saw many comparisons such as all the 3-digit and 4-digit times were longer than 75% of the 2-digit times. The median of 3-digit exceeded all 2-digit. So, overall, it was clear that the times to sort the cards are longer as the number of digits in the numbers increases.

It was interesting that the medians (22.0, 25.8, and 28.3) were about the same as the means (22.1, 25.8, 28.7) even though the distributions had all those gaps. We guessed the possible outliers kind of balanced out the distributions. We checked to see if the outliers we saw in the stemplots were also outliers by the 1.5*IQR calculation for boxplots and none were. Different graphs illustrate different things. Finally, we compared the medians of the 2-digit and 3-digit groups by calculating how many common IQRs separated them. We used the maximum IQR of 2.5 for the value of the IQRs and saw that the medians 22.0 and 25.8 differed by (25.8 – 22.0)/2.5 = 1.5 IQRs. We don't really have a number to compare 1.5 to, but it seems to us that 1.5 IQRs is large enough to say the means differ from each other, since they are really separated when we look at the boxplots.

Assessment with Answers

A class of sixth-grade students conducted an experiment involving LEGO blocks to compare the effect of the type of directions provided to a student on the time needed to complete a task. The task was to build a tower from a given set of blocks. A bag of LEGO blocks contained one of the following three sets of directions:

> **Directions Set 1:** Construct a tower using all the blocks in this bag. The longest blocks should be on the bottom and go up in order to the shortest LEGO blocks at the top.
>
> **Directions Set 2:** Construct a tower using all the blocks in this bag according to the picture. (Figure 4.2.5)
>
> **Directions Set 3:** Build a tower with the blocks.

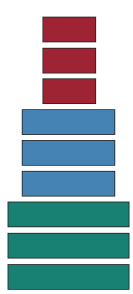


Figure 4.2.5 Diagram shown on directions for set 2

The class was randomly divided into three groups; the results of the experiment are shown in table 4.2.3.

Table 4.2.3 Time to Build Tower

Time (sec) to Build Tower with Directions for Set 1	Time (sec) to Build Tower with Directions for Set 2	Time (sec) to Build Tower with Directions for Set 3
18.1	22.1	11.6
17.5	21.3	15.5
16.3	18.9	15.4
18.8	19.5	15.6
16.2	20.1	15.3
16.0	21.0	15.7
16.6	19.4	13.8
14.8	16.5	16.1
18.1	22.7	15.9
19.8	19.1	16.8
17.6	21.6	14.3
16.5	20.0	12.9
16.7	20.0	17.0

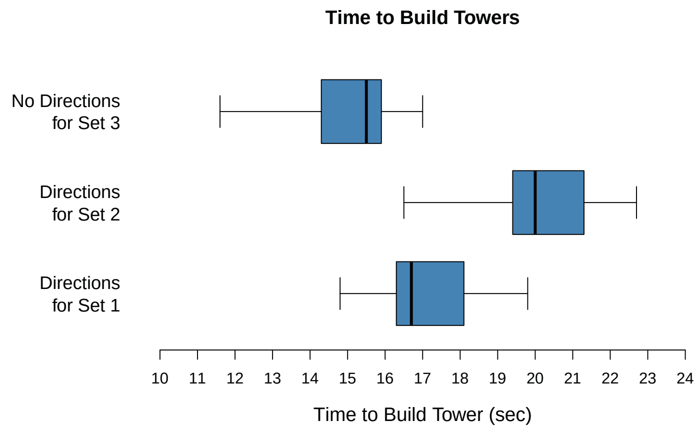
1. What is an appropriate statistical question in the context of this study?
   Does the average time it takes to build a tower with blocks vary with the type of directions given?

2. Find the mean for each group. Set 1 mean = 17.1 sec. Set 2 mean = 20.2 sec. Set 3 mean = 15.1 sec.

3. Find the five-number summary for each of the groups.

	Set 1	Set 2	Set 3
Minimum	14.8	16.5	11.6
Q1	16.3	19.3	14.1
Median	16.7	20.0	15.5
Q3	18.1	21.5	16.0
Maximum	19.8	22.7	17.0

4. Construct side-by-side boxplots of the three groups.



5. Which of the three groups was able to build the tower faster? Using words, numbers, and graphs, explain why you chose the group you did.
   Group 3 was able to build the tower the fastest. The median of this group is less than the other two. About 75% of the times for Group 3 are less than all of Group 2 times and 75% of Group 1.

