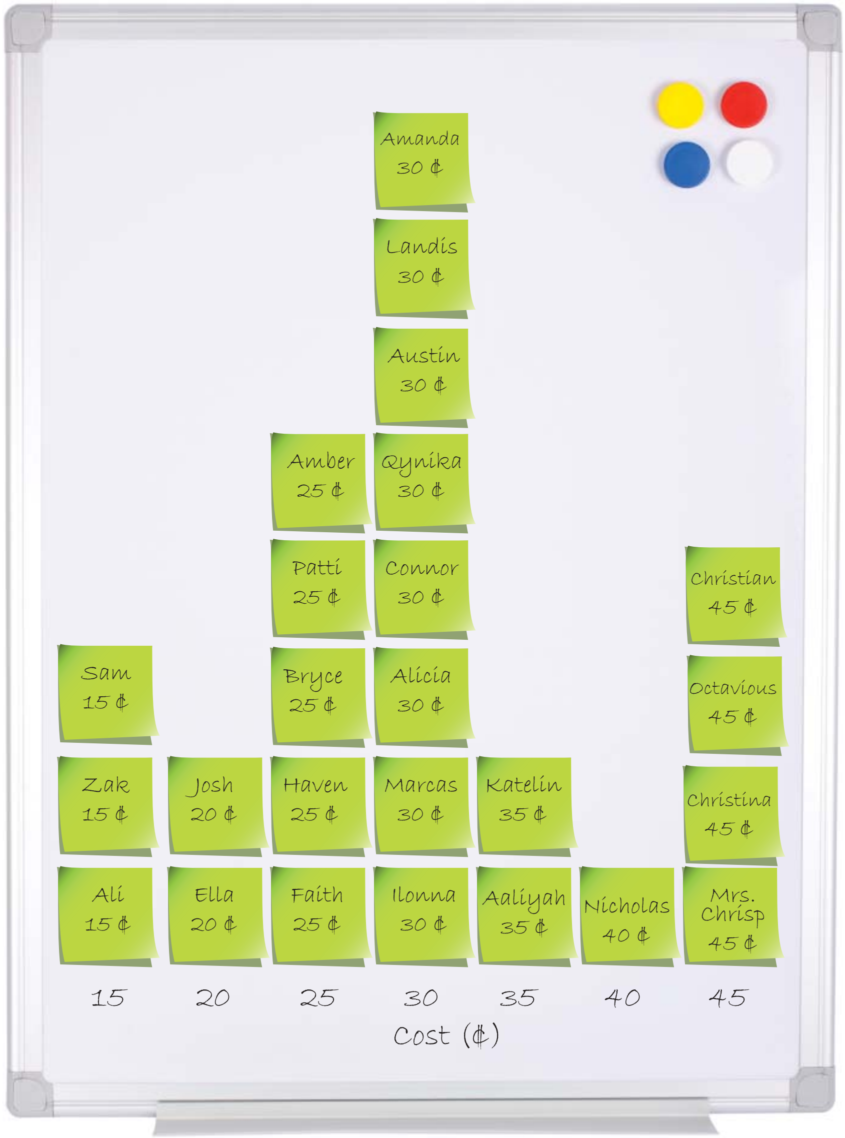

Figure 3.3.3 Dot plot of the cost of sewing on letters

Name: ______________________ Date: ______________

Investigation 3.4: How Long Are Our Shoes?

Assessment

Chris, a seventh-grader, collected the shoe length of a group of 10 students from the eighth grade to investigate the question regarding the shoe length of eighth graders. The data are shown in the following frequency table:

Shoe Length (cm)	16	17	18	19	20
Frequency	1	2	4	2	1

1. Draw a dotplot of the data and describe the shape of the distribution.

2. Draw a boxplot of the data and describe the spread of the distribution.

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Investigation 3.4: How Long Are Our Shoes?

3. Find the mean of the data.

4. Is the mean a useful measure of center?

5. Explain how to interpret the mean in this context.

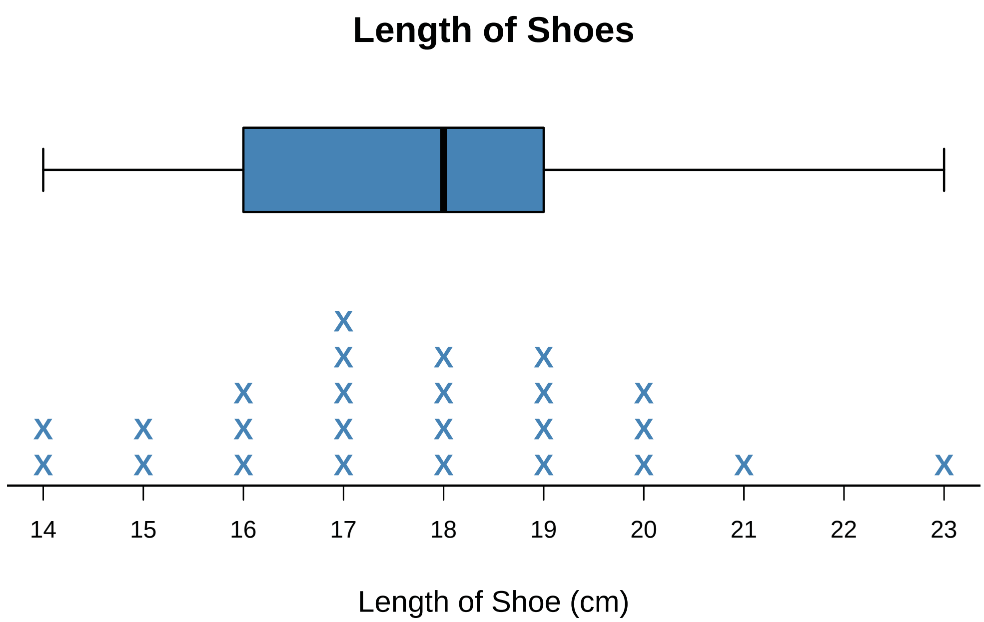
6. Find the mean absolute deviation of the shoe lengths and interpret it in the context of this problem.

Shoe Length	Deviations from the Mean	Absolute Deviations
16		
17		
17		
18		
18		
18		
18		
19		
19		
20		

Investigation 3.4: How Long Are Our Shoes?

Example of 'Interpret the Results'

We wanted to know about the lengths of our shoes. From collecting our shoe lengths in a frequency table, we drew the following dotplot and boxplot.



It was neat to put the two graphs together to see what one of them showed that the other didn't. For example, the dotplot showed a possible outlier at 23 because of a gap between 21 and 23, but when we did the IQR calculation to detect outliers in a boxplot, 23 was not an outlier.

The dotplot shows the shape of the data to be fairly symmetrical. It's a little hard to see that in the boxplot because the distance from the median to Q3 is short and the distance from Q3 to the max is long. That means 25% of us, or about 6 of us, were close together between the median of 18 and Q3 = 19, but 25% of us, or about 6 of us, were spread out between Q3 = 19 and the max of 23. That's not too symmetric.

Regarding how spread out our shoe lengths are, we calculated three measures of spread. The first is the range, which is the overall distance from the minimum to the maximum, 23 - 14 = 9 cm. The second is the range of the middle 50% of the data. It is called the interquartile range. Its value is the distance from the first quartile to the third quartile, which is 19 - 16 = 3 cm. So, half our shoe lengths occupy an interval of length 3 cm. That's pretty closely packed. The third measure of spread is based on the distance each point is from the mean. This distance in statistics is called a deviation. We discovered that if you calculate all the deviations above the mean, they will be positive and the shoe lengths below the mean will be negative. It was kind of neat to see that when we added them all together, the answer was 0. We now have two meanings for the mean: It's the value everyone would have if everyone were to have the same value and a balance point of the data set put on a line.

Investigation 3.4: How Long Are Our Shoes?

To find another measure of spread, we took the mean of the absolute values of the deviations and called it MAD—that stands for the mean absolute deviation. It was 1.73 cm for our shoe lengths, which means, on average, all our shoe lengths are 1.73 cm away from the mean shoe length of 17.72 cm.

Our teacher told us that when we get to high school, we will learn another really cool measure of spread that is important and we will be able to understand it because of our working with MAD.

Investigation 3.4: How Long Are Our Shoes?

Table 3.4.1 Sample Set of Shoe Length Data (cm)

20	17	17	19	20	17	19	14	17	20	15	19	
18	15	21	17	14	16	23	16	18	19	16	18	18

Investigation 3.4: How Long Are Our Shoes?

Table 3.4.2 Frequency Table of Sample Sixth-Grade Class Data

Length	14	15	16	17	18	19	20	21	22	23
Tally	$\vert\vert$	$\vert\vert$	$\vert\vert\vert$	$\cancel{\vert\vert\vert\vert}$	$\vert\vert\vert\vert$	$\vert\vert\vert\vert$	$\vert\vert\vert$	$\vert$		$\vert$
Frequency	2	2	3	5	4	4	3	1	0	1

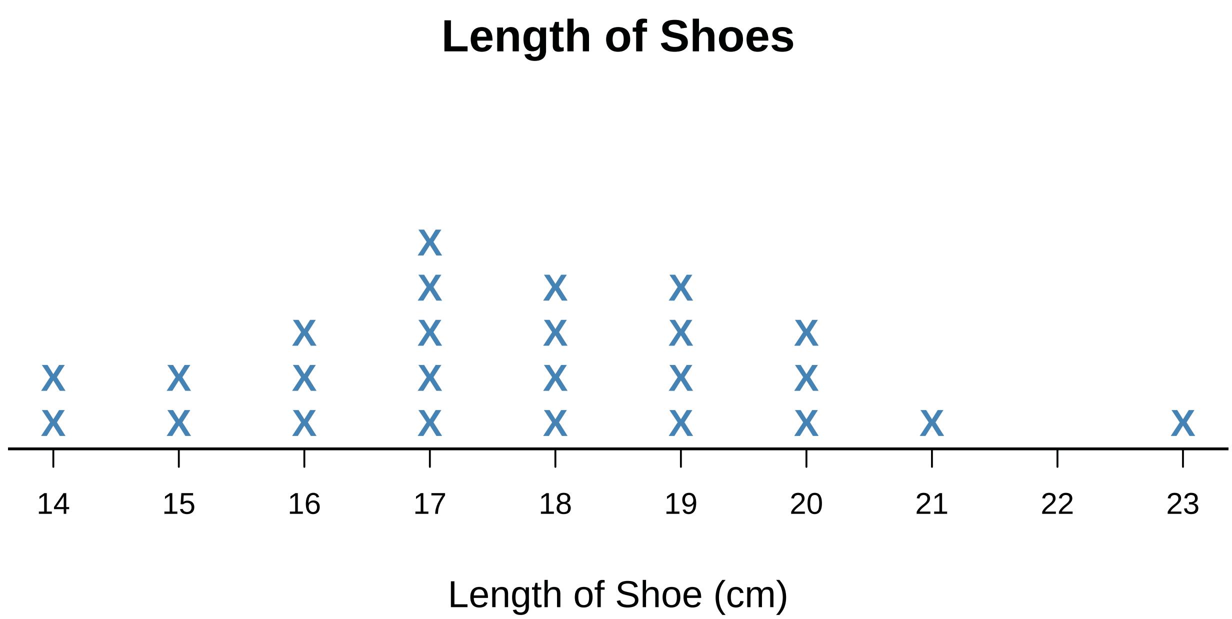


Figure 3.4.1 Dotplot of sample sixth-grade class data

Table 3.4.3 Five-Number Summary of Sample Sixth-Grade Class Data

Min	Q1	Median	Q3	Max
14	16	18	19	23

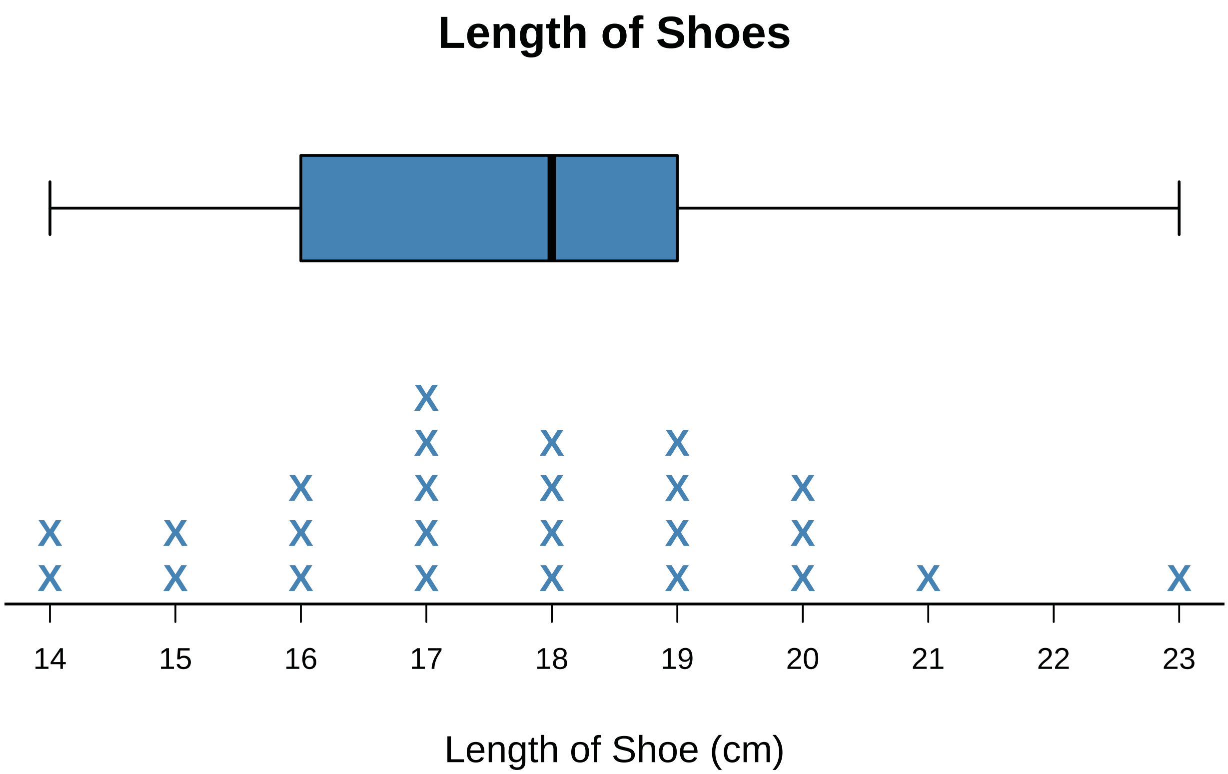


Figure 3.4.2 Boxplot and dotplot of the sample sixth-grade class data

Table 3.4.4 Table of Calculations to Find the Mean Absolute Deviation

Shoe Length	Length - Mean	|Length - Mean|	Shoe Length	Length - Mean	|Length - Mean|
20	20 - 17.72 = 2.28	2.28	18	18 - 17.72 = 0.28	0.28
17	17 - 17.72 = -0.72	0.72	15	15 - 17.72 = -2.72	2.72
17	17 - 17.72 = -0.72	0.72	21	21 - 17.72 = 3.28	3.28
19	19 - 17.72 = 1.28	1.28	17	17 - 17.72 = -0.72	0.72
18	18 - 17.72 = 0.28	0.28	14	14 - 17.72 = -3.72	3.72
20	20 - 17.72 = 2.28	2.28	16	16 - 17.72 = -1.72	1.72
17	17 - 17.72 = -0.72	0.72	23	23 - 17.72 = 5.28	5.28
19	19 - 17.72 = 1.28	1.28	16	16 - 17.72 = -1.72	1.72
14	14 - 17.72 = -3.72	3.72	18	18 - 17.72 = 0.28	0.28
17	17 - 17.72 = -0.72	0.72	19	19 - 17.72 = 1.28	1.28
20	20 - 17.72 = 2.28	2.28	16	16 - 17.72 = -1.72	1.72
15	15 - 17.72 = -2.72	2.72	18	18 - 17.72 = 0.28	0.28
19	19 - 17.72 = 1.28	1.28	Sum	0	43.28

Name: ______________________________ Date: ______________

Investigation 4.1: How Far Can You Jump?

Recording Sheet

Student Number	Group 1 - No Targeted Jump (cm)	Group 2 - Targeted Jump (cm)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
Summary Measures		
Mean		
Median		
Minimum		
Maximum		
Q1		
Q3		

Name: ____________________ Date: ____________

Investigation 4.1: How Far Can You Jump?

Assessment

A group of students conducted an experiment to compare the effect of where the target line is placed for the standing long jump. Target lines were placed at 100 cm and 300 cm. Table 4.1.4 shows the length of the jumps in cm for each group.

Table 4.1.4 Jump Lengths (cm) for Groups with Target of 100 cm and 300 cm

100 cm Target	149	141	161	114	116	142	129	149	138	158	145
300 cm Target	168	185	194	167	147	151	169	178	167	166	139

1. Does the distance a target line is from the start line affect the distance students jump in the standing long jump?

2. Use words, numbers, and graphs to justify your answer by using at least one graph, a measure of center, and a measure of spread.

Summary

	100 cm target	300 cm target
Mean		
Minimum		
Q1		
Median		
Q3		
Maximum		
IQR		

Investigation 4.1: How Far Can You Jump?

Example of 'Interpret the Results'

We conducted a comparative experiment in which some students did a standing long jump with no target in front of them and others did a standing long jump with a target 200 cm in front of them to answer the statistical question, "Will students jump farther if they are given a fixed target in front of them?" (Our gym teacher suggested 200 cm would be a good target for 12-year-olds.)

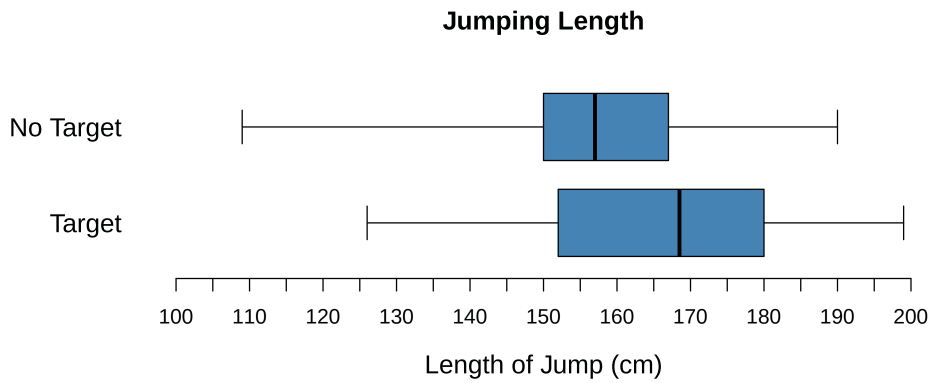
To determine which of us would be in the No Target group and which would be the Target group, we put our names in a hat. The first name randomly drawn from the hat was assigned to the No Target group. The second name drawn was assigned to the Target group. We went back and forth like that until everyone had been assigned to a group.

We measured our distances in centimeters from the starting line to where the closer heel of our shoes landed to the start line. (Everyone landed on their feet.) We tried to make sure everyone did the jump the same way to avoid introducing any sort of bias, like measurement bias, into our results. We drew two comparative graphs of our data.

Jumping Length

No Target		Target
9	10	
	11	
8	12	6
7	13	9
6	14	7
7 7 6 5 4	15	2 4 8
7 7 2	16	7
1	17	0 1 9
1	18	0 1 5
0	19	9

Key: 16|7 represents 167 cm



From the stemplot—except for one possible outlier (109) in the No Target group, it looked like the data sets were spread about the same. But the IQR for the No Target group is 21 and a larger 28 for the Target group, so the middle 50% of the No Target group data is more compact than for the Target group.

Actually, it's better for a data set to have a small variation because it makes us more confident about the centering value. We thought the target group should be more compact because those jumpers had something to concentrate on, but it didn't turn out that way. Regarding the 109, it is an outlier looking at the gap in the stemplot, and it is also an outlier using the Q1 - 1.5*IQR rule for the box-plot. Any value below 146 – 1.5 * (167 – 146) = 114.5 is considered an outlier.

So, did those in the Target group jump farther than the No Target group? From the stemplots, the Target group is shifted to the right compared to the No Target group. Because the No Target group has an outlier, we decided to compare the two groups with medians, rather than means. Based on medians, the answer would be yes, since the median for the Target group was 168.5 cm compared to the median for the No Target group of 157 cm. The Target group jumped a full 11.5 cm longer. In fact, half (seven students) of the Target group jumped farther than 168 cm, but only 3 of the 15 No Target group (20%) jumped that far. Having a target produces higher standing long jump distances. We were wondering if the same conclusion would be made for other age groups. Our guess is that no matter what age groups do this experiment, the results will be similar, since it seems better to have a target as a goal to achieve.