Extension

1. Vary the background of the cards. Using a standard deck of playing cards, create three stacks. Each stack contains the cards ace to 10 with one stack having cards that are all of the same suit, one stack having cards from the two black suits, and one stack having mixed red and black suits. Students would investigate the statistical question, "Does the mixture of suits of cards relate to the amount of time needed to place the cards in order?"

2. Consider Step 11 of the Analysis of the original question in this investigation. Instead of calculating how many common IQRs separate two medians, the separation between means also can be calculated in terms of MADs. Note that the MADs for each group have to close in value so a common value can be determined. Ask your students to calculate mean absolute deviations for the three groups to see if any are similar and, if so, to do Step 11 using MAD in place of IQR.

References

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry, and R. Scheaffer. 2007. *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k–12 curriculum framework*. Alexandria, VA: American Statistical Association. *www.amstat.org/education/gaise*.

National Council of Teachers of Mathematics. 2000. *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Common Core State Standards for Mathematics. www.corestandards.org.

Investigation 4.3
How High Does a Ball Bounce?

Overview

This investigation focuses on students conducting an experiment to determine the bounce height two kinds of balls will reach when dropped from various heights. Students will collect data using a tennis ball and a golf ball that will be dropped from 30, 60, and 90 cm. They will display the data in a **scatterplot** and interpret the results to answer the statistical question, "Does the height from which a ball is dropped affect how high it bounces?"

Note: The Common Core State Standards do not specifically address measurement error, but this experiment has many areas in which error can occur and that can lead to increased variability within height groups. It could be a topic of extended discussion.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Pose investigative questions
- Design and conduct an experiment to investigate questions
- Collect data by conducting an experiment and organize the results in a scatterplot
- Recognize linear relationships and use that information to interpret the data

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

Common Core State Standards Grade Level Content

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.5 Summarize numerical data sets in relation to their context, such as by doing the following:

a. Reporting the number of observations

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

8.SP.1 Construct and interpret scatterplots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6–8 All students should find, use, and interpret measures of center and spread—including mean and interquartile range—and discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stemplots, boxplots, and scatterplots.

Materials

- Two types of balls that bounce well (e.g., tennis ball, golf ball, basketball, or table tennis ball)
- Meter sticks
- Calculators
- Recording sheet for each type of ball (available on the CD)
- Graph paper

Estimated Time

Two days

Instructional Plan

Formulate a Statistical Question

Note: This investigation will use a tennis ball and golf ball, but any two types of balls can be used.

Begin by holding a tennis ball and golf ball up for the students. Have them generate questions about the differences between a tennis ball and golf ball. The following are some of the questions students may come up with:

How are they made?

Which one weighs more?

What are they made out of?

Which ball could you throw farther?

Which ball bounces higher?

Have students generate their own statistical questions. This investigation focuses on two questions: "Does the height from which a ball is dropped affect how high it bounces?" and "Do tennis balls bounce higher, lower, or the same as golf balls?"

Collect Appropriate Data

1. Ask your students how they think the data should be collected.

2. Point out that to make comparisons, dropping balls must be done in the same way and onto the same type of surface (i.e., experimental conditions must be the same).

3. Following is the procedure for the experiment:

   a. Divide your students into groups of three. One person will drop the ball, a second will observe the height of the bounce, and a third will record the results in a table. Discuss why it would be beneficial to have the same student doing all the ball bouncing and the same student doing the measuring for all heights.

   b. Tape the meter stick to the wall with the 1 cm end on the floor and the 100 cm end at the top.

   c. Hold the tennis ball so that the bottom of the ball is at the 30 cm mark. Drop the ball; don't "throw" the ball down.

   d. Watch carefully to see how high it bounces back up. Record the height in Table 4.3.1. Repeat the drop two more times, recording each trial. Note that students may find it difficult to gauge the height accurately. The ball bounces back very fast. You may want to have two students watch the height of the bounce and compare their numbers. They need to agree or the drop is repeated.

   e. Next, drop the tennis ball from 60 cm three times. Record the height of each of the three bounces in Table 4.3.1. Repeat for a 90 cm height.