Investigation 4.1: How Far Can You Jump?

Table 4.1.1 An Example of Data Collected from a Group of 12-Year-Olds

Length in Centimeters for No Target Group														
146	190	109	181	155	167	154	171	157	156	128	157	167	162	137
Length in Centimeters for Target Group														
199	167	147	180	185	170	171	139	154	126	179	158	181	152	

Jumping Length

No Target		Target
9	10	
	11	
8	12	6
7	13	9
6	14	7
7 6 7 4 5	15	4 8 2
2 7 7	16	7
1	17	0 1 9
1	18	0 5 1
0	19	9

Key: 16|7 represents 167 cm

Figure 4.1.1 Back-to-back stemplot comparing length of jumps for No Target group and Target group

Jumping Length

No Target		Target
9	10	
	11	
8	12	6
7	13	9
6	14	7
7 7 6 5 4	15	2 4 8
7 7 2	16	7
1	17	0 1 9
1	18	0 1 5
0	19	9

Key: 16|7 represents 167 cm

Figure 4.1.2 Back-to-back stemplot comparing length of jumps for No Target group and Target group with the digits in order

Table 4.1.2 Five-Number Summary for Target and No Target Group

	Min	Max	Median	Q1	Q3
No Target Group	109	190	157	146	167
Target Group	126	199	168.5	152	180

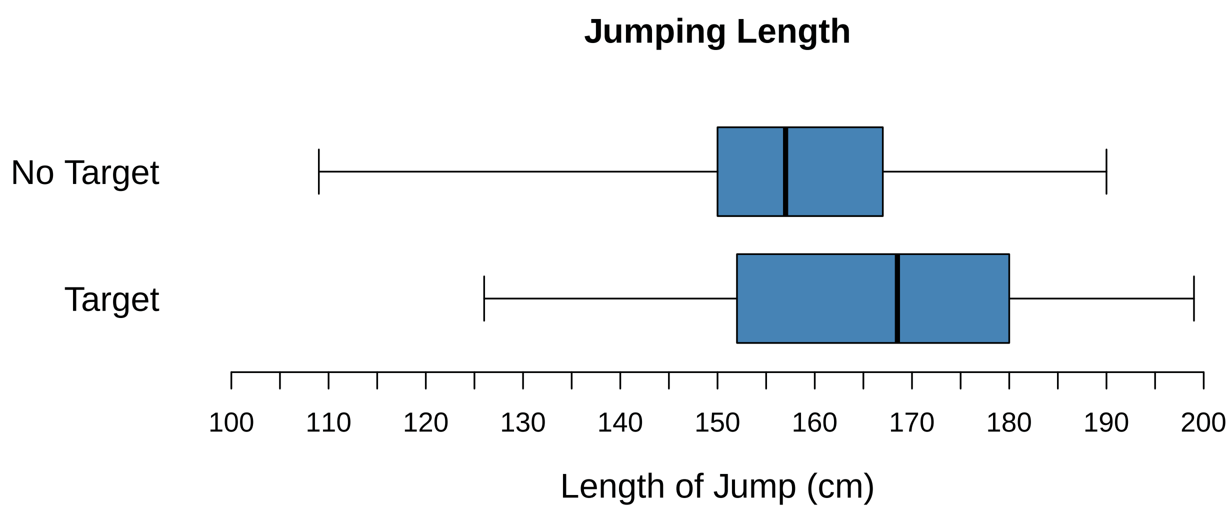


Figure 4.1.3 Side-by-side boxplots comparing length of jumps for No Target group and Target group

Investigation 4.1: How Far Can You Jump?

Table 4.1.3 Example Recording Sheet

Student Number	Group 1 - No Targeted Jump (cm)	Group 2 - Targeted Jump (cm)
1	146	199
2	190	167
3	109	147
4	181	180
5	155	185
6	167	170
7	154	171
8	171	139
9	157	154
10	156	126
11	128	179
12	157	158
13	167	181
14	162	152
15	137	
Summary Measures		
Mean	155.8	164.8
Median	157	168.5
Minimum	109	126
Maximum	190	199
Q1	146	152
Q3	167	180

Name: ____________________ Date: ____________

Investigation 4.2: How Fast Can You Sort Cards?

Recording Sheet

Time (sec) to Sort 2 Digits	Time (sec) to Sort 3 Digits	Time (sec) to Sort 4 Digits

Investigation 4.2: How Fast Can You Sort Cards?

Master for Card Labels

Write these numbers on a set of 3 x 5 index cards.

Stack 1	Stack 2	Stack 3
20	140	1050
21	141	1051
22	142	1052
23	143	1053
24	144	1054
25	145	1055
26	146	1056
27	147	1057
28	148	1058
29	149	1059
30	150	1060
31	151	1061
32	152	1062
33	153	1063
34	154	1064
35	155	1065
36	156	1066

Name: ______________________________ Date: ______________

Investigation 4.2: How Fast Can You Sort Cards?

Assessment

A class of sixth-grade students conducted an experiment involving LEGO blocks to compare the effect of the type of directions provided to a student on the time needed to complete a task. The task was to build a tower from a given set of blocks. A bag of LEGO blocks contained one of the following three sets of directions:

Directions Set 1: Construct a tower using all the blocks in this bag. The longest blocks should be on the bottom and go up in order to the shortest LEGO blocks at the top.

Directions Set 2: Construct a tower using all the blocks in this bag according to the picture. (Figure 4.2.5)

Directions Set 3: Build a tower with the blocks.

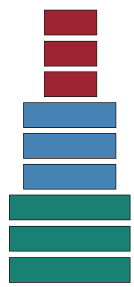


Figure 4.2.5 Diagram shown on directions for set 2

The class was randomly divided into three groups; the results of the experiment are shown in table 4.2.3.

Table 4.2.3 Time to Build Tower

Time (sec) to Build Tower with Directions for Set 1	Time (sec) to Build Tower with Directions for Set 2	Time (sec) to Build Tower with Directions for Set 3
18.1	22.1	11.6
17.5	21.3	15.5
16.3	18.9	15.4
18.8	19.5	15.6
16.2	20.1	15.3
16.0	21.0	15.7
16.6	19.4	13.8
14.8	16.5	16.1
18.1	22.7	15.9
19.8	19.1	16.8
17.6	21.6	14.3
16.5	20.0	12.9
16.7	20.0	17.0

Name: ______________________________ Date: ______________

Investigation 4.2: How Fast Can You Sort Cards?

1. What is an appropriate statistical question in the context of this study?

2. Find the mean for each group.

3. Find the five-number summary for each of the groups.

	Set 1	Set 2	Set 3
Minimum			
Q1			
Median			
Q3			
Maximum			

4. Construct side-by-side boxplots of the three groups.

Name: ______________________________ Date: ______________

Investigation 4.2: How Fast Can You Sort Cards?

5. Which of the three groups was able to build the tower faster? Using words, numbers, and graphs, explain why you chose the group you did.

Investigation 4.2: How Fast Can You Sort Cards?

Example of 'Interpret the Results'

We investigated how fast it took us to sort cards that had two-, three-, or four-digit numbers on them. There were 17 cards in each group. We were assigned to one of the groups. To avoid introducing bias into the experimental procedure, we put all our names in a container and then drew them out randomly, one at a time, assigning the first name to the two-digit group, the second to the three-digit group, and the third to the four-digit group. We repeated this until everyone was assigned. After getting our data, we drew stemplots and boxplots.

Sort Times for 4-Digit Numbers

25	2
26	2 8
27	2 9
28	3 6 9
29	
30	
31	2 3
32	
33	
34	3

Key: 31|2 represents 31.2 sec.

Sort Times for 2-Digit Numbers

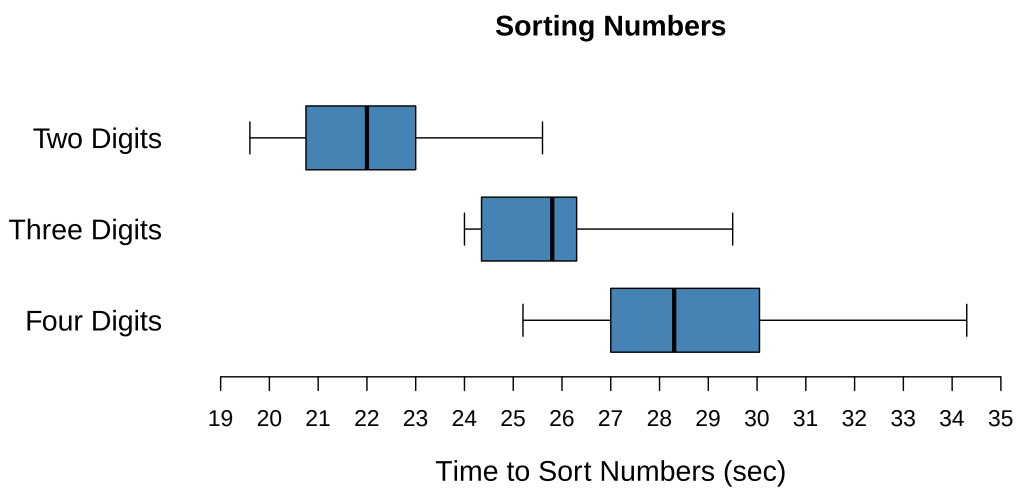
19	6
20	5 6 9
21	8
22	0 2 9
23	1 6
24	
25	6

Key: 25|6 represents 25.6 sec.

Sort Times for 3-Digit Numbers

24	0 1 3 4
25	1 8 9
26	2 4
27	
28	4
29	5

Key: 26|2 represents 26.2 sec.



Each stemplot had at least one gap, indicating there were possible outliers. The two-digit shape had a dip in the middle, but looked symmetric. The three-digit shape was definitely skewed to the right. The four-digit one looked like a triangle for the lower values and then had a couple big gaps. We should have put the stemplots side by side on the same scale like we did with the boxplots. It was really clear from the boxplots that the medians increased and the spread of the middle 50% measured by IQR of the 2-digit and 3-digit data sets was similar, with the spread of the 4-digit about twice as much. We saw many comparisons such as all the 3-digit and 4-digit times were longer than 75% of the 2-digit times. The median of 3-digit exceeded all 2-digit. So, overall, it was clear that the times to sort the cards are longer as the number of digits in the numbers increases.

It was interesting that the medians (22.0, 25.8, and 28.3) were about the same as the means (22.1, 25.8, 28.7) even though the distributions had all those gaps. We guessed the possible outliers kind of balanced out the distributions. We checked to see if the outliers we saw in the stemplots were also outliers by the 1.5*IQR calculation for boxplots and none were. Different graphs illustrate different things. Finally, we compared the means of the 2-digit and 3-digit groups by calculating how many common IQRs separated them. We used the maximum IQR of 2.5 for the value of the IQRs and saw that the means 22.1 and 25.8 differed by (25.8 – 22.1)/2.5 = 1.5 IQRs. We don't really have a number to compare 1.5 to, but it seems to us that 1.5 IQRs is large enough to say the means differ from each other, since they are really separated when we look at the boxplots.

Investigation 4.2: How Fast Can You Sort Cards?

Table 4.2.1 Example of Class Data

Time (sec) to Sort 2 Digits	Time (sec) to Sort 3 Digits	Time (sec) to Sort 4 Digits
20.6	26.2	31.2
22.9	25.8	28.6
20.9	24.1	28.3
22.2	24.3	31.3
25.6	25.9	26.8
23.1	24.4	27.9
19.6	26.4	28.9
23.6	29.5	27.2
20.5	28.4	34.3
22.0	25.1	26.2
21.8	24.0	25.2

Investigation 4.2: How Fast Can You Sort Cards?

Sort Times for 2-Digit Numbers

19	6
20	5 6 9
21	8
22	0 2 9
23	1 6
24	
25	6

Key: 25|6 represents 25.6 sec.

Figure 4.2.1 Stemplot of sort times for 2-digit numbers

Sort Times for 3-Digit Numbers

24	0 1 3 4
25	1 8 9
26	2 4
27	
28	4
29	5

Key: 26|2 represents 26.2 sec.

Figure 4.2.2 Stemplot of sort times for 3-digit numbers

Sort Times for 4-Digit Numbers

25	2
26	2 8
27	2 9
28	3 6 9
29	
30	
31	2 3
32	
33	
34	3

Key: 31|2 represents 31.2 sec.

Figure 4.2.3 Stemplot of sort times for 4-digit numbers

Table 4.2.2 Five-Number Summary for Each Group

	Min	Max	Range	Q1	Q3	IQR
Two-Digit Group	19.6	25.6	6.0	20.6	23.1	2.5
Three-Digit Group	24.0	29.5	5.5	24.3	26.4	2.1
Four-Digit Group	25.2	34.3	9.1	26.8	31.2	4.4

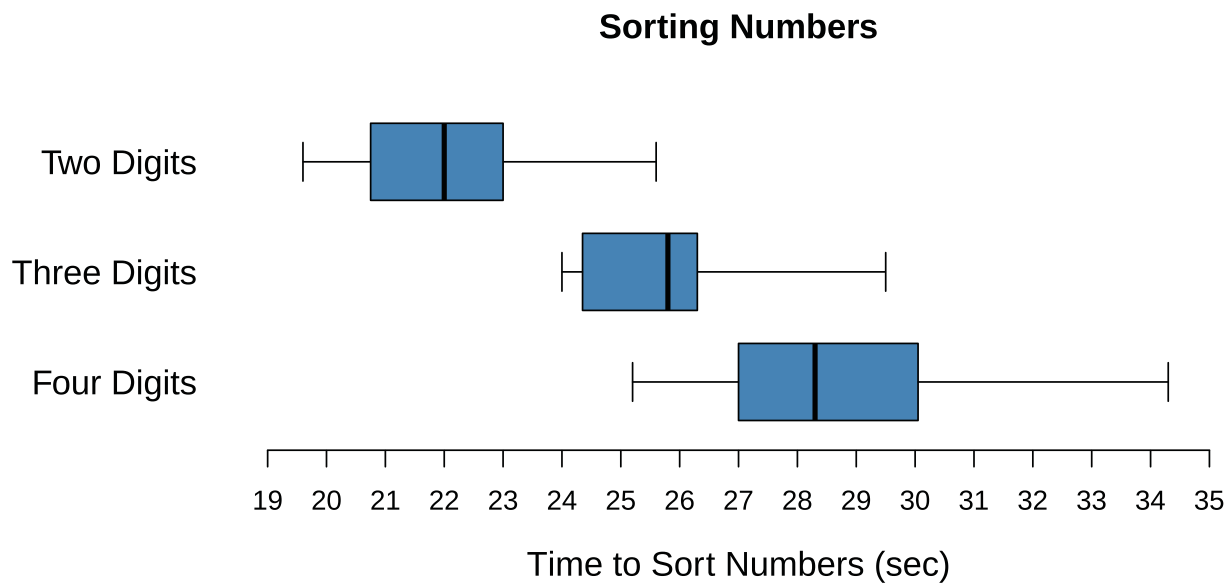


Figure 4.2.4 Side-by-side boxplots of the example class data

Name: ______________________ Date: ______________

Investigation 4.3: How High Does a Ball Bounce?

Table 4.3.1 Tennis Ball Recording Sheet

	Tennis Ball		
	30 cm	60 cm	90 cm
Trial 1			
Trial 2			
Trial 3			

Table 4.3.2 Golf Ball Recording Sheet

	Golf Ball		
	30 cm	60 cm	90 cm
Trial 1			
Trial 2			
Trial 3			

Name: ______________________________ Date: ______________

Investigation 4.3: How High Does a Ball Bounce?

Ratio of Bounce Height to Drop Height Recording Sheet

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height

Name: ____________________________ Date: ______________

Investigation 4.3: How High Does a Ball Bounce?

Assessment

A group of students conducted the ball drop experiment using a basketball. Table 4.3.8 contains the results of their experiment when they dropped a basketball from 30, 60, and 90 cm.

Table 4.3.8 Results of Dropping a Basketball

	Basketball		
	30 cm	**60 cm**	**90 cm**
Trial 1	22.0	45.0	67.0
Trial 2	23.0	44.0	68.0
Trial 3	22.0	44.0	67.0
Mean			
Median			
Ratio			

1. Find the mean and median bounce height for each drop height and record them in the chart above.

2. Find the ratio of the median bounce height to the median drop height and record them in the chart above.

3. Discuss how the mean and median bounce heights relate to the drop height. Include the ratio of median bounce height to median drop height in your discussion.

Name: ______________________________ Date: ______________

Investigation 4.3: How High Does a Ball Bounce?

4. Construct a scatterplot that shows the relationship between the heights from which the basketball was dropped and the median height of the bounce.

5. Describe the graph and the relationship between drop height and median bounce height.

6. Is the bounce height of a basketball higher than either the tennis ball or golf ball that you used in the investigation? Explain your answer.

Name: ______________________ Date: ______________

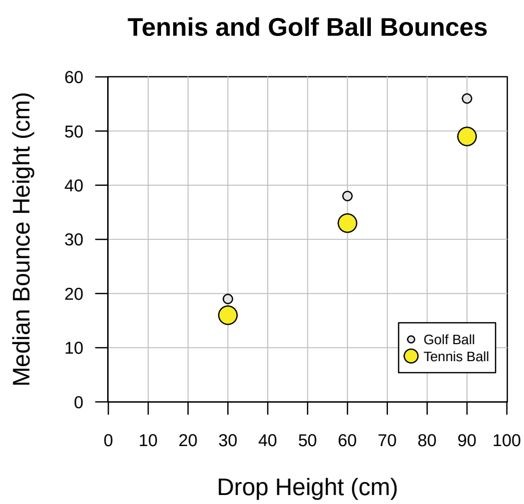
Investigation 4.3: How High Does a Ball Bounce?

Alternative Assessment

Find a ball at your house and replicate what was done in class with a family member. Drop the ball from 30, 60, and 90 cm. Record your data, find the mean and median, and create a scatterplot. Describe the relationship between drop heights and bounce height and compare your results with the results from the class experiment.

Example of 'Interpret the Results'

We think the height from which a ball is dropped does and does not affect the bounce height too much. It depends on what you are looking at. If it's the actual height of the bounce, then it goes up if the drop height goes up. But if it's the ratio of the bounce height to the drop height, then the ratio is constant for a tennis ball or a golf ball—about .54 for the tennis ball and .63 for the golf ball. Our conclusion is based on data we got from dropping a tennis ball three times each from heights of 30, 60, and 90 cm. We dropped the ball three times from each height to get an accurate result. We then took the median of the three data points to represent the bounce height for each drop height. We did the same thing for a golf ball. Here is our scatterplot of median bounce height for each drop height:



Looking at the scatterplot, we see there is a positive relationship between drop height and bounce height for both tennis and golf balls. We also see that the relationship is pretty linear for both the tennis and the golf ball and that the golf ball bounces higher than the tennis ball at each drop height. The gap between the heights gets wider as the drop gets higher. We now see why our teacher asked us to calculate the ratio of bounce height to drop height. Here are the calculations:

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height
30	16	16/30= 0.53	19	19/30= 0.63
60	33	33/60 = 0.55	38	38/60 = 0.63
90	49	49/90 = 0.54	56	56/90 = 0.62

It's interesting to see that the tennis ball bounces back around 54% of its drop height and the golf ball does better—at around 63%. It's probably because of the composition of a golf ball. We wonder what ratio a "super ball" would have.

Investigation 4.3: How High Does a Ball Bounce?

Table 4.3.3 Example of Tennis Ball Bounce Height

	Tennis Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	16 cm	33 cm	50 cm
Trial 2	17 cm	32 cm	49 cm
Trial 3	16 cm	33 cm	49 cm

Investigation 4.3: How High Does a Ball Bounce?

Table 4.3.4 Example of Golf Ball Bounce Height

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	19 cm	37 cm	55 cm
Trial 2	17 cm	32 cm	49 cm
Trial 3	16 cm	33 cm	49 cm

Investigation 4.3: How High Does a Ball Bounce?

Table 4.3.5 Sample Results for Tennis Ball Drop

	Tennis Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	16.0	33.0	50.0
Trial 2	17.0	32.0	49.0
Trial 3	16.0	33.0	49.0
Mean	16.3	32.6	49.3
Median	16.0	33.0	49.0

Investigation 4.3: How High Does a Ball Bounce?