Table 4.3.1 Tennis Ball Recording Sheet

	Tennis Ball		
	30 cm	60 cm	90 cm
Trial 1			
Trial 2			
Trial 3			

4. Repeat the tennis ball procedure with a golf ball. Drop a golf ball three times from 30, 60, and 90 cm. Each time, record the height of the bounce. Record the data in a table similar to Table 4.3.2.

Table 4.3.2 Golf Ball Recording Sheet

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1			
Trial 2			
Trial 3			

5. Tables 4.3.3 and 4.3.4 contain data collected by a group of students.

Table 4.3.3 Example of Tennis Ball Bounce Height

	Tennis Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	16 cm	33 cm	50 cm
Trial 2	17 cm	32 cm	49 cm
Trial 3	16 cm	33 cm	49 cm

Table 4.3.4 Example of Golf Ball Bounce Height

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	19 cm	37 cm	55 cm
Trial 2	17 cm	32 cm	49 cm
Trial 3	16 cm	33 cm	49 cm

6. Ask your students what the variables are in this investigation. Students should realize that the first variable of interest is the type of ball (tennis versus golf ball) and the second variable of interest is the height of drop (30, 60, 90 cm).

7. Ask your students why they think they had to drop the balls from each height three times? Students should realize that by taking more measurements, the final heights could be more accurate.

Analyze the Data

1. Ask your students how the information gathered for each ball at each height can be consolidated. For example, we dropped the tennis ball from 30 cm three times. How can we determine a representative height for the bounce of the tennis ball from 30 cm? Students should suggest that they could use either the mean or the median.

2. Discuss with your students whether to use the mean or median. The median is more robust in that it is less influenced by extreme values. The mean is influenced by extreme values, but includes all the information in a calculation. Students should realize that both measures of center might be valuable here.

3. Have your students calculate the mean for their three drops for each ball at each height. Have them find the median for their three drops for each ball at each height. Students should record the mean and median on their recording sheet. Table 4.3.5 and Table 4.3.6 show results from an example experiment.

Table 4.3.5 Sample Results for Tennis Ball Drop

	Tennis Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	16.0 cm	33.0 cm	50.0 cm
Trial 2	17.0 cm	32.0 cm	49.0 cm
Trial 3	16.0 cm	33.0 cm	49.0 cm
Mean	16.3 cm	32.6 cm	49.3 cm
Median	16.0 cm	33.0 cm	49.0 cm

Table 4.3.6 Sample Results for Golf Ball Drop

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	19.0 cm	37.0 cm	55.0 cm
Trial 2	18.0 cm	38.0 cm	57.0 cm
Trial 3	19.0 cm	38.0 cm	56.0 cm
Mean	18.6 cm	37.6 cm	56.0 cm
Median	19.0 cm	38.0 cm	56.0 cm

4. Have each group of students construct a scatterplot of their results. Note that it is customary to put the independent variable on the x-axis, which is the height of the drop for this experiment. The variable on the y-axis should be the height of the bounce (since this is the dependent variable—dependent upon the height of the drop). Instruct your students to graph the median bounce height for each drop height. They should graph the data for both types of balls by using two colors or symbols. See Figure 4.3.1 for an example of a scatterplot of the sample results. **Note:** It might be valuable to graph the raw data and the median. Since there are only three trials, it would provide a nice visual connection of where the median fits into the raw data and how variable the original data are.

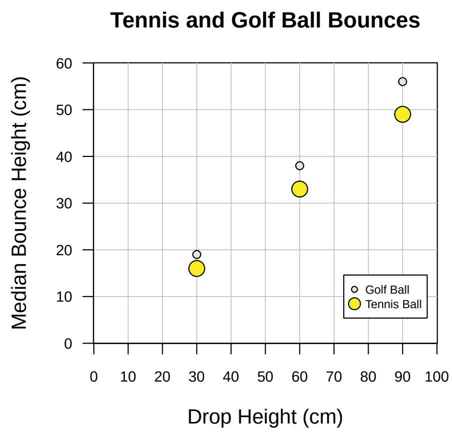


Figure 4.3.1 Scatterplot of median bounce height versus drop height for a tennis ball and golf ball

5. Ask your students to describe any patterns they observe in their scatterplots. Students should be able to use words such as positive or negative relationship. A positive relationship means that data points go from the lower left of a scatterplot to the upper right, whereas a negative relationship means the data points go from the upper left of a scatterplot to the lower right.