Table 4.3.6 Sample Results for Golf Ball Drop

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	19.0	37.0	55.0
Trial 2	18.0	38.0	57.0
Trial 3	19.0	38.0	56.0
Mean	18.6	37.6	56.0
Median	19.0	38.0	56.0

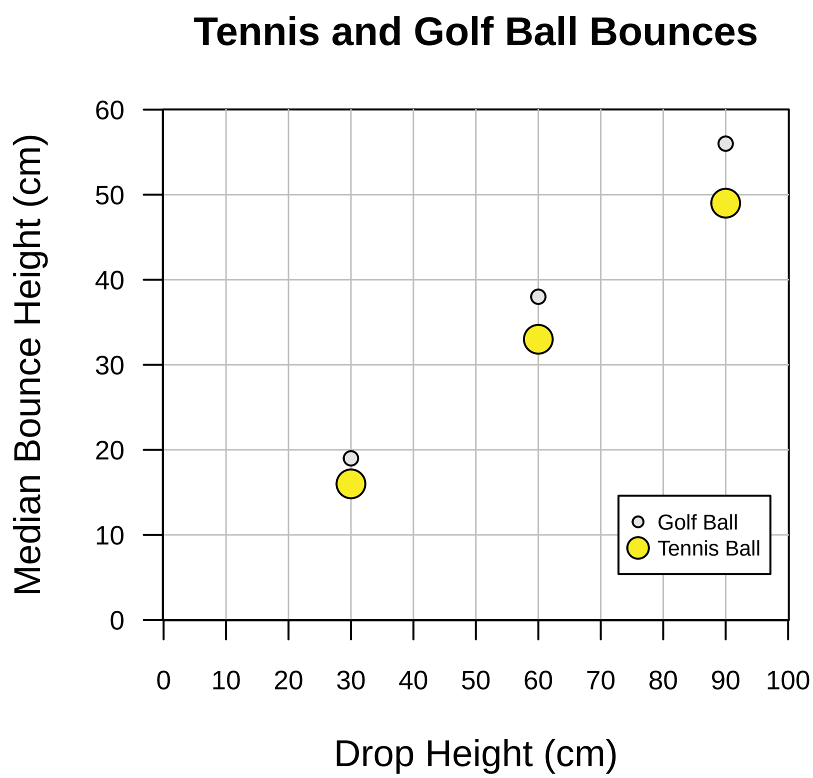


Figure 4.3.1 Scatterplot of median bounce height versus drop height for a tennis ball and golf ball

Table 4.3.7 Ratios of Bounce Height to Drop Height

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height
30	16	16/30= 0.53	19	19/30= 0.63
60	33	33/60 = 0.55	38	38/60 = 0.63
90	49	49/90 = 0.54	56	56/90 = 0.62

Investigation 4.4: Can You Roll Your Tongue?

Data Collection Sheet

Boy __________

Girl __________

Can roll tongue __________

Can't roll tongue __________

Name: ______________________________ Date: ______________

Investigation 4.4: Can You Roll Your Tongue?

Recording Sheet

Student	Boy or Girl	Roll Your Tongue Yes or No?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

Name: ______________________________ Date: ______________

Investigation 4.4: Can You Roll Your Tongue?

Assessment

A survey asked a group of students if they participated in a sport and if they played a musical instrument. Table 4.4.7 shows the survey results.

Table 4.4.7 Survey Results

	Music Yes	Music No	Total
Sport Yes	18	2	20
Sport No	8	22	30
Total	**26**	**24**	**50**

Use the table to answer the following questions:

1. How many students said they participated in a sport?

2. How many students said they did not play a musical instrument?

3. What does the number 8 represent in the table?

4. What percentage of those who said they participated in a sport also played a musical instrument?

5. What percentage of those who said they did not participate in a sport played a musical instrument?

Name: ______________________________ Date: ______________

Investigation 4.4: Can You Roll Your Tongue?

6. If a student participates in a sport, are they more likely to play a musical instrument than a student who does not participate in a sport? Use words, numbers, and graphs to explain your answer.

Example of 'Interpret the Results'

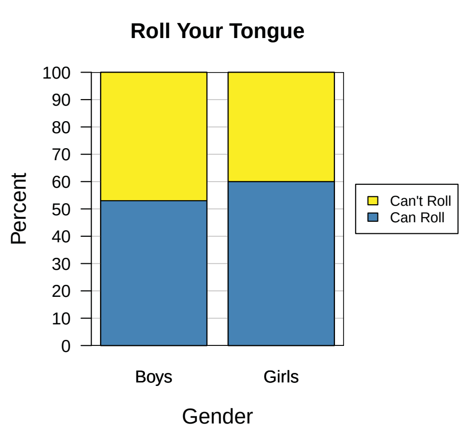
In our biology class, we often talk about genetics, so we thought a good statistics project in our mathematics class would be to take a genetic trait and see if it is associated with gender. We chose rolling our tongues. (After our study was complete, we found out that rolling one's tongue is not actually genetic. It is a learned trait. But it was fun doing the experiment anyway.) Our statistical question was "Is gender associated with ability to roll one's tongue?" We collected data by making a list of boys or girls and whether they could roll their tongue. We then counted how many there were in each of the four categories and organized the data in a two-way table like this one.

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	15
Girl	6	4	10
Total	**14**	**11**	**25**

So, to answer the question, some of us say boys are more likely to roll their tongues than girls are. But, we messed up because there were more boys in class than girls. So, we should be looking at percentages, not counts. When we calculated the percentages, we almost based them on 25, but realized they had to be calculated within boys' and girls' totals. So, here is our table of row percentages.

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8/15 = .53 = 53%	7/15 = .47 = 47%	15/15 = 1.00 = 100%
Girl	6/10 = .60 = 60%	4/10 = .40 = 40%	10/10 = 1.00 = 100%
Total			

The actual answer to our question is that a higher percentage of girls can roll their tongues as compared to boys. Sixty percent of girls could roll their tongues compared to 53% of boys. Our teacher showed us how to visualize these results in what is called a segmented bar graph. It makes it clear that the percentage of girls is higher.



Investigation 4.4: Can You Roll Your Tongue?

But we debated whether gender and ability to roll one's tongue are associated because some of us thought that 53% and 60% are kind of close and so the variables are not associated. Others thought the percentages were far enough apart to claim the variables are associated. Our teacher said we will learn more about association in high school.

Investigation 4.4: Can You Roll Your Tongue?

Table 4.4.2 Frequency Table

Possibilities	**Count/Frequency**
Boy – Yes	
Boy – No	
Girl – Yes	
Girl – No	
Total	

Investigation 4.4: Can You Roll Your Tongue?

Table 4.4.3 Two-Way Table

	Yes – Can Roll Tongue	**No – Can't Roll Tongue**	**Total**
Boy			
Girl			
Total			

Investigation 4.4: Can You Roll Your Tongue?

Table 4.4.4 Example of Completed Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	a	b	
Girl	c	d	
Total			

Investigation 4.4: Can You Roll Your Tongue?

Table 4.4.5 Row of the Boys' Data from the Two- Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	**15**
Girl	6	4	**10**
Total	**14**	**11**	**25**

Table 4.4.6 Row of the Boys' Data from the Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	15

Investigation 4.4: Can You Roll Your Tongue?

Table 4.4.7 Row of the Girls' Data from the Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Girl	6	4	10

Table 4.4.8 Example of Row Percentages

	Yes – Can Roll Tongue	**No – Can't Roll Tongue**	**Total**
Boy	8/15 = .53 = 53%	7/15 = .47 = 47%	15/15 = 1.00 = 100%
Girl	6/10 = .60 = 60%	4/10 = .40 = 40%	10/10 = 1.00 = 100%
Total			

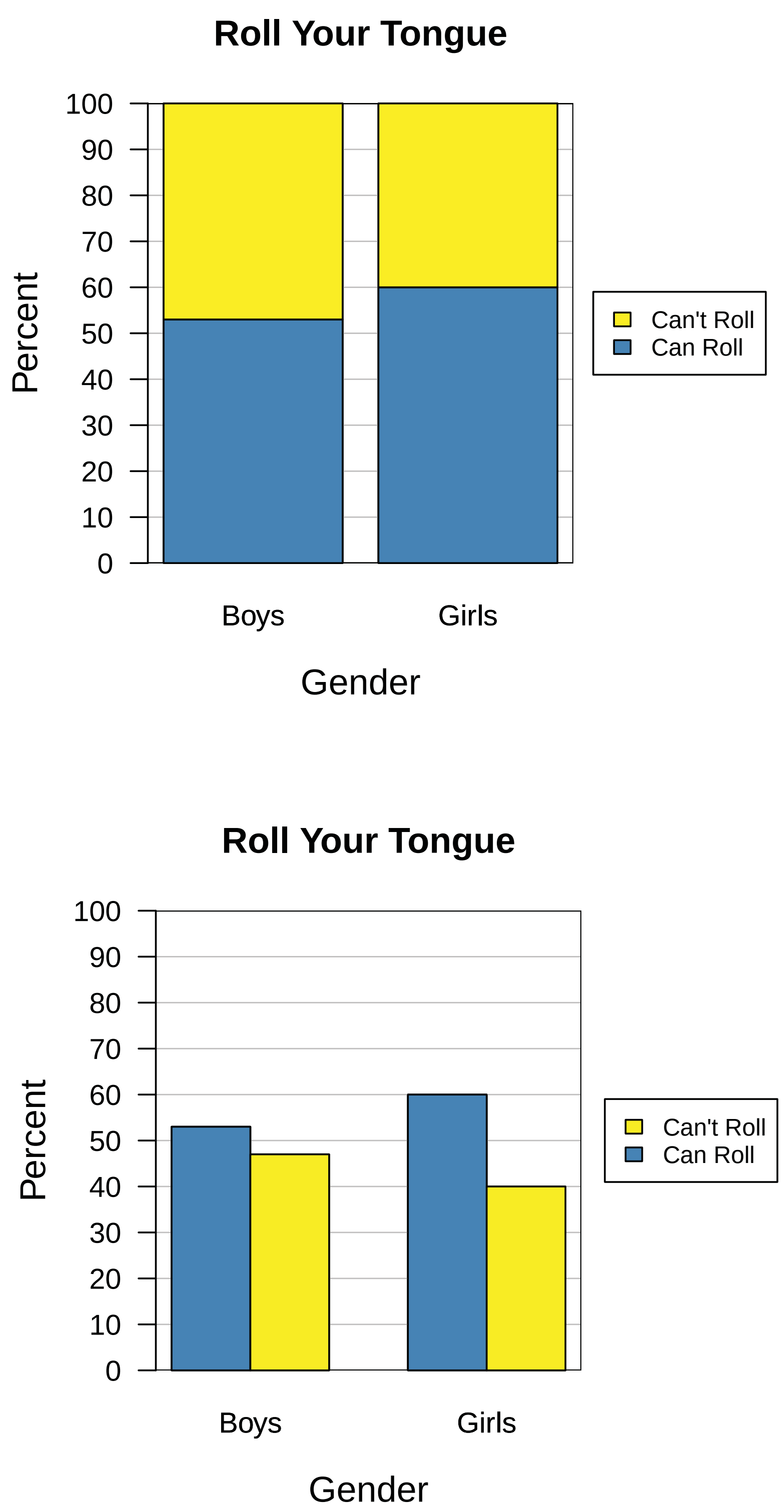


Figure 4.4.2 Segmented bar graph and side-by-side bar graph of example class data

Name: ____ Date: ____________________

Investigation 5.1: Do Names and Cost Relate?

Data Collection Sheet

Student	Length of First Name	Cost of First Name (¢)	Length of Last Name	Cost of Last Name (¢)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				

Name: ______________________________ Date: ______________

Investigation 5.1: Do Names and Cost Relate?

Assessment

A group of 8th-grade students investigated the statistical question, "Is there a relationship between the length of their last name and the cost of their last name." Figure 5.1.5 is a scatterplot of the length of the students' last names and the cost of their last names.

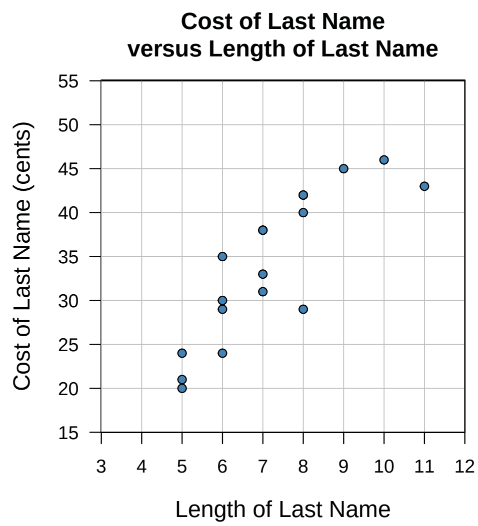


Figure 5.1.5 Scatterplot of cost (¢) of last name versus length of last name

1. Choose a point on the graph and describe what it means in the context of the variables.

2. If a student has a long last name, does that student tend to have a more or less expensive cost for their last name? Explain your answer.

3. Overall, is there a relationship between the length of a student's last name and the cost of their last name? Use words and numbers to explain your answer.

Name: ______________________________ Date: ______________

Investigation 5.1: Do Names and Cost Relate?

Alternative Assessment

March is National Reading Month and a teacher wanted to know if her students read more books in March than in February. Figure 5.1.6 is a scatterplot of the number of books sixth-graders each read during February and March.

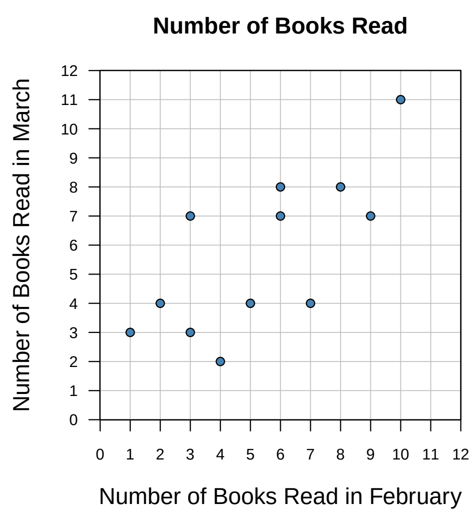


Figure 5.1.6 Scatterplot of number of books read by sixth-graders in March versus number of books read by sixth-graders in February

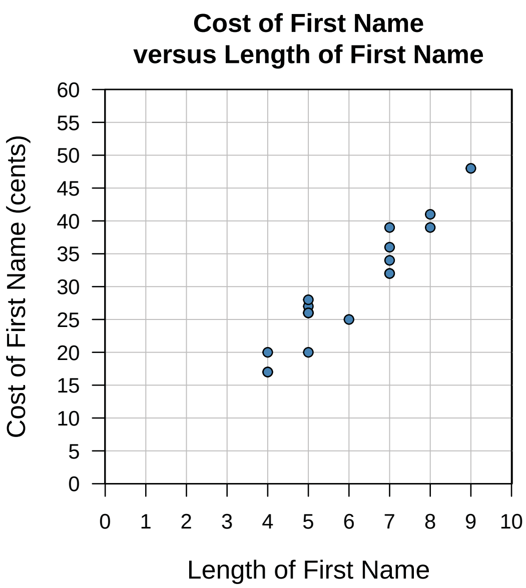
1. Choose a point and describe what it means in the context of the variables.

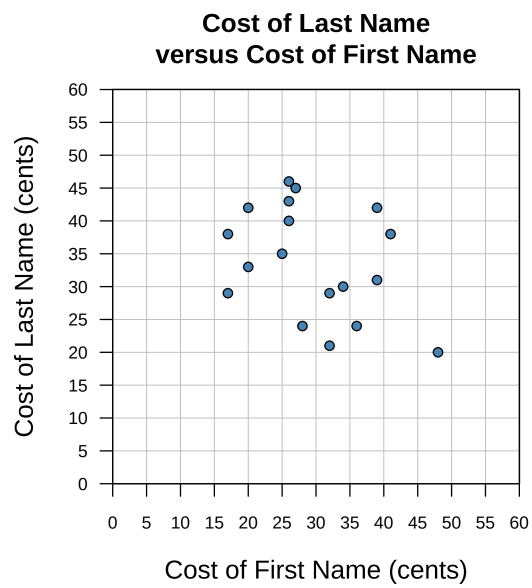
2. If a student read many books in February, what did that student tend to do in March? Explain your answer.

3. Overall, is there a relationship between the number of books sixth-graders read in February and the number of books they read in March? Use words and/or numbers to explain your answer.

Example of 'Interpret the Results'

In this investigation, we looked at two questions. One was on relating the length of our first names and the cost of monogramming them on T-shirts. The second question was on investigating if the cost of monogramming our first names and last names were related. The assignment of costs to letters was based on the frequency of usage of letters in English. High-frequency letters cost more and low-frequency letters cost less. In Scrabble, it's the opposite, with the letters that don't occur very often being worth more. To see how the length of our first name and its cost are related, we displayed the length and cost of our first names on a scatterplot using sticky notes. We did the same for the cost of our first and last names. Here were our graphs.





We saw that the cost of the name was higher for longer names, which meant there was a positive association between the length of our first name and its cost. We also investigated the relationship between the cost of our first and cost of our last name. We displayed a scatterplot of the cost of both names and observed that the higher costs of the first names were associated with lower costs of the last names. This meant there was a negative relationship between the cost of the first and last names.

We are going to continue this study by analyzing names in foreign countries such as China and Russia. For example, John in Chinese is Yue Han, which has a cost of 29 cents. John in English was 16 cents. Maybe Chinese names have more vowels, so they might cost more. We'll see.

Name: ______________________________ Date: ______________

Investigation 5.1: Do Names and Cost Relate?

Table 5.1.1 Occurrence of Letter Percentages

Letter	a	b	c	d	e	f	g	h	i
Percentage	8.2%	1.4%	2.8%	4.2%	12.7%	2.2%	2.0%	6.1%	7.0%
Letter	j	k	l	m	n	o	p	q	r
Percentage	0.2%	0.8%	4.0%	2.4%	6.7%	7.5%	1.9%	0.1%	6.0%
Letter	s	t	u	v	w	x	y	z	
Percentage	6.3%	9.1%	2.7%	1.0%	2.4%	0.2%	2.0%	0.1%	

Name: ______________________________ Date: ______________

Investigation 5.1: Do Names and Cost Relate?

Table 5.1.2 Cost of Letters (cents)

A	B	C	D	E	F	G	H	I	J	K	L	M
6¢	3¢	3¢	4¢	7¢	3¢	3¢	5¢	5¢	1¢	2¢	4¢	3¢
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
5¢	5¢	3¢	1¢	5¢	5¢	6¢	3¢	2¢	3¢	1¢	3¢	1¢

Investigation 5.1: Do Names and Cost Relate?

Table 5.1.4 Sample Data of Costs of First and Last Names

Student	Length of First Name	Cost of First Name (¢)	Length of Last Name	Cost of Last Name (¢)
1	5	26	11	43
2	4	17	7	38
3	5	20	8	42
4	8	39	8	42
5	6	25	6	35
6	8	41	7	38
7	4	20	7	33
8	7	34	6	30
9	4	17	6	29
10	5	27	9	45
11	5	26	10	46
12	9	48	5	20
13	7	32	5	21
14	5	26	8	40
15	7	36	5	24
16	5	28	6	24
17	7	39	7	31
18	7	32	8	29

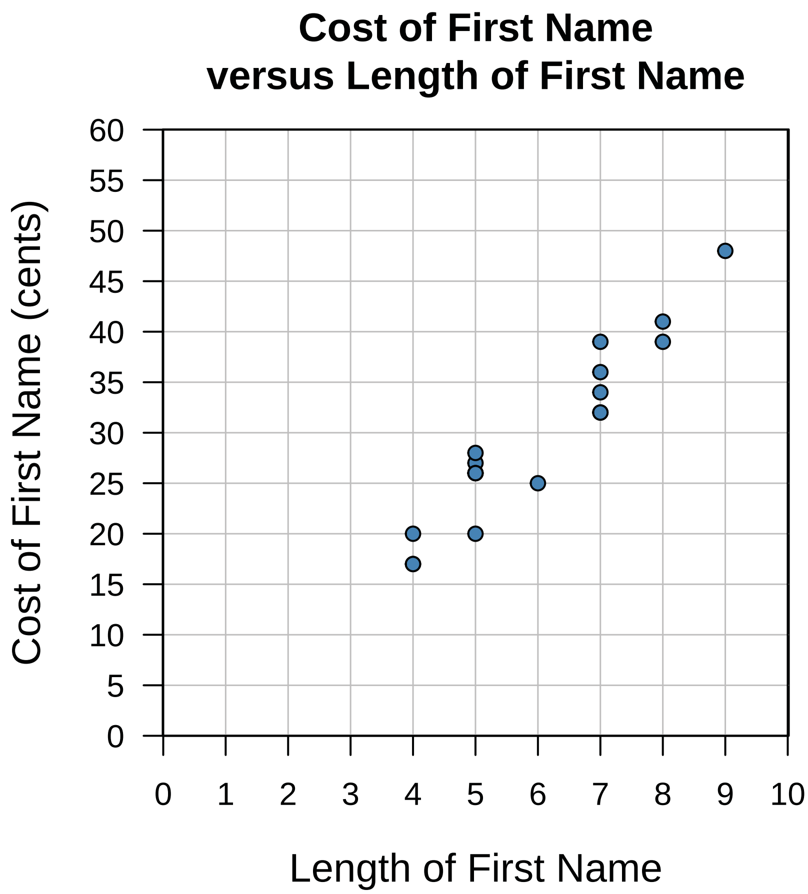


Figure 5.1.2 Scatterplot of cost (¢) of first name versus length of first name. **Note:** There are two data points at coordinate (4,17), three at (5,26), and two at (7,32).

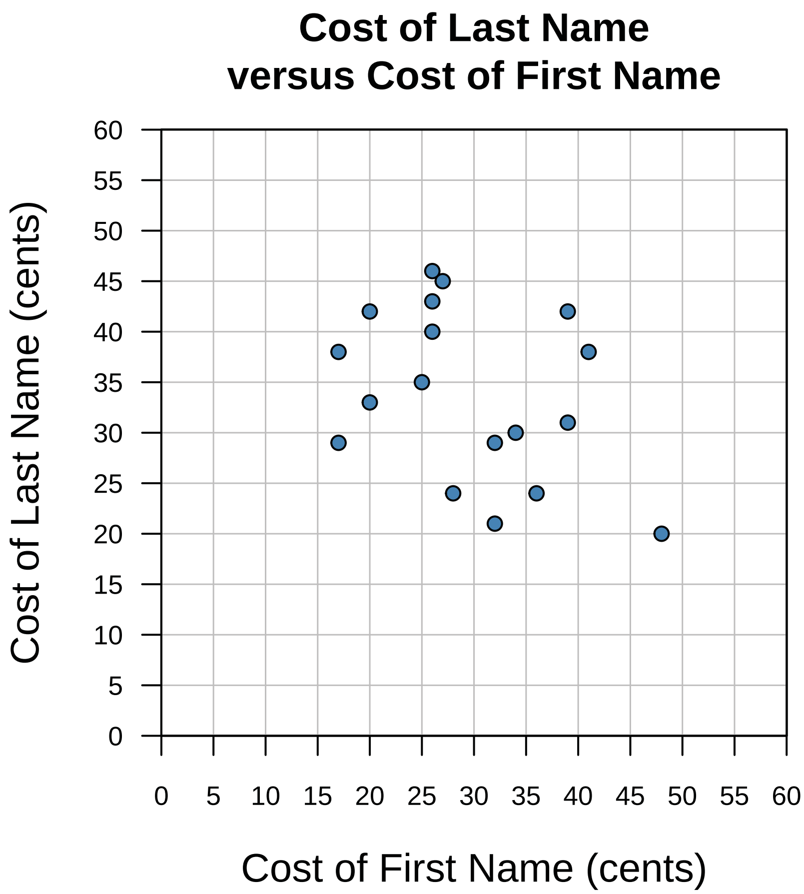


Figure 5.1.4 Scatterplot of cost (¢) of last name versus cost (¢) of first name

Name: ______________________________ Date: ______________

Investigation 5.2: How Tall Were the Ancestors of Laetoli?

Data Collection Sheet

Student Number	Foot Length cm	Height cm
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		
26		

Name: ______________________ Date: ______________

Investigation 5.2: How Tall Were the Ancestors of Laetoli?

Assessment

A group of students measured their height and arm span in centimeters. Table 5.2.3 shows the data they collected, and the scatterplot of the data is shown in Figure 5.2.5.

Table 5.2.3 Height and Arm Span (cm)

Height	Arm Span	Height	Arm Span
155	151	173	170
162	162	175	166
162	161	176	171
163	172	176	173
164	167	178	173
164	155	178	166
165	163	181	183
165	165	183	181
166	167	183	178
166	164	183	174
168	165	183	180
171	164	185	177
171	168	188	185

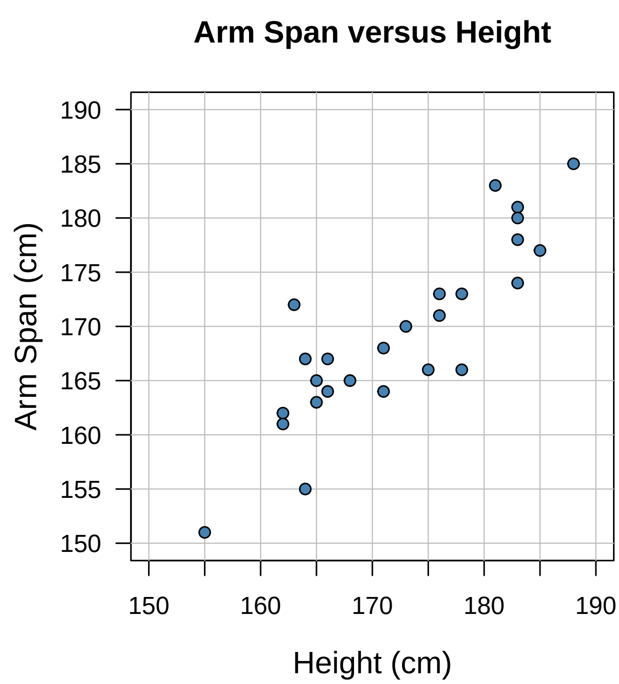


Figure 5.2.5 Scatterplot of arm span versus height

Name: ______________________________ Date: ______________

Investigation 5.2: How Tall Were the Ancestors of Laetoli?

1. Describe the relationship between arm span and height.

2. Find the mean height and the mean arm span.

3. Locate the point (mean height, mean arm span) on the graph and draw a horizontal line and a vertical line through the point.

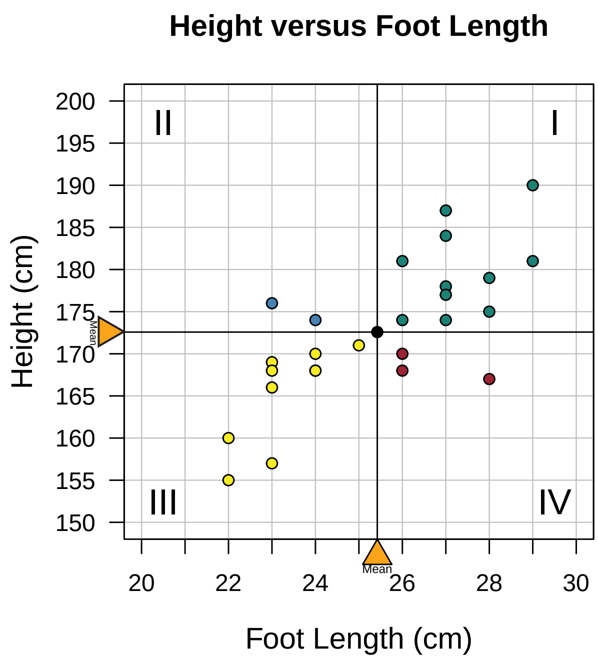
4. Find the value of the QCR.

5. Interpret the value of the QCR.

Example of 'Interpret the Results'

For a statistics project, we got an idea from an anthropological study by Dr. Leakey, who found footprints of 3.6 million–old ancestors in Laetoli, Tanzania. The study had the ancestors footprint lengths, and we were wondering how tall they might have been. One of the set of footprints had a mean footprint of 21.5 cm. Our statistical question was, "Is there a relationship between human height and foot length?" Our data were the lengths of our right foot and our height. There were 26 paired data points in our class.

The first thing we did was to draw a picture, a scatterplot, with height on the vertical axis and foot length on the horizontal axis. It looked like people with longer feet were taller and those with shorter feet were shorter. To see that, we gave out sticker dots and placed them on a big scatterplot on the board. The dots were determined by whether our height was above or below the mean height of 172.6 cm and how our foot length compared to the mean 25.4 cm. Green dots were for (above 25.4 foot length, above 172.6 height); blue for (below, above); orange for (below, below); and red for (above, below). We added vertical and horizontal lines through the paired mean point. The scatterplot looked like this:



We could see a definite trend from the lower left to the upper right. In statistics, single numbers called summary statistics are often calculated to indicate the degree of some characteristic. So, our teacher suggested we count the number of points in the first and third quadrants and subtract the numbers in quadrants two and four, and then take the mean and call the result the Quadrant Count Ratio (QCR). For our data, QCR = ((11+10) – (2+3))/26 = 0.62. If all the data had been in quadrants one and three, the QCR would have been 1. So, we decided that .62 was pretty good and that it reflected a positive relationship. We then decided that our Laetoli ancestors would have had orange stickers, since the mean footprint we had for them was 21.5 and, from our scatterplot, there was no way the sticker could be blue. We were thinking about doing this study on all our teachers to get a new data set and see if it differs from ours. There's a difference of opinion. Some of us think it would have more variation because the ages of the teachers are more spread out than our ages.

Investigation 5.2: How Tall Were the Ancestors of Laetoli?

Laetoli, Tanzania

There is a place in Tanzania, Africa, known as Laetoli. It is a special place because it is where scientists believe our ancestors of long ago walked side-by-side. It is where scientists have worked to get an understanding of the past.

In the late 1970s, two sets of footprints were discovered at Laetoli. There were 70 footprints in two side-by-side lines 30 meters long, preserved in volcanic ash. Apparently, a volcano exploded sending ash everywhere and the two individuals just happened to walk through the area, preserving their footprints. Fossil remains in the area tell scientists that the ancestors who left the footprints found at Laetoli lived about 3.5 million years ago.

We know the size of the feet because Dr. Mary Leakey, an anthropologist, and her team made copies of the prints using plaster casts. The locations of the footprints were put on a map, so the length of stride (distance between footprints) also can be determined. Based on these observations, foot dimensions and stride length for the two ancestors are given in Table 5.2.1. These are averages based on the 70 observed footprints.

Table 5.2.1 Footprint Data Collected by Dr. Leakey at Laetoli

	Ancestor 1	**Ancestor 2**
Length of Footprint	21.5 cm	18.5 cm
Width of Footprint	10 cm	8.8 cm
Length of Stride	47.2 cm	28.7

Much has been learned from these footprints. They share many characteristics with the prints made by modern human feet.

A research question of interest to the scientists was "How tall were these ancestors at Laetoli?" The foot length, foot width, and length of stride can be used to produce estimates of the heights of these ancestors.

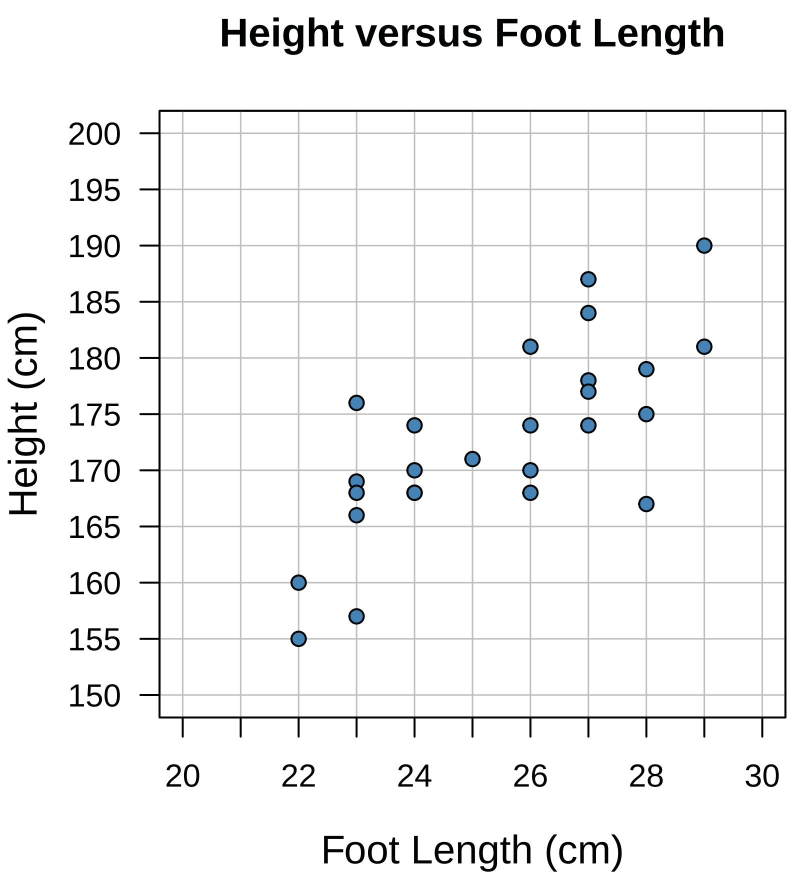


Figure 5.2.7 Scatterplot of height versus foot length. **Note:** There is a duplicate data point at (24,168).

Name: ______________________________ Date: ______________

Investigation 5.2: How Tall Were the Ancestors of Laetoli?

Table 5.2.2 Sample Set of 8th-Grade Class Data

Student Number	Foot Length cm	Height cm	Student Number	Foot Length cm	Height cm
1	28	175	14	24	168
2	26	181	15	23	168
3	24	168	16	23	176
4	26	168	17	27	177
5	27	178	18	25	171
6	24	174	19	22	160
7	28	179	20	27	187
8	23	157	21	28	167
9	29	190	22	27	184
10	26	170	23	29	181
11	23	169	24	27	174
12	23	166	25	22	155
13	26	174	26	24	170

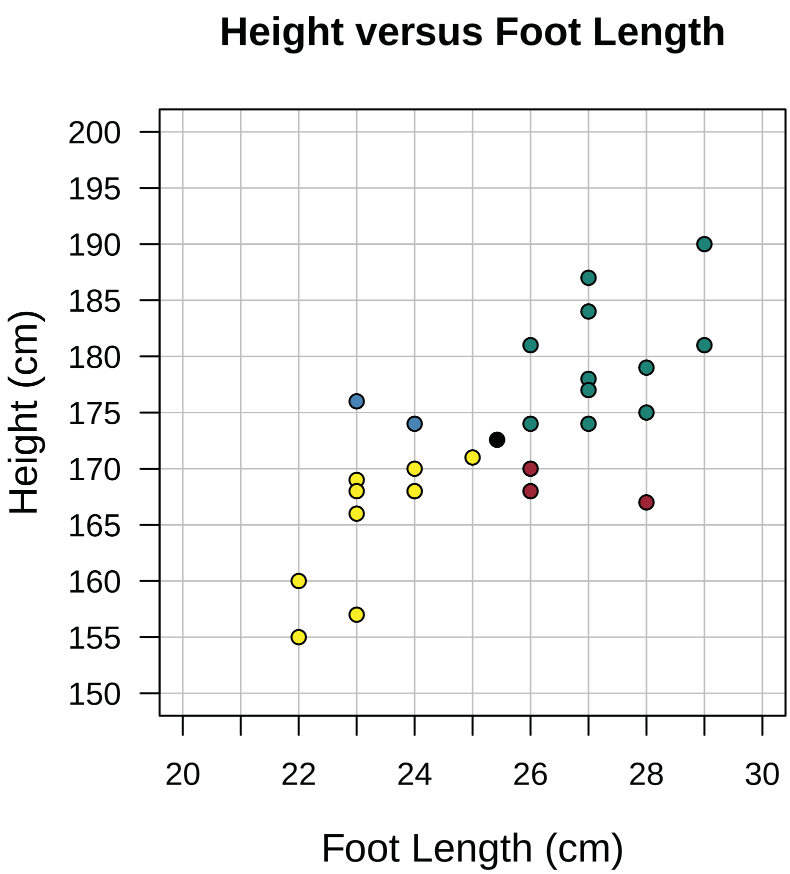


Figure 5.2.1 Class scatterplot of height versus foot length. **Note:** There is a duplicate data point at (24, 168).