6. Ask your students to calculate the ratio of the bounce height to the drop height for each drop height and for both balls. Record the answers in a table similar to Table 4.3.7 (template available on the CD), which shows the ratio of bounce height to drop height for the sample data.

Table 4.3.7 Ratios of Bounce Height to Drop Height

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height
30	16	16/30= 0.53	19	19/30= 0.63
60	33	33/60 = 0.55	38	38/60 = 0.63
90	49	49/90 = 0.54	56	56/90 = 0.62

7. Ask your students to interpret the ratio of the bounce height to the drop height for the tennis ball and golf ball. Note that students' responses should center on a tennis ball bounces back around 54% of the height from which it was dropped. The golf ball bounces back more, somewhere around 63% of the height.

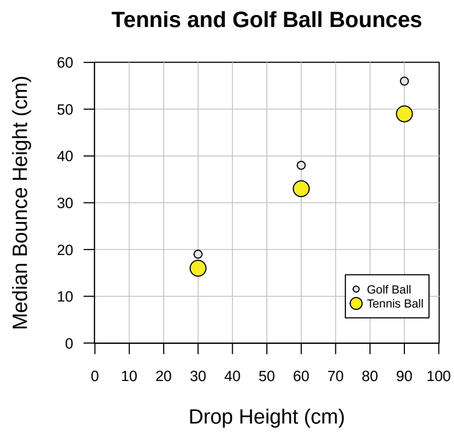
Interpret the Results in the Context of the Original Question

Ask your students to discuss in their groups their answer to the question, "Does the height from which a ball is dropped affect how far it bounces?" Have each group write a summary of the experiment that starts with stating an answer to the question and then supporting their answer by using their analysis. Your students should base their answer on the data collected, key calculations, their scatterplot, and the ratios.

Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

We think the height from which a ball is dropped does and does not affect the bounce height too much. It depends on what you are looking at. If it's the actual height of the bounce, then it goes up if the drop height goes up. But if it's the ratio of the bounce height to the drop height, then the ratio is constant for a tennis ball or a golf ball—about .54 for the tennis ball and .63 for the golf ball. Our conclusion is based on data we got from dropping a tennis ball three times each from heights of 30, 60, and 90 cm. We dropped the ball three times from each height to get an accurate result. We then took the median of the three data points to represent the bounce height for each drop height. We did the same thing for a golf ball. Here is our scatterplot of median bounce height for each drop height:



Looking at the scatterplot, we see there is a positive relationship between drop height and bounce height for both tennis and golf balls. We also see that the relationship is pretty linear for both the tennis and the golf ball and that the golf ball bounces higher than the tennis ball at each drop height. The gap between the heights gets wider as the drop gets higher. We now see why our teacher asked us to calculate the ratio of bounce height to drop height. Here are the calculations:

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height
30	16	16/30= 0.53	19	19/30= 0.63
60	33	33/60 = 0.55	38	38/60 = 0.63
90	49	49/90 = 0.54	56	56/90 = 0.62

It's interesting to see that the tennis ball bounces back around 54% of its drop height and the golf ball does better—at around 63%. It's probably because of the composition of a golf ball. We wonder what ratio a "super ball" would have.

Assessment with Answers

A group of students conducted the ball drop experiment using a basketball. Table 4.3.8 contains the results of their experiment when they dropped a basketball from 30, 60, and 90 cm.