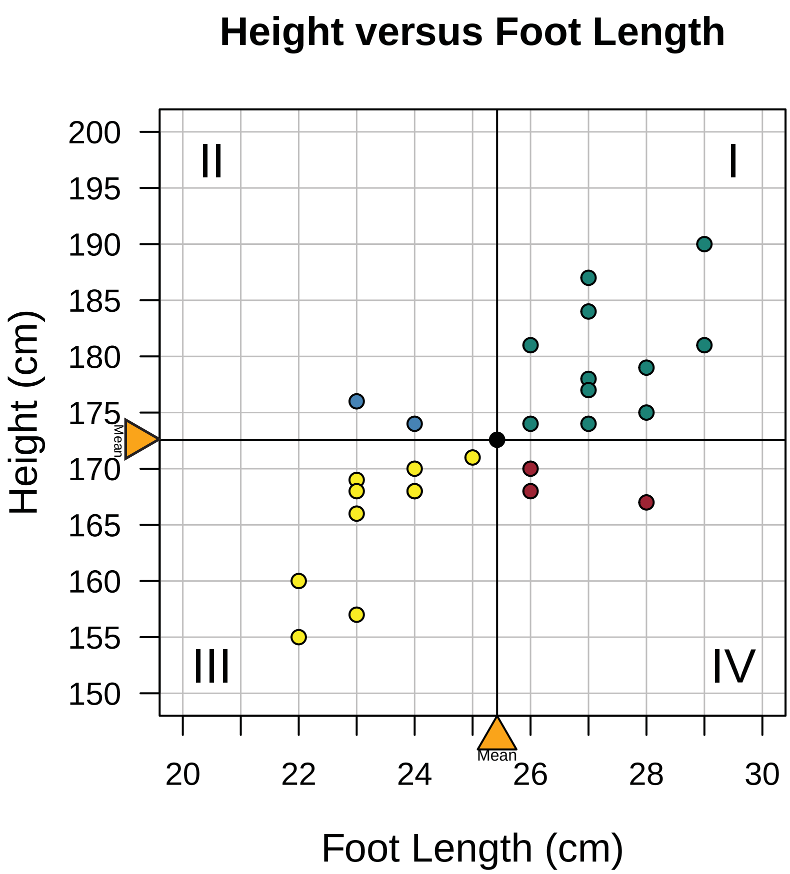


Figure 5.2.2 Scatterplot of height versus foot length showing the quadrants

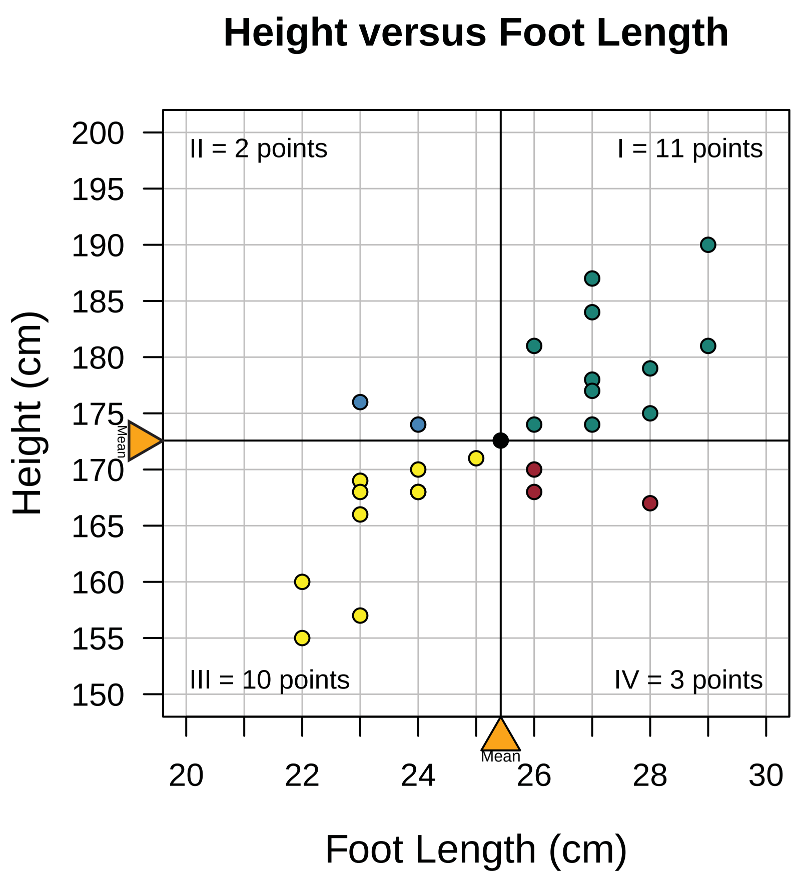


Figure 5.2.3 Scatterplot showing number of ordered pairs in each quadrant

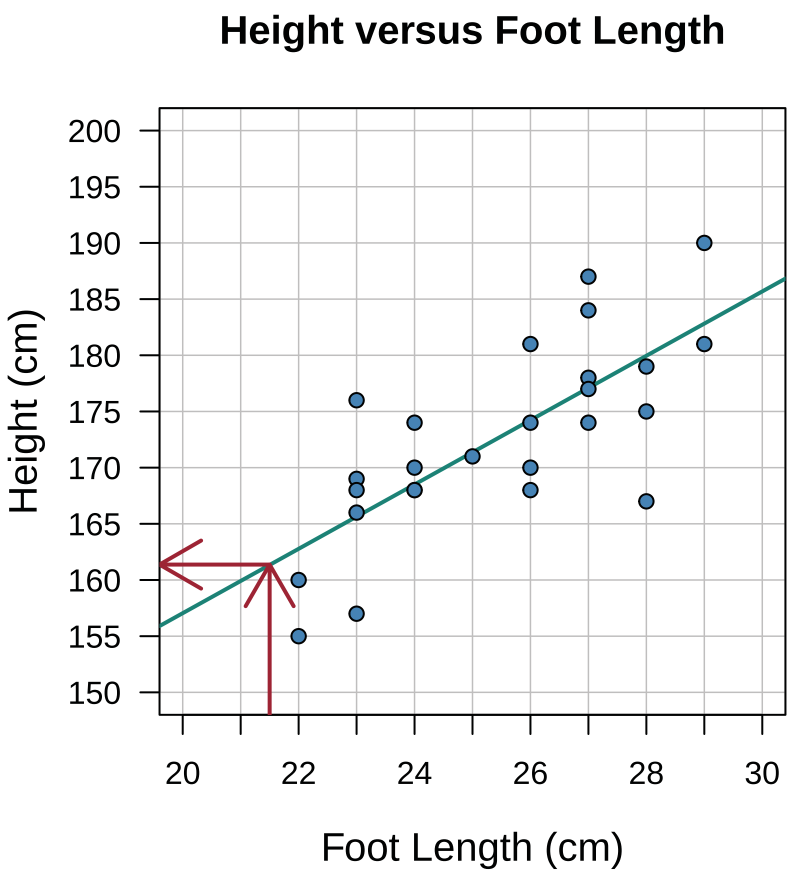


Figure 5.2.8 Scatterplot of height versus foot length with eyeball fit line. **Note:** There is a duplicate data point at (24,168).

Name: ____________________ Date: ____________

Investigation 5.3: How Long Does It Take to Perform the Wave?

Data Collection Sheet

Number of Students	Time (sec) to Complete the Wave
3	
6	
9	
12	
15	
18	
21	
24	
27	
30	
33	
36	
39	
42	
45	
48	
51	
54	
57	
60	
63	
66	
69	
72	
75	

Name: ____________________ Date: ____________

Investigation 5.3: How Long Does It Take to Perform the Wave?

Data Collection Sheet

Number of Students	Time (sec) to Complete the Wave	Change in Time
3		
6		
9		
12		
15		
18		
21		
24		
27		
30		
33		
36		
39		
42		
45		
48		
51		
54		
57		
60		
63		
66		
69		
72		
75		

Name: ______________________________ Date: ______________

Investigation 5.3: How Long Does It Take to Perform the Wave?

Assessment

A group of 8th-grade students wanted to investigate the relationship between how long it takes to perform the wave and the number of people participating. The table below shows the results of an experiment that students conducted. The experiment started with a group of five students. The timer said "Go" and the five students made a wave. The first student stood up, threw his/her hands in the air, turned around, and sat down. The second student did the same, and so on. The last student said "Stop" when he/she sat down. The timer recorded the elapsed time in seconds. The experiment was repeated with 9, 13, 17, 21, and 25 students.

Table 5.3.4 Number of Students and Length of Time to Perform the Wave

Number of Students	Time (sec)
5	16
9	28
13	42
17	54
21	66
25	78

1. Draw a scatterplot of the length of time (sec) versus the number of students.

2. Is there a relationship between the number of students and the length of time to perform the wave? Describe the relationship.

Name: ______________________________ Date: ______________

Investigation 5.3: How Long Does It Take to Perform the Wave?

3. Describe any patterns you observe in the collected data for both the number of students and the length of time.

4. Draw a line that matches the pattern in the data as closely as you can. List an ordered pair that lies on the line. Describe what the coordinates of the ordered pair represent.

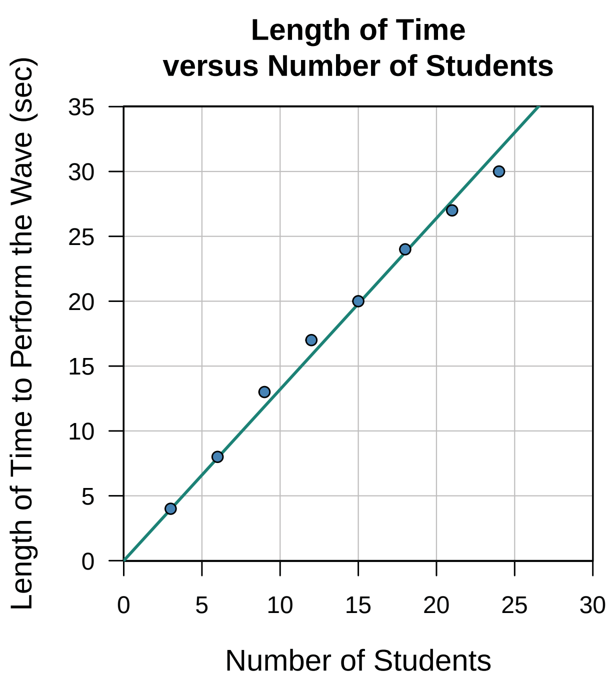
5. For each additional student added, how much longer does it take to perform the wave? Use words, numbers, and/or graphs to explain your answer.

Investigation 5.3: How Long Does It Take to Perform the Wave?

Example of 'Interpret the Results'

This activity was really fun because we got to perform the wave in class. The statistical question we came up with was "Is there a relationship between the number of people and the length of time to perform the wave?" We actually collected data in our classroom, starting with timing how long it took three of us to perform the wave.

First, we all had to practice so we were doing the procedure the same. Otherwise, we would bias our data. We also had one timekeeper maintain all the times so no bias would enter there, either. We made a data chart by increasing the number of us performing the wave by three each time and the time it took us. We calculated that it took a median increased time of 4 seconds for every three students we added, so the rate of change is an increase of 4/3 seconds for every additional person. We also figured out that if our whole grade level of 243 students lined up to perform the wave and our rate of change was accurate, it would take 243*(4/3) = 324 seconds or about 5.4 minutes to perform the wave. Wow. We showed our data in another way by graphing the points in a scatterplot. Here it is.



We eyeballed a line through the data. We decided the line should go through the origin because it made sense that if there are no people, then the time to perform the wave is 0. We calculated a rate of change by finding a point that was on our line. The point (20,26) looked like it was on our line. So, the rate of change or slope is 26/20 = 1.3, which is about what we got before for the rate of change, 4/3. This rate means that for every additional person added, the time to perform the wave goes up about 1.3 seconds.

Investigation 5.3: How Long Does It Take to Perform the Wave?

Table 5.3.2 Results of the Wave Experiment for a Group of 8th-Graders

Number of Students	Time (sec)
3	4
6	8
9	13
12	17
15	20
18	24
21	27
24	30

Investigation 5.3: How Long Does It Take to Perform the Wave?

Table 5.3.3 Change in Time

Number of Students	Time (sec)	Change in Time
3	4	
6	8	4
9	13	5
12	17	4
15	20	3
18	24	4
21	27	3
24	30	3

Investigation 5.3: How Long Does It Take to Perform the Wave?

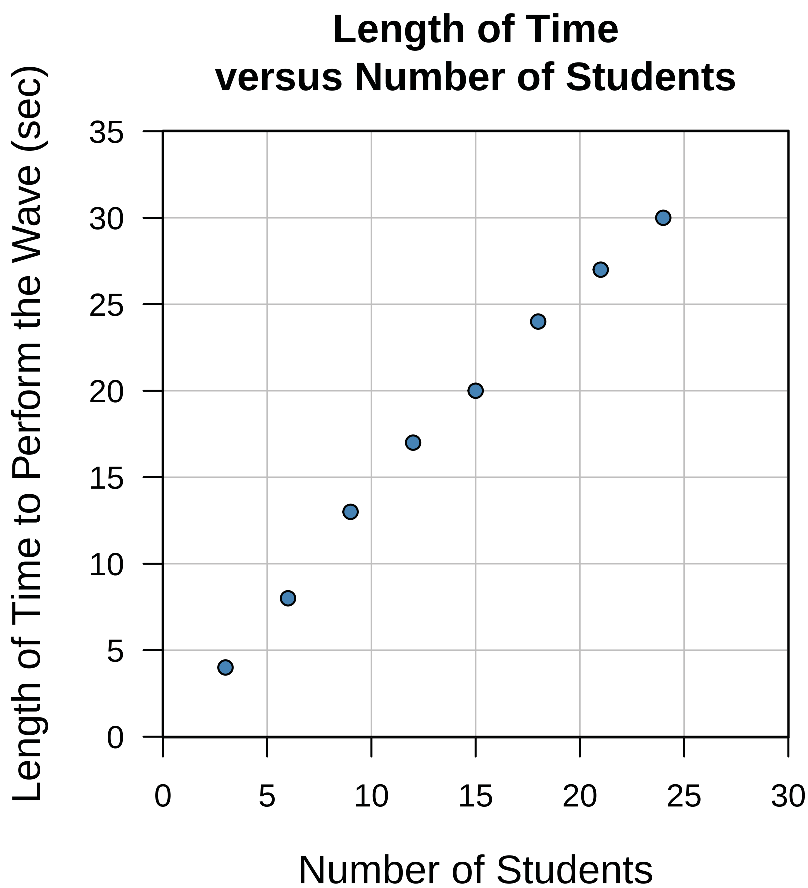


Figure 5.3.1 Scatterplot of length of time versus number of students

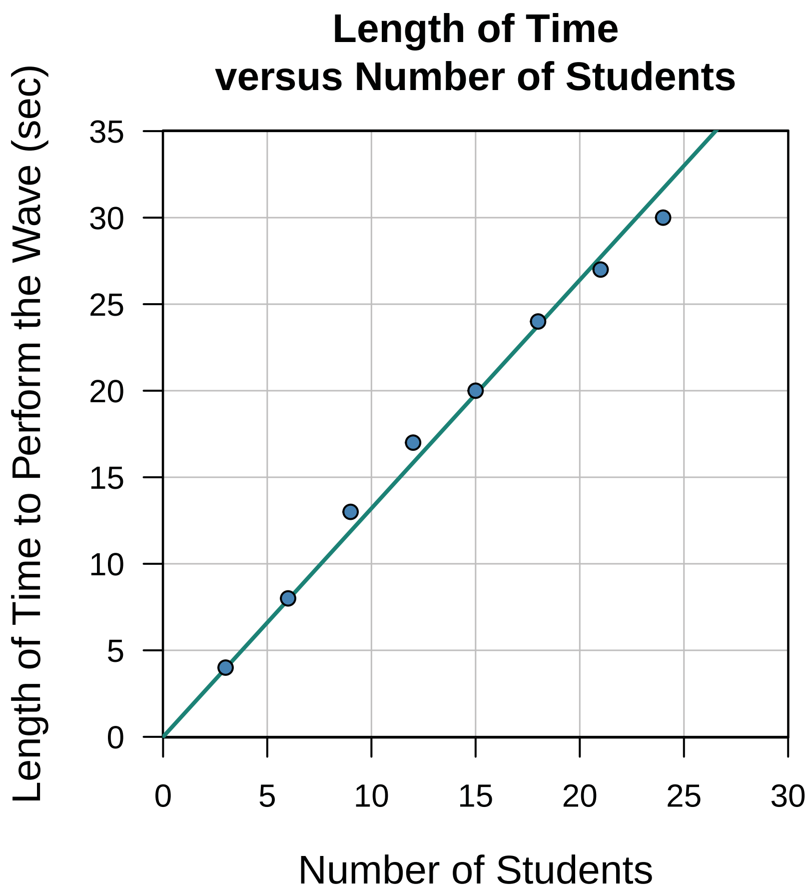


Figure 5.3.2 Scatterplot with line drawn through the (0,0)

Name: ______________________________ Date: ______________

Investigation 5.4: How Do Events Change Over Time?

Find the mean gross receipts of the top three movies for each of the years.

Activity Sheet

Year	First	Second	Third	Mean
2010	415.0 Toy Story 3	334.2 Alice in Wonderland	312.1 Iron Man 2	
2009	760.5 Avatar	402.1 Transformers: Revenge of the Fallen	302.0 Harry Potter and the Half-Blood Prince	
2008	533.3 The Dark Knight	318.3 Iron Man	317.0 Indiana Jones and the Kingdom of the Crystal Skull	
2007	336.5 Spider-Man 3	320.7 Shrek the Third	318.8 Transformers	
2006	423.0 Pirates of the Caribbean: Dead Man's Chest	250.9 Night at the Museum	244.1 Cars	
2005	380.3 Star Wars: Episode III – Revenge of the Sith	291.7 The Chronicles of Narnia: The Lion, The Witch, and the Wardrobe	290.0 Harry Potter and the Goblet of Fire	
2004	436.5 Shrek 2	373.4 Spider-Man 2	370.3 The Passion of the Christ	
2003	377.0 The Lord of the Rings: The Return of the King	339.7 Finding Nemo	305.4 Pirates of the Caribbean: The Curse of the Black Pearl	
2002	403.7 Spider-Man	340.5 The Lord of the Rings: The Two Towers	310.7 Star Wars: Episode II – Attack of the Clones	
2001	317.6 Harry Potter and the Sorcerer's Stone	313.8 The Lord of the Rings: The Fellowship of the Ring	267.7 Shrek	
2000	260.0 How the Grinch Stole Christmas	233.6 Cast Away	215.4 Mission: Impossible II	

Name: ______________________________ Date: ______________

Investigation 5.4: How Do Events Change Over Time?

Assessment

A group of students was interested in answering the question, "By how much have the winning times of the past 15 Olympic Games men's 100-meter dash decreased? Table 5.4.6 shows the data they collected for the years 1952–2008.

Table 5.4.6 Winning Times for Men's Olympic 100-Meter Dash

Winning Times (seconds) - Olympic Games 100-Meter Dash – Men								
Year	1952	1956	1960	1964	1968	1972	1976	
Time	10.40	10.50	10.20	10.00	9.95	10.14	10.06	
Year	1980	1984	1988	1992	1996	2000	2004	2008
Time	10.25	9.99	9.92	9.96	9.84	9.87	9.85	9.69

Note that, in 1992, the original winner was Ben Johnson of Canada, who ran the dash in 9.79 s, but he was stripped of the medal after testing positive for steroid use. Figure 5.4.4 is a time series plot with the year on the horizontal axis and the dash time on the y-axis.

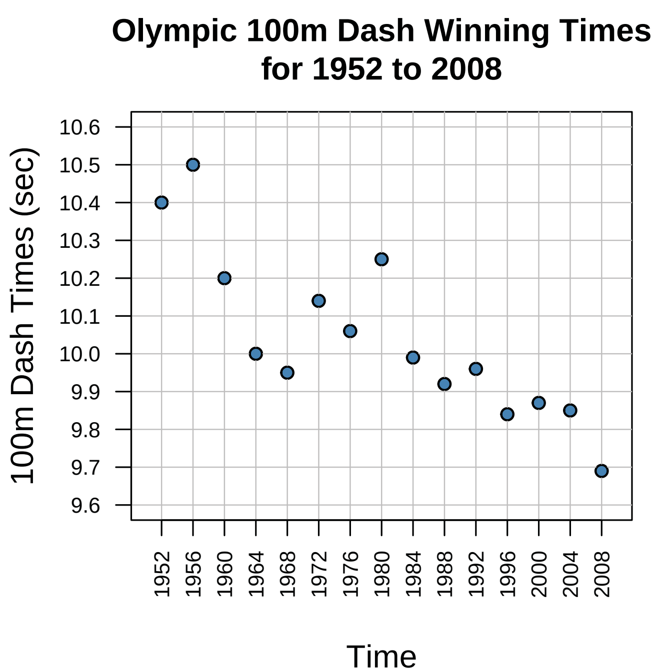


Figure 5.4.4 Time series plot of men's Olympic 100 m dash winning times for 1952–2008

Name: ________________________________ Date: ______________

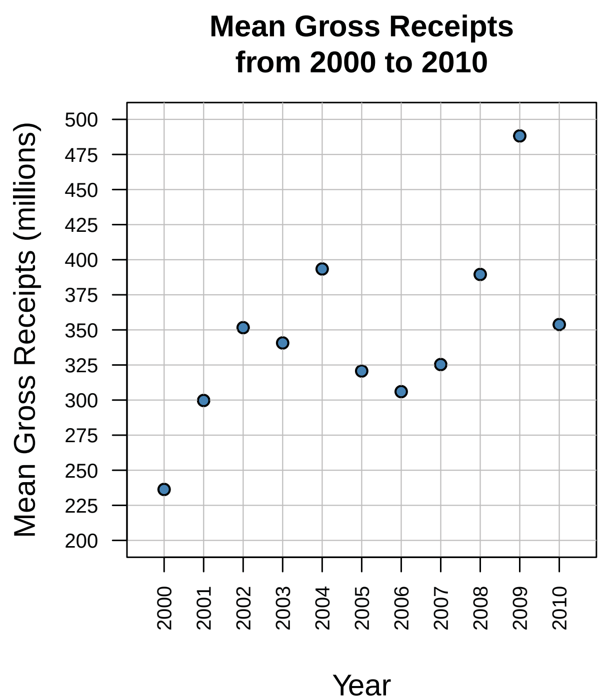
Investigation 5.4: How Do Events Change Over Time?

Write a report starting with answering the question, "By how much have the winning times of the past 15 Olympic Games men's 100-meter dash decreased?" Include the following:

- A description of the trend you observe in the data.
- Identification of the years in which the Olympic 100-meter time was higher than the previous Olympic 100-meter time.
- An appropriate graph with a line drawn through the points (1952, 10.4) and (2008, 9.69) and, by using these two points, the rate of change of the Olympic 100-meter times.
- A written explanation of what the rate of change of the times represents in the context of this investigation.

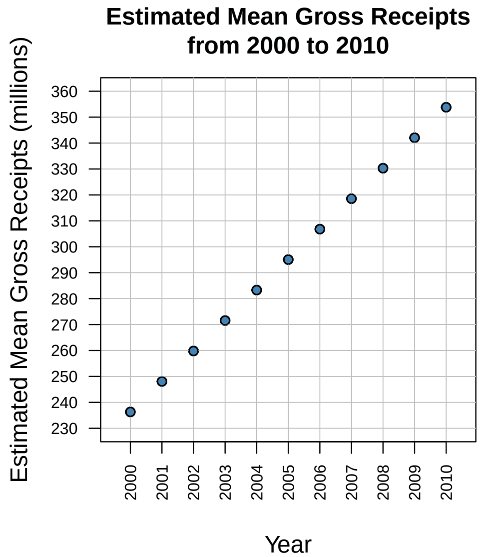
Example of 'Interpret the Results'

In our communications class last week, we were looking at old silent movies and comparing them to the high-tech ones of today. We were wondering how much movies make. We decided that a neat question to investigate in our mathematics class would be, "By how much, if any, are the average gross receipts for movies increasing over time?" From a website, we found a listing of gross receipts for movies year by year. We decided to look at the top three money-making films for the years 2000–2010 and then take the mean of the three to use as our data. The data are in millions of dollars by the way. To see if there was any relationship or trend, we drew a scatterplot, which is called a time series plot since time would be on the horizontal axis. Here is our plot:

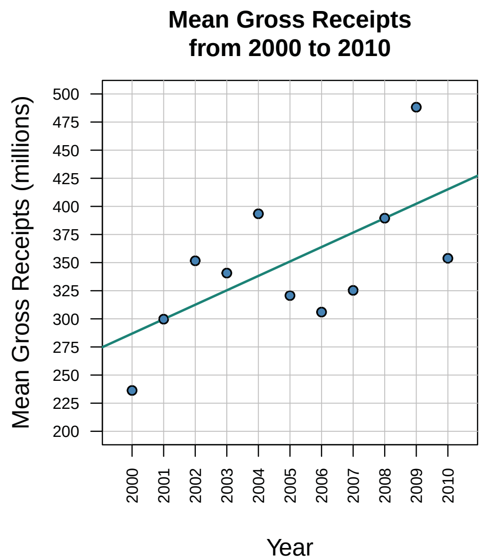


We concluded from the graph that there is a positive relationship between time and mean gross receipts. That means, as we look at years from 2000 going up to 2010, mean gross receipts for those years generally increase. Of course, in some years, receipts went down, but overall there was an upward trend. To see the ups and downs, we calculated them and then found their mean. The average change in mean gross receipts was \$11.75 million. In words, if the gross receipts changed a constant amount from year to year between 2000–2010, then that constant amount would be \$11.75 million.

So, we also looked at this by saying that if all our data points fell on a straight line exactly, then the slope of that line would be 11.75. Here's a graph of what that constant situation would look like:



But, of course, our real data did not fall on a straight line. So, as a final part of our analysis, we looked at our original data in its plot and drew a line through the data that we thought would fit the data pretty well. We went through the points and picked on (2001, 299.7) and (2008, 389.5). Here is the plot with our prediction line on it:



The slope for our estimated real data line was (389.5 – 299.7) / (2008 – 2001) = $12.83 million. It is higher than the average one of $11.75 million. It's kind of hard to say which method is right. The constant method averaged over the ups and downs, which smoothed things over. The picking two points method is very dependent on which points were chosen, but we think we did a good job because, looking at the graph, the line balances the points fairly well. So, we like our two-point method better.

To use our line to predict what mean gross receipts might be in 2011, we see from the graph that a prediction would be around $425 million. To be more correct, our slope is $12.83 increase per year. We know that (2008,389.5) lies on our line. Since 2011 is three years from 2008, our prediction for 2011 is 389.5 + 3*12.83 = $427.99 million.

We checked the website and found that the actual top movies in 2011 grossed a mean of $337.90, considerably less than our prediction. One reason is that the economy is not very good and people don't have as much money to spend on going out.

Investigation 5.4: How Do Events Change Over Time?

Table 5.4.1 Top Three Money-Making Movies for Years 2000–2010 in Millions of Dollars

Year	First	Second	Third	Mean
2010	415.0 Toy Story 3	334.2 Alice in Wonderland	312.1 Iron Man 2	353.8
2009	760.5 Avatar	402.1 Transformers: Revenge of the Fallen	302.0 Harry Potter and the Half-Blood Prince	488.2
2008	533.3 The Dark Knight	318.3 Iron Man	317.0 Indiana Jones and the Kingdom of the Crystal Skull	389.5
2007	336.5 Spider-Man 3	320.7 Shrek the Third	318.8 Transformers	325.3
2006	423.0 Pirates of the Caribbean: Dead Man's Chest	250.9 Night at the Museum	244.1 Cars	306.0
2005	380.3 Star Wars: Episode III – Revenge of the Sith	291.7 The Chronicles of Narnia: The Lion, The Witch, and the Wardrobe	290.0 Harry Potter and the Goblet of Fire	320.7
2004	436.5 Shrek 2	373.4 Spider-Man 2	370.3 The Passion of the Christ	393.4
2003	377.0 The Lord of the Rings: The Return of the King	339.7 Finding Nemo	305.4 Pirates of the Caribbean: The Curse of the Black Pearl	340.7
2002	403.7 Spider-Man	340.5 The Lord of the Rings: The Two Towers	310.7 Star Wars: Episode II – Attack of the Clones	351.6
2001	317.6 Harry Potter and the Sorcerer's Stone	313.8 The Lord of the Rings: The Fellowship of the Ring	267.7 Shrek	299.7
2000	260.0 How the Grinch Stole Christmas	233.6 Cast Away	215.4 Mission: Impossible II	236.3

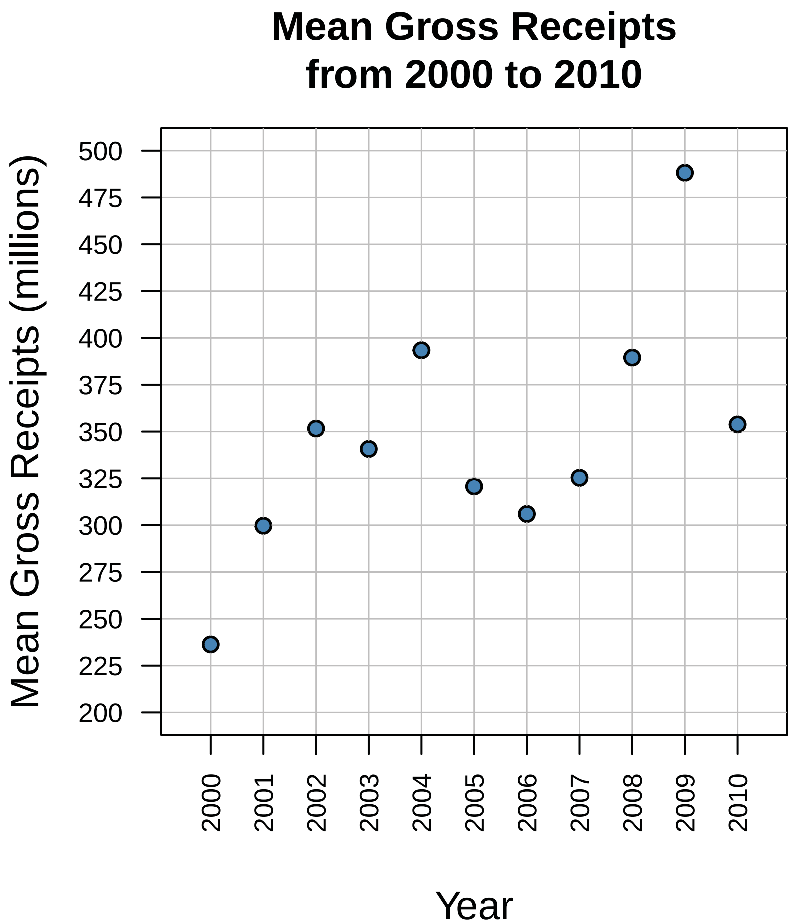


Figure 5.4.1 Time series plot of mean gross receipts (in millions) from 2000–2010

Name: ______________________ Date: ______________

Investigation 5.4: How Do Events Change Over Time?

Table 5.4.2 Change in Mean Gross Receipts

	2000–2001	**2001–2002**	**2002–2003**	**2003–2004**	**2004–2005**
Mean Change (millions)	+63.4				
	2005–2006	**2006–2007**	**2007–2008**	**2008–2009**	**2009–2010**
Mean Change (millions)					-134.4

Table 5.4.3 Change in Mean Gross Receipts (Completed Table)

	2000–2001	2001–2002	2002–2003	2003–2004	2004–2005
Mean Change (millions)	+63.4	+51.9	-10.9	+52.7	-72.7
	2005–2006	**2006–2007**	**2007–2008**	**2008–2009**	**2009–2010**
Mean Change (millions)	-14.7	+19.3	+64.2	+98.7	-134.4

Name: ______________________________ Date: ______________

Investigation 5.4: How Do Events Change Over Time?

Table 5.4.4 Estimated Mean Gross Receipts (Assuming a Constant Increase)

Estimated Mean Gross Receipts (Assuming a Constant Increase in Receipts from Year to Year)						
Year	2000	2001	2002	2003	2004	2005
Time	236.3					
Year	2006	2007	2008	2009	2010	
Time						

Table 5.4.5 Complete Table of the Estimated Mean Gross Receipts (Assuming a Constant Increase)

Estimated Mean Gross Receipts (Assuming a Constant Increase in Receipts from Year to Year)						
Year	2000	2001	2002	2003	2004	2005
Time	236.3	248.05	259.8	271.55	283.3	295.05
Year	2006	2007	2008	2009	2010	
Time	306.8	318.55	330.3	342.05	353.8	

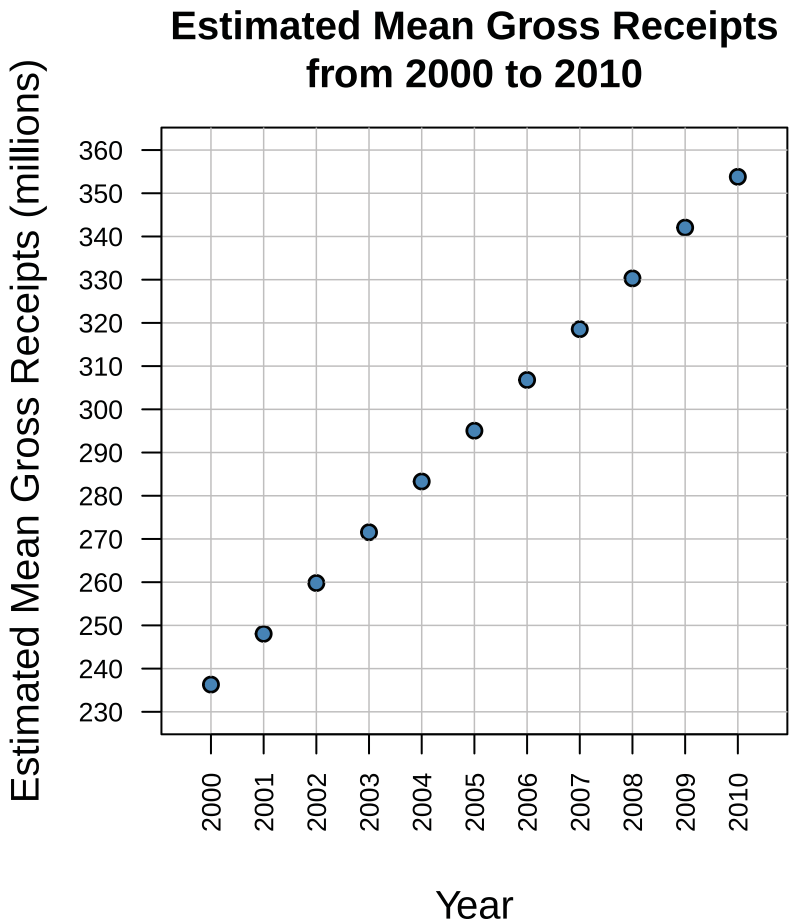


Figure 5.4.2 Time series plot of estimated mean gross receipts for 2000–2010 (assuming a constant change)

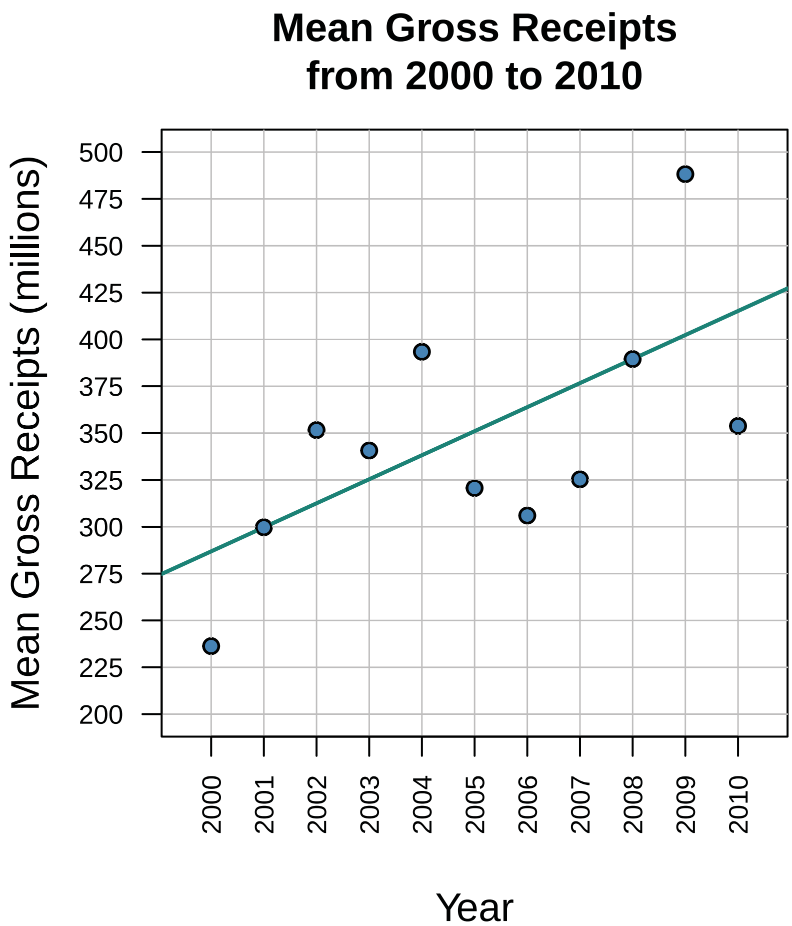


Figure 5.4.3 Time series plot of estimated mean gross receipts from 2000–2010 with trend line

Name: ______________________________ Date: ______________

Investigation 6.1: How Likely Is It?

Place each event listed in Table 6.6.1 under impossible, unlikely, neither unlikely nor likely, likely, or certain.

Table 6.1.1 Chance Events

Classify each of these chance events as being impossible to occur, unlikely to occur, neither unlikely nor likely to occur, likely to occur, or certain to occur.

a. The class will watch TV in school today.

b. We will all use computers sometime today.

c. We will have lunch today.

d. The class will be in school on Saturday.

e. The class will go to the movies this week.

f. We will go outside for recess today.

g. If the teacher were to put the names of all the students in our class in a hat and draw one name, a boy's name will be chosen.

h. If I have a bag of 10 blue cubes and one red cube and draw one cube, the red cube will be drawn.

Impossible	**Unlikely**	**Neither Unlikely nor Likely**	**Likely**	**Certain**

Name: ______________________________ Date: ______________

Investigation 6.1: How Likely Is It?

Assessment

1. Think about each of the following events. Decide where each event would be located on the scale below. Place the letter for each event below on the appropriate place on the scale.

   What are the chances for each event?

   A. The next roll of a fair number cube will be a 2.

   B. You will be successful in four of your next 10 free throw shots.

   C. You will meet a dinosaur on your way home from school.

   D. You will read at least three books this month.

   E. A coin will come up heads five times in a row.

   F. A word chosen randomly from this sentence has four letters.

   G. It will be sunny tomorrow.

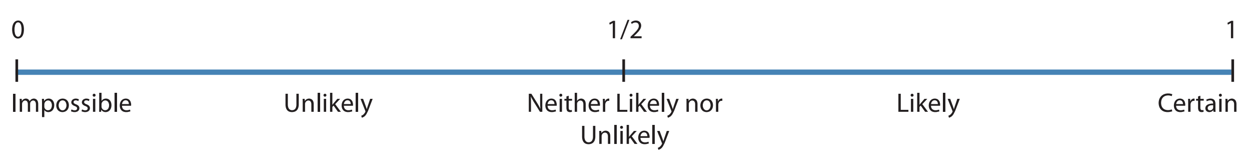
   H. You will eat something the color blue today.

   I. A spinner with 10 equal parts numbered 1 through 10 will come up an even number in the next spin.

   J. You will have math homework tonight.

   K. If the names of all the teachers at our school are in a hat, my teacher's name will be picked.

Probability Scale



Name: ______________________________ Date: ______________

Investigation 6.1: How Likely Is It?

2. Write two events that are impossible to occur, two that are unlikely to occur, two that are neither unlikely nor likely to occur, two that are likely to occur, and two that are certain to occur. Give reasons for your answers.

Example of 'Interpret the Results'

Our teacher asked us to give him examples of things that are impossible for us to do. Impossible means that the event cannot happen. Some of the events we said were hit a baseball 500 feet in the air, fly like a bird, and run 100 miles an hour. He also asked us for events we are certain will happen. These are events that have to happen. Some of us said the sun will rise in the east, I will sit at my school desk today, and I will eat lunch in the lunchroom today. We also talked about events that we were not certain of happening. We said it was likely that we would eat a dessert today and drink milk today, but we said that eating something blue was unlikely because we didn't think it would happen, but it might if someone ate a blue sucker.

After we assigned a word to each of the chance events as to how often we thought they would happen, our teacher had us assign numbers to the events. These numbers are called probabilities. A probability for a chance event is how likely the event will occur. The probability numbers go from 0, which means impossible, to 1, which means certain. So, we assigned 0 to the event that we could run 100 miles an hour, and we assigned 1 to the event the sun will rise in the east. We didn't assign a number to the events we thought were unlikely, but we suggested that they would be between 0 and the middle. The likely ones would be between the middle and 1. Our teacher told us that the halfway number between 0 and 1 is the fraction ½, and it would mean neither unlikely nor likely.

Our teacher put tape on the floor that showed 0, ½, and 1 spread out. All of us in the class stood on the "walk-on probability scale" to show how likely we thought the event "I can jump a jump rope 20 times in a row" would be. It was neat to see that some of us didn't think we could do it. They were down toward 0 and others were spread out between 0 and 1. I was pretty sure I could do it, so I stood about halfway between ½ and 1 on the likely part.

Investigation 6.1: How Likely Is It?

Table 6.1.1 Chance Events

Classify each of these chance events as being impossible to occur, unlikely to occur, neither unlikely nor likely to occur, likely to occur, or certain to occur.

a. The class will watch TV in school today.

b. We will all use computers sometime today.

c. We will have lunch today.

d. The class will be in school on Saturday.

e. The class will go to the movies this week.

f. We will go outside for recess today.

g. If the teacher were to put the names of all the students in our class in a hat and draw one name, a boy's name will be chosen.

h. If I have a bag of 10 blue cubes and one red cube and draw one cube, the red cube will be drawn.

Impossible	Unlikely	Neither Unlikely nor Likely	Likely	Certain
Saturday school	Red cube drawn	Draw boy's name	Watch TV	Use computers
Go to movie			Recess outside	Lunch

Figure 6.1.1 Chance events

Name: ______________________ Date: ______________

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

Tally Chart/Frequency Table

Name of Animal	Tally	Count/Frequency
Bear		
Camel		
Elephant		
Lion		
Monkey		
Mountain Goat		
Owl		
Penguin		
Rabbit		
Rhinoceros		
Tortoise		
Zebra		

Name: ______________________ Date: ______________

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

Assessment

Chris sorted a bag of animal crackers and drew the bar graph shown in figure 6.2.4.