Table 4.3.8 Results of Dropping a Basketball

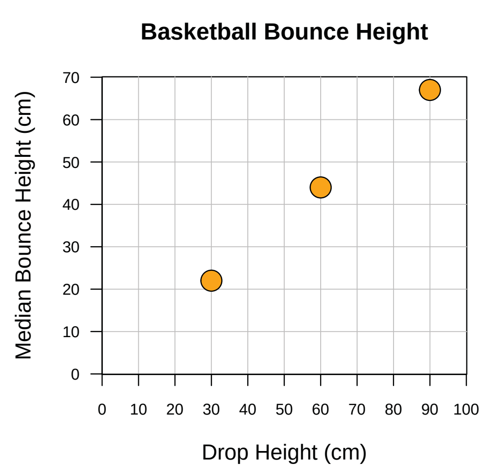
	Basketball		
	30 cm	**60 cm**	**90 cm**
Trial 1	22.0	45.0	67.0
Trial 2	23.0	44.0	68.0
Trial 3	22.0	44.0	67.0
Mean	22.3	44.3	67.3
Median	22.0	44.0	67.0
Ratio	0.73	0.73	0.74

1. Find the mean and median bounce height for each drop height and record them in the chart above.

2. Find the ratio of the median bounce height to the median drop height and record them in the chart above.

3. Discuss how the mean and median bounce heights relate to the drop height. Include the ratio of median bounce height to median drop height in your discussion. The height that the basketball bounces is about 73% of the height from which it was dropped.

4. Construct a scatterplot that shows the relationship between the heights from which the basketball was dropped and the median height of the bounce.



5. Describe the graph and the relationship between drop height and median bounce height. The higher the ball is dropped, the higher the bounce.

6. Is the bounce height of a basketball higher than either the tennis ball or golf ball that you used in the investigation? Explain your answer. Based on the sample data given in the investigation, the bounce height of the basketball was higher than both the tennis and golf ball. The median bounce height at the drop heights of 30 cm, 60 cm, and 90 cm were all higher for the basketball than both tennis and golf ball. Based on your class data collected, the answer may differ.

Alternative Assessment

Find a ball at your house and replicate what was done in class with a family member. Drop the ball from 30, 60, and 90 cm. Record your data, find the mean and median, and create a scatterplot. Describe the relationship between drop heights and bounce height and compare your results with the results from the class experiment.

Extensions

1. Ask your students how high they think the ball would bounce if it were dropped from 25 cm? 75 cm? 150 cm? Ask them what they are basing this on? After they make their predictions based on the collected data, have them drop the balls from 25, 75, and 150 cm and compare the results to their predictions. Note that predictions should not be made outside the interval of the original data. However, this should make for an interesting extension, such as seeing if the data trend changes at a certain point.

2. Ask your students how they would carry out this experiment if they wanted to find the ratio of the table tennis ball bounce height to its drop height. Their description should include what drop heights they would use, how many drops from each height, and how they would calculate their answer.

3. Have your students look at their scatterplots. Ask them if it would be possible to imagine a line being drawn to represent the trend in our data? How would drawing this line help us answer the two questions above?

References

Bereska, C., L. C. Bolster, C. A. Bolster, and R. Scheaffer. 1998. *Exploring statistics in the elementary grades: Book one, grades k–6*. White Plains, NY: Dale Seymour.

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Common Core State Standards for Mathematics. www.corestandards.org.

Investigation 4.4
Can You Roll Your Tongue?

Overview

This investigation focuses on students examining an **association** between two **categorical variables**. Specifically, they will investigate whether there is an association between gender and whether a person can roll their tongue. As part of this investigation, students will collect, organize, and analyze data in a **two-way table**; construct and analyze **segmented bar graphs**; and calculate the **percentages** of boys and girls who can roll their tongue. This investigation is based on an activity in *Probability Through Data*, a module in the Data-Driven Mathematics series (1999).

Association

Two categorical variables are associated if certain values of one variable are more likely to occur with certain values of the other variable.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Organize data collected into a two-way table
- Analyze data in a two-way table

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Common Core State Standards Grade Level Content

6RP3c Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

6SP3 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

8.SP.4 Understand that patterns of association also can be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6-8 Students should understand and use ratios and percentages to represent quantitative relationships and formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population.

Materials

- Data collection sheet (available on the CD)
- Data recording sheet (available on the CD)
- Grid paper
- Color markers

Estimated Time

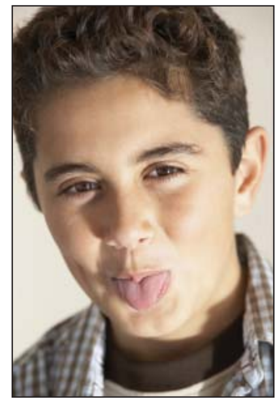
One day

Instructional Plan

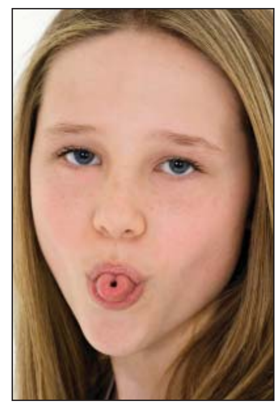
Formulate a Statistical Question

Ask your students to look around their classroom. Pose the question, "Is anyone in the room exactly like you?" "Are identical twins exactly the same?" Discuss that there are many traits or characteristics that make us different from each other. Have students list some of these traits. Examples are hair color, eye color, skin color, blood type, having double-jointed elbows, having "free" earlobes or "attached" earlobes, and whether they can roll their tongue. Discuss with your students that many of these traits are genetic (i.e., inherited or passed on from their parents). Ask which of the traits might have been inherited from their parents.

Tell students there are many traits they could investigate. Indicate that, for this activity, they will be investigating rolling one's tongue (even though it isn't genetic). The statistical question is, "Is gender associated with ability to roll one's tongue?"



Can't roll tongue



Can roll tongue

Collect Appropriate Data

1. Have one student demonstrate how he/she is able to roll his/her tongue and another demonstrate that he/she is unable to roll his/her tongue.

2. Hand out a data collection sheet to each student. Your students should check whether they are a boy or girl and whether they can roll their tongue. Collect each of the data collection sheets. Figure 4.4.1 is an example of a data collection sheet.

Boy ________

Girl ________

Can roll tongue ________

Can't roll tongue ________

Figure 4.4.1 Data Collection Sheet

3. Hand out a recording sheet (available on the CD) to each student. Take each of the data collection sheets and read whether the sheet is checked boy or girl and whether the student can roll their tongue. As you read each data collection sheet, students should record the data on the recording sheet as shown in Table 4.4.1. Suggest that they write B for boy, G for girl, Y for yes they can roll their tongue, and N for no they cannot roll their tongue.

Table 4.4.1 Example of Class Recording Sheet

Student	Boy or Girl	Roll Your Tongue Yes or No?
1	B	N
2	B	Y
3	G	Y
…	…	…

Analyze the Data

1. Discuss with your students that one way to help analyze the data is to organize the data into a table. Ask them what answers they could record when they were reading the data collection sheets. On the board, display Table 4.4.2. Ask your students to fill in the frequencies (counts) for the four possibilities based on their recording sheet.

Table 4.4.2 Frequency Table

Possibilities	Count/Frequency
Boy – Yes	
Boy – No	
Girl – Yes	
Girl – No	
Total	

2. Explain to your students that their frequency table can be displayed in a different way, called a two-way table. A two-way table organizes data about two categorical variables with rows labeled with the categories of one variable and the columns labeled with the categories of the other variable. In this investigation, the rows of the table are labeled with gender—boys and girls—and the columns are labeled with whether a person can roll their tongue. Demonstrate drawing and labeling the two-way table. The general form is shown in Table 4.4.3. Note that the two-way format is useful when investigating whether there is an association between two categorical variables.

Table 4.4.3 Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy			
Girl			
Total			

3. Label each cell in Table 4.4.3 with letters representing frequencies, as shown in Table 4.4.4.

Table 4.4.4 Example of Completed Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	a	b	
Girl	c	d	
Total			

4. Explain to your students that the cell labeled "a" will contain the number of students who are both a boy and who said they could roll their tongue. Ask your students what the cell labeled "b" represents. Cell "c"? Cell "d"?

5. Ask your students how many boys are in the sample, using the letters in Table 4.4.4. **Note:** There are "a+b" boys. How many girls? There are "c+d." How many students can roll their tongues? "a+c" can roll their tongues. How many can't? "b+d" can't.

6. Have your students fill in the two-way table based on their class data as recorded in their frequency table, Table 4.4.2. An example of what their table may look like is given in Table 4.4.5.

Table 4.4.5 Row of the Boys' Data from the Two- Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	**15**
Girl	6	4	**10**
Total	**14**	**11**	**25**

7. Ask your students to use Table 4.4.5 to answer the following questions. As students answer each question, have them point to the appropriate cell.

   a. How many students were in the class?

   b. How many students could roll their tongue?

   c. How many students were girls?

   d. How many students were boys?

   e. How many girls could roll their tongue?

   f. How many boys could roll their tongue?

   g. How many boys could not roll their tongue?