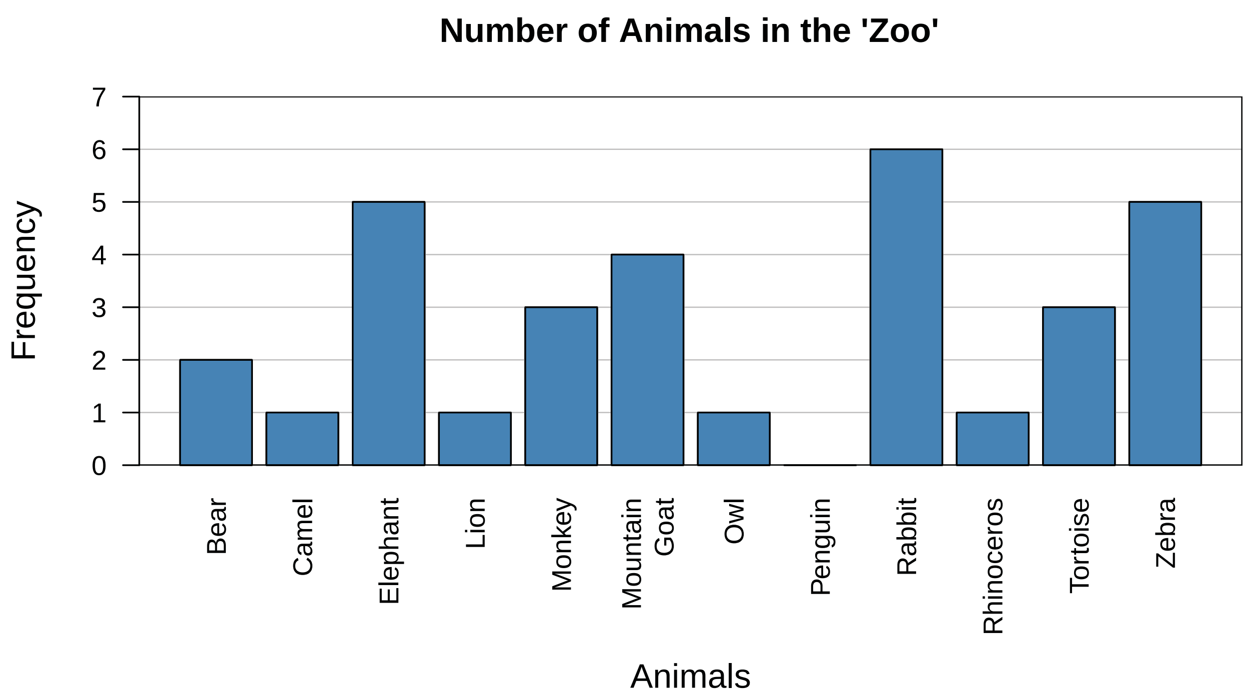


Figure 6.2.4 Bar graph of Chris's zoo

1. How many elephants were in Chris's bag?

2. How many rabbits were in Chris's bag?

3. How many more rabbits were in the bag than penguins?

4. How many more rabbits were in the bag than monkeys?

Name: ______________________________ Date: ______________

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

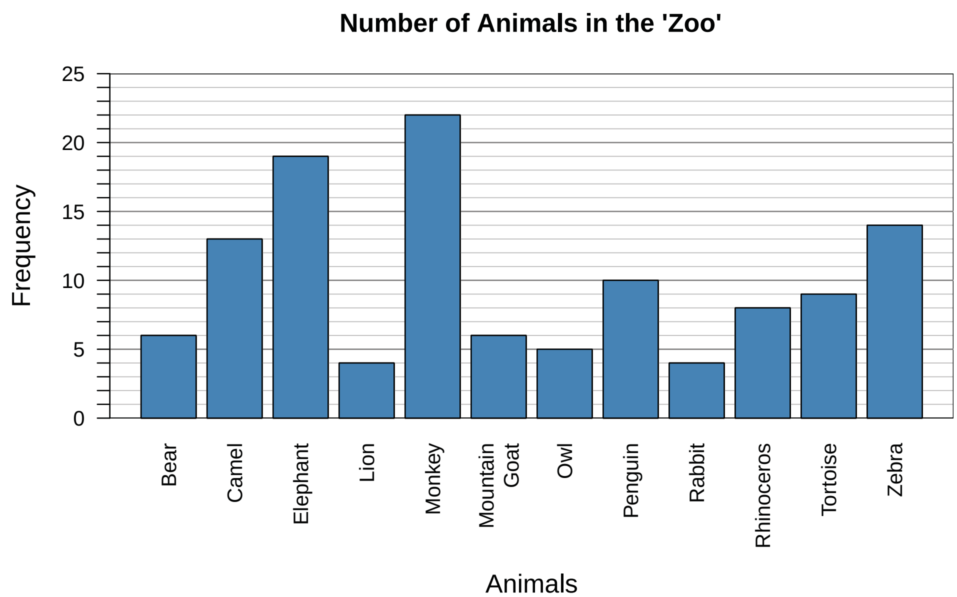
5. If you reach into Chris's bag and randomly picked out one animal, which animal would:

   Most likely be chosen?

   Least likely be chosen?

Example of 'Interpret the Results'

Our teacher talked to us about animals in a zoo. We made a list and then our teacher handed out a bag of animal crackers that represented our zoo. We worked in pairs. We investigated the question, "If an animal were to be chosen at random from a bag of zoo animals, which type of animal would be the most likely or least likely to be chosen?" We made a tally chart of the animals in our bag and then made a bar graph. Based on our bar graphs, we decided which animal was most likely by looking at the heights of the bars. The animals that had the highest bars were the ones we thought would be the most likely to be chosen. Also, the animal that occurred the most is called the mode. We combined the results from all the groups. The following bar graph shows our class results:



Our class data showed that the monkey was the most likely to be chosen, with the elephant also a good possibility. So the monkey is the mode animal, while the lion, owl, and rabbit were the least likely animals to be chosen. Also, we concluded that Kellogg's does not bake the same number of each animal. If they did, the bars would be more even. Statistics is kind of fun because we were allowed to eat our data when we were done.

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

Figure 6.2.1 Twelve animals in Austin Zoo Animal Crackers

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

Table 6.2.1 Example of a Frequency Distribution of a 'Zoo'

<table>
<tr><th>Name of Animal</th><th>Tally</th><th>Count/Frequency</th></tr>
<tr><td>Bear</td><td>|</td><td>1</td></tr>
<tr><td>Camel</td><td>||</td><td>2</td></tr>
<tr><td>Elephant</td><td>||||</td><td>4</td></tr>
<tr><td>Lion</td><td>|</td><td>1</td></tr>
<tr><td>Monkey</td><td>~~||||~~ |</td><td>6</td></tr>
<tr><td>Mountain Goat</td><td>||</td><td>2</td></tr>
<tr><td>Owl</td><td>||</td><td>2</td></tr>
<tr><td>Penguin</td><td>|||</td><td>3</td></tr>
<tr><td>Rabbit</td><td></td><td>0</td></tr>
<tr><td>Rhinoceros</td><td>||</td><td>2</td></tr>
<tr><td>Tortoise</td><td>|||</td><td>3</td></tr>
<tr><td>Zebra</td><td>||||</td><td>4</td></tr>
</table>

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

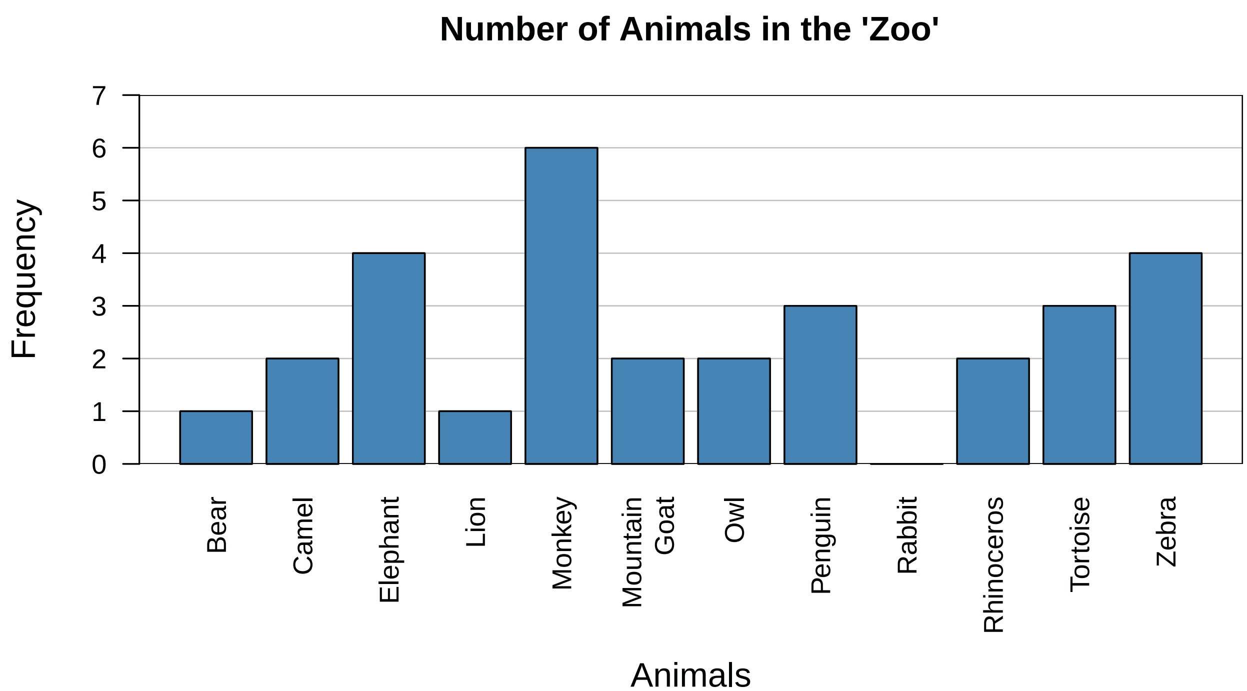


Figure 6.2.2 Bar graph of the example zoo

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

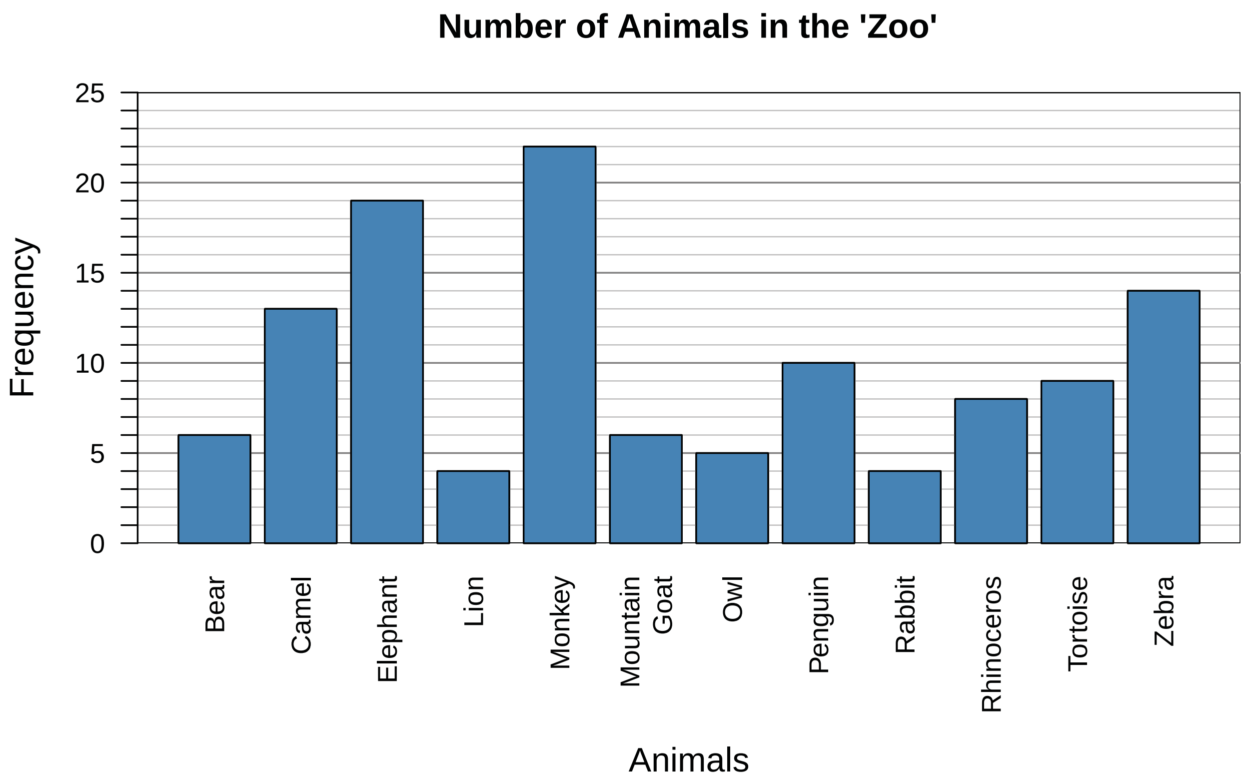


Figure 6.2.3 Bar graph of sample class zoo

Investigation 6.2: What's the Chance of Seeing an Elephant at the Zoo?

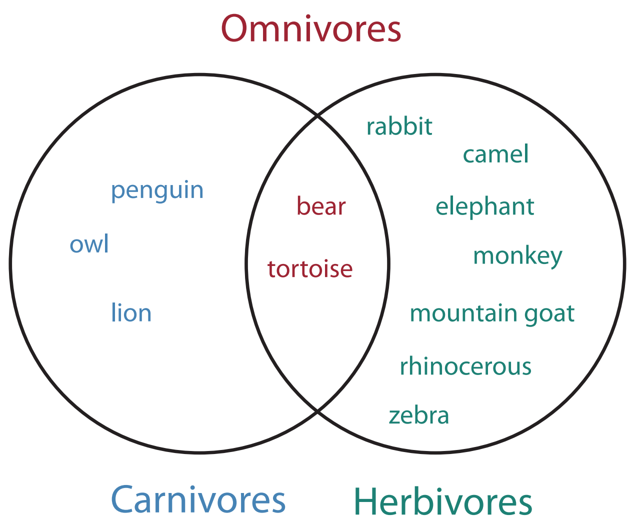


Figure 6.2.5 Venn diagram of carnivores, herbivores, and omnivores

Name: ______________________________ Date: ______________

Investigation 6.3: What Do Frogs Eat?

Recording Form

Bag Number ________	
Trial Number	**Results**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Name: ______________________________ Date: ______________

Investigation 6.3: What Do Frogs Eat?

Table 6.3.2 Frequency Table of Experimental Trial Results Drawn from Bag 1

Bag Number 1		
Color	**Frequency**	**Relative Frequency**
Blue		
Green		
Yellow		
TOTAL		

Name: ________________________________ Date: ________________

Investigation 6.3: What Do Frogs Eat?

Assessment

One group of students drew a cube from each of three bags that were labeled Bag 1 Marsh, Bag 2 Stream, Bag 3 Tropical Garden. They repeated the drawing 20 times for each bag. They got the results shown in Table 6.3.3.

Table 6.3.3 Results of 20 Draws from 3 Bags

Color	Count for Bag 1 (Marsh)	Color	Count for Bag 2 (Stream)	Color	Count for Bag 3 (Tropical Garden)
Blue	9	Blue	5	Blue	5
Green	5	Green	7	Green	9
Yellow	6	Yellow	8	Yellow	6
TOTAL	**20**	**TOTAL**	**20**	**TOTAL**	**20**

1. Complete the following table by converting each color count into relative frequency.

Color	Count for Bag 1	Rel. Freq.	Color	Count for Bag 2	Rel. Freq.	Color	Count for Bag 3	Rel. Freq.
Blue	9		Blue	5		Blue	5	
Green	5		Green	7		Green	9	
Yellow	6		Yellow	8		Yellow	6	
TOTAL	**20**		**TOTAL**	**20**		**TOTAL**	**20**	

2. Draw a bar graph of the results for each of the three bags. Be sure to include a title for the graph and label the axes.

Name: ______________________________ Date: ______________

Investigation 6.3: What Do Frogs Eat?

3. If you reach into the student's first bag and randomly choose a single cube, which color would you most likely choose? Explain your answer.

4. If you reach into the student's second bag and randomly choose a single cube, which color would you most likely choose? Explain your answer.

5. If you reach into the student's third bag and randomly choose a single cube, which color would you most likely choose? Explain your answer.

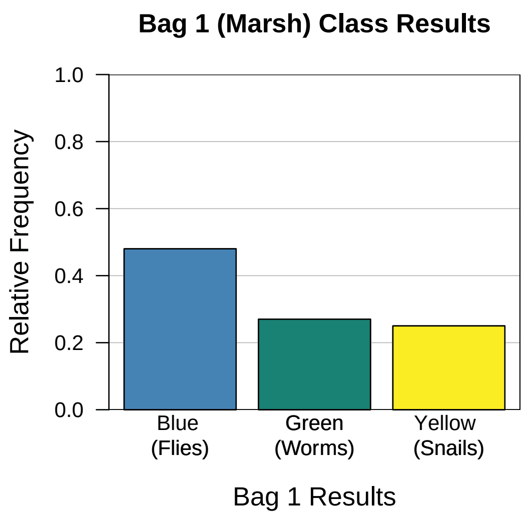
6. Based on these sample results, what do you think the proportion of blue, green, and yellow cubes in each of the three bags is? Explain your answer.

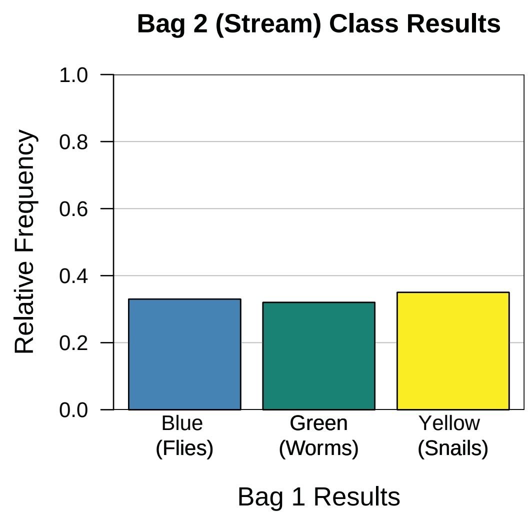
Example of 'Interpret the Results'

Our biology and mathematics teachers must have gotten together and decided to give us a statistical problem involving trying to determine the distribution of food types for different habitats in which frogs live. The food types that frogs eat are flies, worms, and snails and the frog habitats we used were marshes, streams, and tropical gardens.

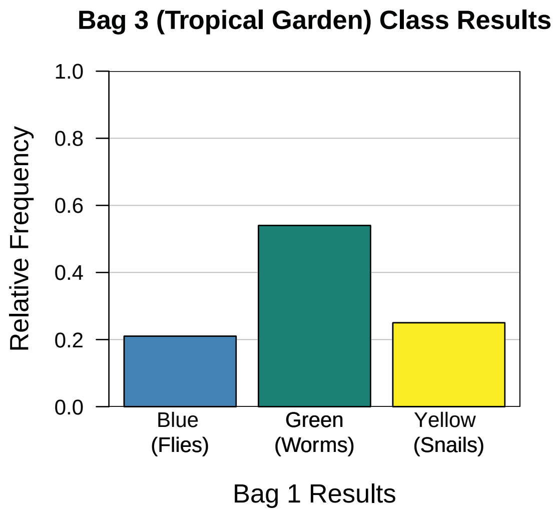
To collect data, flies were represented by blue cubes, worms by green ones, and snails by yellow. The habitats were paper bags labeled 1, 2, and 3. The question we investigated was, "Is the probability of choosing blue, green, yellow cubes different from bag to bag?" Or, in terms of the frog scenario, "Does habitat have any influence on a frog's food choice?"

We worked in groups of four and each group was given three bags. The bags had different proportions of colored cubes in them. We were not allowed to look inside the bags. To determine experimental probabilities for the food types per bag, each of our groups randomly selected a food type from a bag, wrote it down, replaced the cube, and did it 19 more times. Each time was called a trial. We shook the bag a lot each time so we didn't bias the choices. To get a better idea of what each bag (habitat population) had in it, we put all of our group results together and drew these bar graphs, one for each habitat.





Investigation 6.3: What Do Frogs Eat?



We decided that Blue (flies) was the most likely pick from Bag 1 (marsh habitat) and that the other two foods, worms and snails, were about the same. So we thought the distribution in marshes would be 45% flies, 27.5% worms, and 27.5% snails.

It turned out that Bag 1 contained 2 blue, 1, green, and 1 yellow, so the population proportions were actually 50%, 25%, 25%. Our experimental results were pretty close.

In the stream habitat, our experimental results indicated that all three food types were equal. We were right because Bag 2, the stream habitat, contained one of each of the colors, so the distribution of food types is 33 1/3% each.

For Bag 3, the tropical garden habitat, our bar graph of experimental results looked like it would have been determined from a 1-2-1 distribution (i.e., 25% flies, 50% worms, and 25% snails). We were right on that one, too. By the way, our individual group results were not really close to the actual bag proportions, but we learned that getting a larger data collection by putting all our group results together brought us much closer to the right answers. It was neat to see that biology and statistics go together.