8. Remind your students of the question they are investigating: "Is gender associated with ability to roll one's tongue?" Ask them if they are ready to answer the question. Note that many of your students will say more boys can roll their tongues than girls. Keep asking until someone suggests they should be looking at percentages, not raw counts, as there are more boys than girls in the class.

9. Ask your students to find the percentage of boys who could roll their tongue. To help them answer this question, show them only the row with the boys' data. See Table 4.4.6. Have them find the fraction that answers the question, convert it to a decimal, and then convert it to a percentage. For example, for the boys who can roll their tongue, 8/15 = .53 = 53%.

Table 4.4.6 Row of the Boys' Data from the Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	15

10. Using Table 4.4.7, ask your students to find the percentage of girls who can and cannot roll their tongue.

Table 4.4.7 Row of the Girls' Data from the Two-Way Table

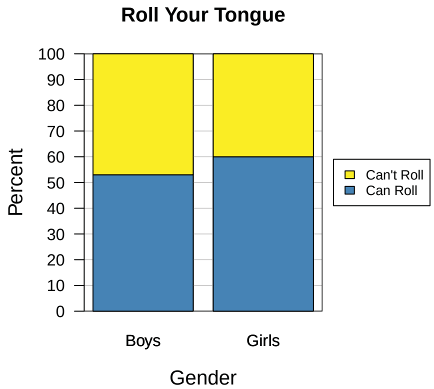
	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Girl	6	4	10

11. Ask your students to put their percentages in a two-way table. See Table 4.4.8. Note that the Total row percentages are each 100%.

Table 4.4.8 Example of Row Percentages

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8/15 = .53 = 53%	7/15 = .47 = 47%	15/15 = 1.00 = 100%
Girl	6/10 = .60 = 60%	4/10 = .40 = 40%	10/10 = 1.00 = 100%
Total			

12. To help your students visualize the different percentages of boys and girls who can and cannot roll their tongue, demonstrate the construction of a segmented bar graph. Using Table 4.4.8, a segmented bar graph is shown in Figure 4.4.2. Note that the percentages could also be visualized in side-by-side bar graphs.



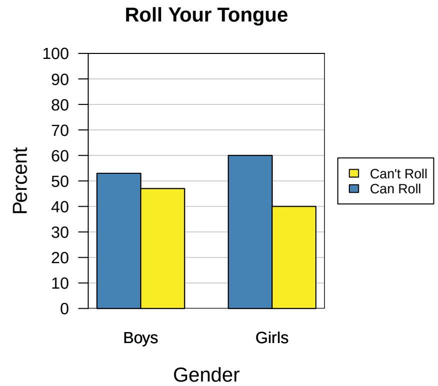


Figure 4.4.2 Segmented bar graph and side-by-side bar graph of example class data

Interpret the Results in the Context of the Original Question

1. Have your students recall the original statistical question, "Is gender associated with ability to roll one's tongue?" Have each group of students write an answer to the question and then justify it using the two-way table, appropriate calculations involving percentages, and the segmented bar graph. Suggest to your students that they should focus on the difference in the percentages and the heights of the bars in the segmented bar graph. Remind your students that an association exists between two categorical variables if knowing the response of one of the variables helps to know what the response might be of the other variable. Does knowing a girl was chosen from the group help know whether she can roll her tongue? Similarly, does knowing a boy was chosen help know whether he has the ability to roll his tongue? Have each group of students present their results to the class.

Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

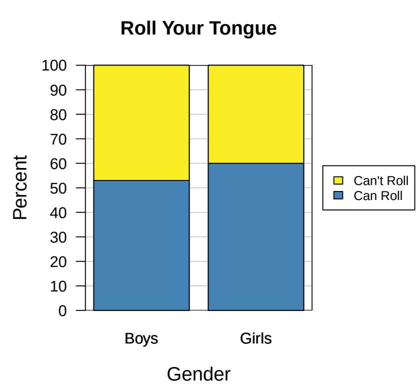
In our biology class, we often talk about genetics, so we thought a good statistics project in our mathematics class would be to take a genetic trait and see if it is associated with gender. We chose rolling our tongues. (After our study was complete, we found out that rolling one's tongue is not actually genetic. It is a learned trait. But it was fun doing the experiment anyway.) Our statistical question was "Is gender associated with ability to roll one's tongue?" We collected data by making a list of boys or girls and whether they could roll their tongue. We then counted how many there were in each of the four categories and organized the data in a two-way table like this one.

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	15
Girl	6	4	10
Total	**14**	**11**	**25**

So, to answer the question, some of us say boys are more likely to roll their tongues than girls are. But, we messed up because there were more boys in class than girls. So, we should be looking at percentages, not counts. When we calculated the percentages, we almost based them on 25, but realized they had to be calculated within boys' and girls' totals. So, here is our table of row percentages.

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8/15 = .53 = 53%	7/15 = .47 = 47%	15/15 = 1.00 = 100%
Girl	6/10 = .60 = 60%	4/10 = .40 = 40%	10/10 = 1.00 = 100%
Total			

The actual answer to our question is that a higher percentage of girls can roll their tongues as compared to boys. Sixty percent of girls could roll their tongues compared to 53% of boys. Our teacher showed us how to visualize these results in what is called a segmented bar graph. It makes it clear that the percentage of girls is higher.



But we debated whether gender and ability to roll one's tongue are associated because some of us thought that 53% and 60% are kind of close and so the variables are not associated. Others thought the percentages were far enough apart to claim the variables are associated. Our teacher said we will learn more about association in high school.

Assessment with Answers

A survey asked a group of students if they participated in a sport and if they played a musical instrument. Table 4.4.7 shows the survey results.

Table 4.4.7 Survey Results

	Music Yes	Music No	Total
Sport Yes	18	2	20
Sport No	8	22	30
Total	**26**	**24**	**50**

Use the table to answer the following questions:

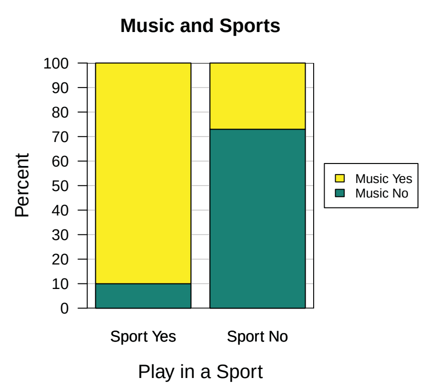
1. How many students said they participated in a sport? Twenty said they participated in a sport.

2. How many students said they did not play a musical instrument? Twenty-four said they did not play a musical instrument.

3. What does the number 8 represent in the table? The number 8 represents the number of students who said no to sports and yes to music.

4. What percentage of those who said they participated in a sport also played a musical instrument? 18/20 = .90.
5. What percentage of those who said they did not participate in a sport played a musical instrument? 8/30 = .27.

6. If a student participates in a sport, are they more likely to play a musical instrument than a student who does not participate in a sport? Use words, numbers, and graphs to explain your answer.



Students who do not participate in a sport are much less likely to play a musical instrument than those who do participate in a sport. Twenty-seven percent of students who do not participate in a sport also played an instrument while 90%

of those that did participate in a sport played an instrument. The segmented bar graph shows the big difference between the groups who do and don't play sports and whether they play an instrument. We can say that participation in sports and playing a musical instrument are associated.

Right-thumbed

Left-thumbed

Extensions

1. Ask students to collect data at home. Each student should ask one parent/guardian if he/she could roll his/her tongue. Collect data in a table during the next class period:

Possibilities	Number
Student yes – Parent/guardian yes	
Student yes – Parent/guardian no	
Student no – Parent/guardian yes	
Student no – Parent/guardian no	

   Your students should organize the data in a two-way table. Based on the table and calculated percents, students should determine if there appears to be an association between whether the parent/guardian can roll his/her tongue and whether the student can roll his/her tongue.

2. Your students could investigate if there appears to be an association between whether a person is left-handed or right-handed and whether they are left-thumbed or right-thumbed. **Note:** To determine whether one is left- or right-thumbed, have your students clasp their hands together immediately without thinking about it. Then look at the pictures to the left to determine the category. Students could collect class data and analyze the data to determine if there appears to be an association.

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