Table 6.3.2 Example of a Frequency Table of Experimental Trial Results Drawn from Bag 1

Bag Number 1		
Color	**Frequency**	**Relative Frequency**
Blue	8	
Green	5	
Yellow	7	
TOTAL	**20**	

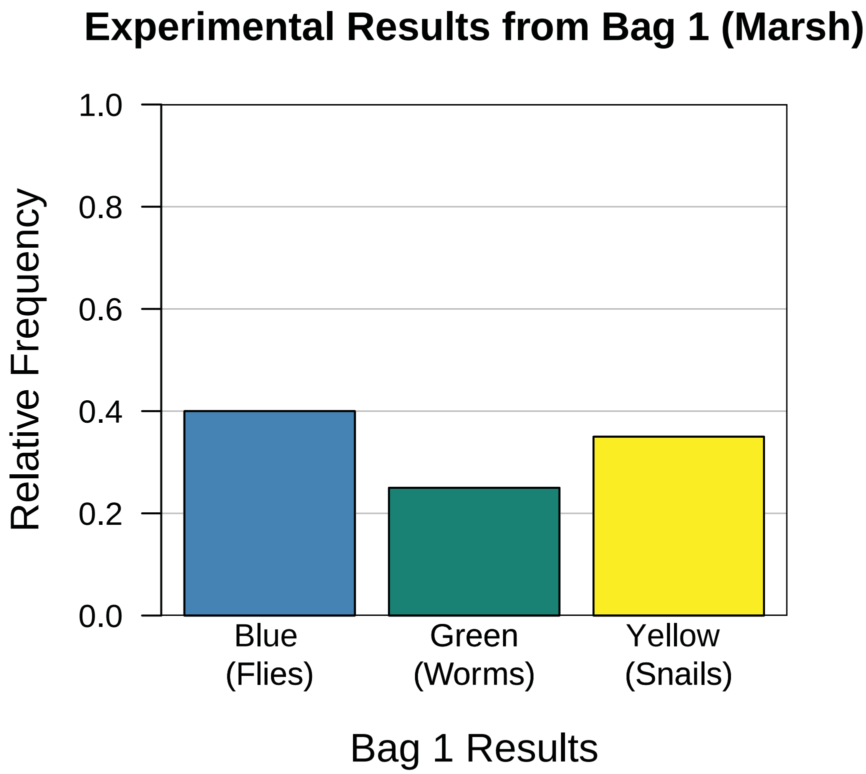


Figure 6.3.1 Bar graph of example experimental results drawn from Bag 1 (Marsh)

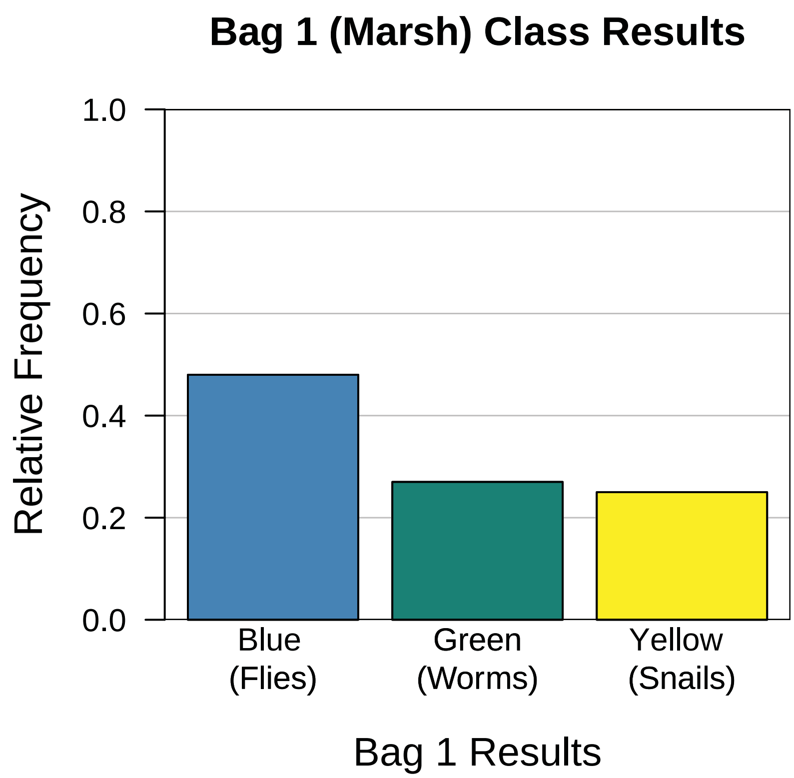


Figure 6.3.2 Bar graph of sample class data for Bag 1 (Marsh)

Name: ______________________________ Date: ______________

Investigation 6.4: How Many Spins to Win the Prize?

Recording Sheet

Spin Number	Color
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

Name: ______________________ Date: ______________

Investigation 6.4: How Many Spins to Win the Prize?

Worksheet: Probabilities Based on the 100 Simulations

Number of Spins Needed to Win (n)	P(n)	P(n or more)
25		
24		
23		
22		
21		
20		
19		
18		
17		
16		
15		
14		
13		
12		
11		
10		
9		
8		
7		
6		
5		
4		
3		
2		
1		

Name: ______________________ Date: ______________

Investigation 6.4: How Many Spins to Win the Prize?

Assessment

A carnival game used a spinner with five equal sections (Figure 6.4.4). A person won a prize if the spinner stopped on the yellow section. One hundred students each played the game until they won a prize. This means they each kept spinning the spinner until it stopped on yellow. Figure 6.4.5 is a dotplot of how many spins it took each of the 100 students to win a prize.

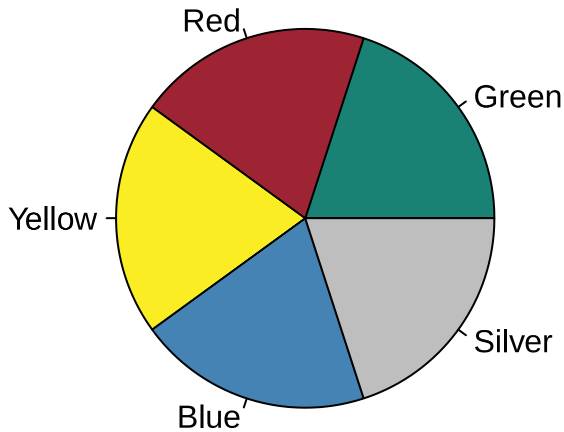


Figure 6.4.4 Spinner

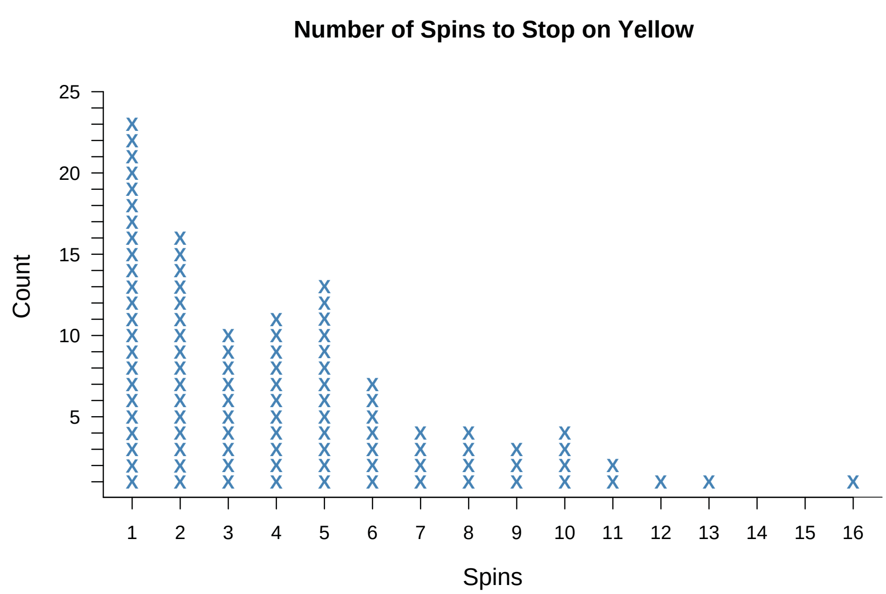


Figure 6.4.5 Dotplot of number of spins needed to stop on yellow

Name: ______________________________ Date: ______________

Investigation 6.4: How Many Spins to Win the Prize?

1. Describe the distribution of the number of spins to stop on yellow.

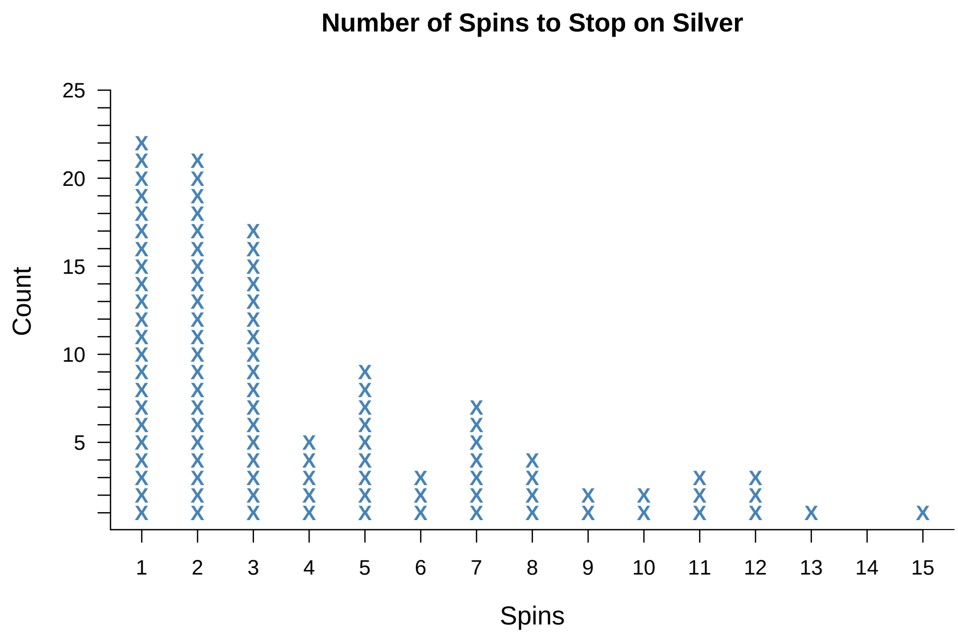
2. Estimate the center of the distribution and explain what this value would represent.

3. Find the probability of each of the following:
   - P(exactly 8 spins) =
   - P(exactly 3 spins) =
   - P(11 or more spins) =
   - P(1 or more spins) =

4. Andrea, a sixth-grade student, played the game. Use words, numbers, and/or drawings to explain how many times you think it would take Andrea to play the game.

Example of 'Interpret the Results'

We were given the Winning a Silver Car scenario, in which a girl named Sarah wanted to win a silver car by spinning a spinner that had four equal sections. One section was silver, and she had to use a ticket for each spin. We wanted to find out how many spins it would take for Sarah to win a silver car. To help answer this question, we played the game by spinning the spinner and recording how many spins it took before the spinner stopped on silver. As we played the game, we recorded on a dotplot how many spins it took to stop on silver. We played the game 100 times, and the results are shown in the following dotplot.



We used our class data and found that the median of our number of spins needed to win was three. We also estimated the mean to be about four. We next calculated the probability of winning for each number of spins and the probabilities for each number of spins or more. Based on the table of probabilities, we thought Sarah would need to buy about seven tickets to win a silver car. About 75% of the time, our dotplot showed Sarah would win if she bought seven or fewer tickets.

Winning a Silver Car

At the school carnival, there is a game in which students spin a large spinner. The spinner has four equal sections: silver, green, blue, and red. Each section represents the color of a toy car that can be won. To play the game, Sarah has to buy some tickets at the ticket booth. She needs one ticket each time she spins the spinner. She also wants to win a silver toy car. If the spinner stops on silver on her first spin, Sarah wins. If not, she has to spin the spinner until it stops on silver. So, she needs to decide how many tickets she should buy to play this game to win a silver toy car.

Investigation 6.4: How Many Spins to Win the Prize?

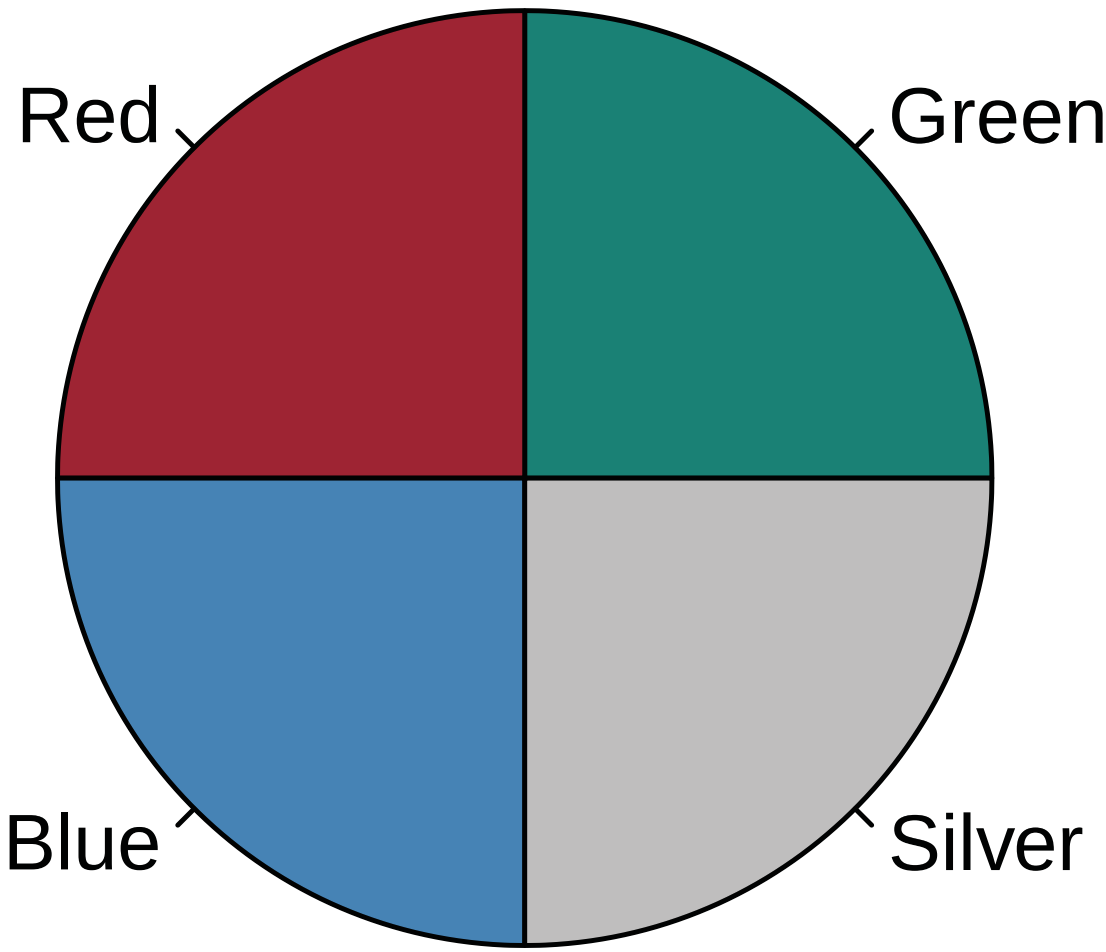


Figure 6.4.1 Spinner

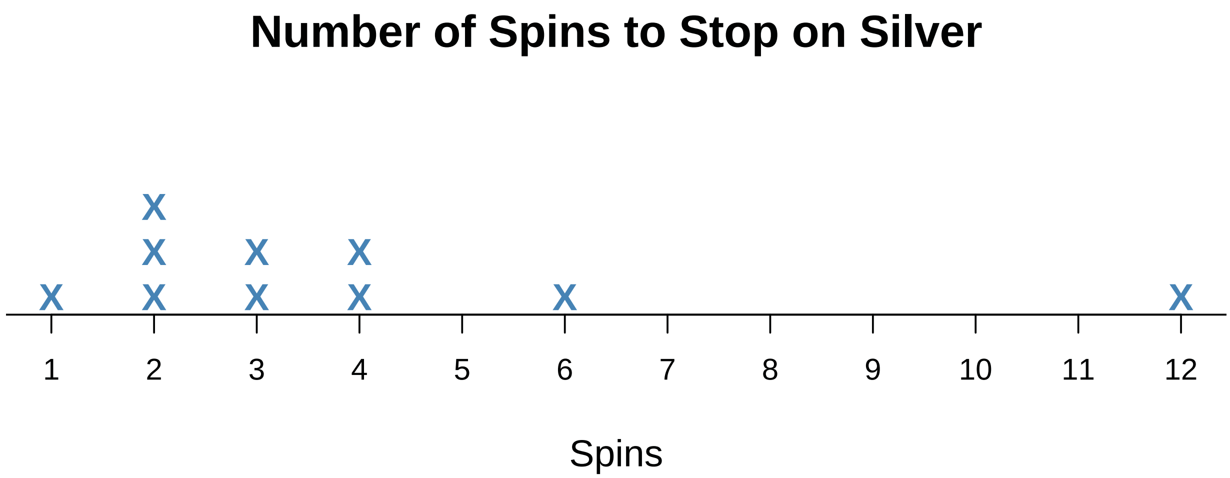


Figure 6.4.2 Dotplot of an example of class data

Investigation 6.4: How Many Spins to Win the Prize?

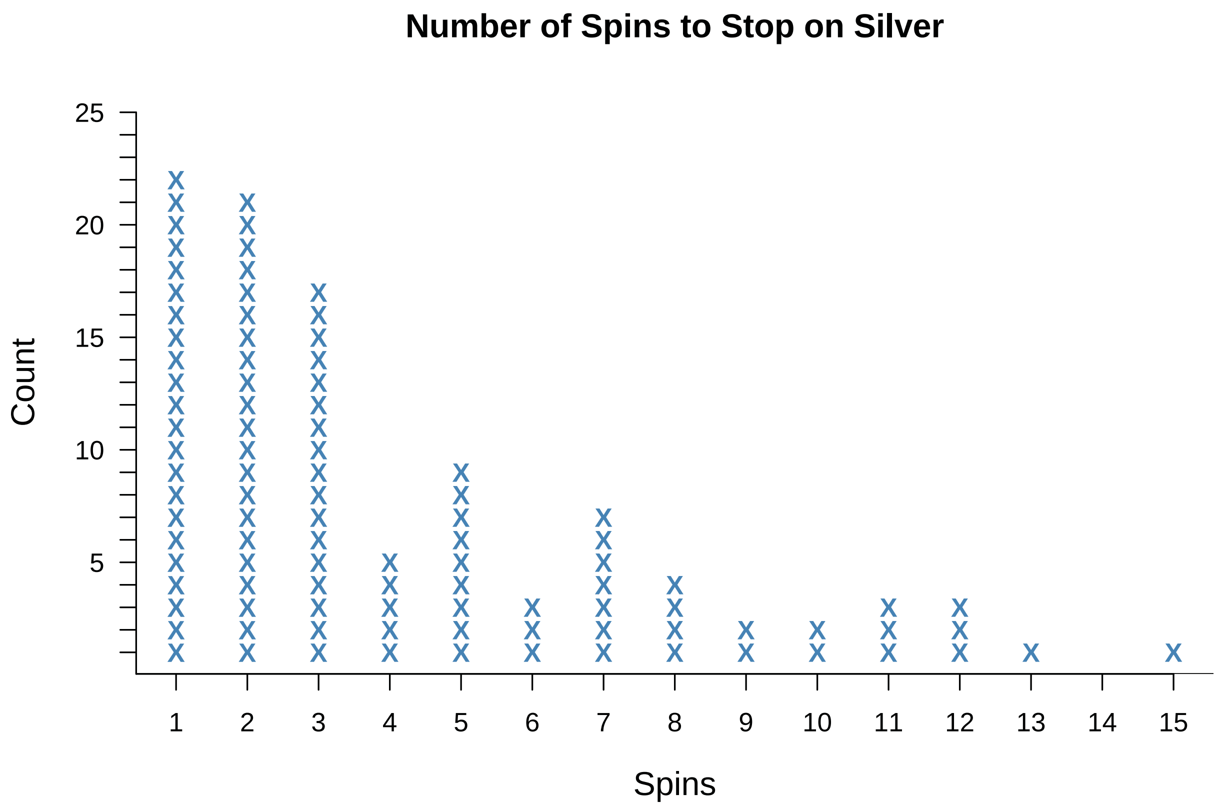


Figure 6.4.3 Dotplot of class data number of spins

Investigation 6.4: How Many Spins to Win the Prize?

Table 6.4.2 Probabilities Based on the 100 Simulations

Number of Spins Needed to Win (n)	P(n)	P(n or more)
15	.01	.01
14	.00	.01
13	.01	.02
12	.03	.05
11	.03	.08
10	.02	.10
9	.02	.12
8	.04	.16
7	.07	.23
6	.03	.26
5	.09	.35
4	.05	.40
3	.17	.57
2	.21	.78
1	.22	1.00