Investigation 5.2
How Tall Were the Ancestors of Laetoli?

Overview

The focus of this investigation is to look for and measure the degree of any **relationship** between two **quantitative** variables, specifically height and foot length. The motivation for this study comes from a science dig. Footprints were found that were determined to be more than 3 million years old. It is of interest to predict how tall those ancestors might have been based on the lengths of their footprints. Students will investigate whether there is a relationship between their own height and foot length. They will collect, organize, and analyze such data and then informally predict what the height of the ancestors might have been.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Learn to make conjectures about the relationship between two quantitative variables
- Demonstrate an ability to organize their data and display them in a scatterplot
- Learn to quantify the degree of relationship between two quantitative variables by developing the Quadrant Count Ratio (QCR)

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

Common Core State Standard Grade level Content

8.SP.1 Construct and interpret scatterplots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6-8 Students should formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; select, create, and use appropriate graphical representations of data, including histograms, boxplots, and scatterplots.

Materials

- Class data recording sheet (available on the CD)
- Sticker dots (3/4" diameter, four colors - green, blue, yellow, and red)
- Metric sticks
- Tape
- Graph paper
- Calculators
- Laetoli background information (available on the CD)

Estimated Time

Two days

Instructional Plan

✏ Formulate a Statistical Question

1. Begin this investigation by asking your students if they think there is any relationship between the size of a person's foot and his/her height. Do people with longer feet tend to be taller? Explain to your students that scientists look for relationships like this so they can estimate the height of people who lived a long time ago. Share with your students the following background information from Wikipedia about Laetoli, Tanzania (available on the CD).

   There is a place in Tanzania, Africa, known as Laetoli. It is a special place because it is where scientists believe our ancestors of long ago walked side-by-side. It is where scientists have worked to get an understanding of the past.

   In the late 1970s, two sets of footprints were discovered at Laetoli. There were 70 footprints in two side-by-side lines 30 meters long, preserved in volcanic ash. Apparently, a volcano exploded sending ash everywhere and the two individuals just happened to walk through the area, preserving their footprints. Fossil remains in the area tell scientists that the ancestors who left the footprints found at Laetoli lived about 3.5 million years ago.

   We know the size of the feet because Dr. Mary Leakey, an anthropologist, and her team made copies of the prints using plaster casts. The locations of the footprints were put on a map, so the length of stride (distance between footprints) also can be determined. Based on these observations, foot dimensions and stride length for the two ancestors are given in Table 5.2.1. These are averages based on the 70 observed footprints.

Table 5.2.1 Footprint Data Collected by Dr. Leakey at Laetoli

	Ancestor 1	Ancestor 2
Length of Footprint	21.5 cm	18.5 cm
Width of Footprint	10 cm	8.8 cm
Length of Stride	47.2 cm	28.7

   Much has been learned from these footprints. They share many characteristics with the prints made by modern human feet.

   A research question of interest to the scientists was "How tall were these ancestors at Laetoli?" The foot length, foot width, and length of stride can be used to produce estimates of the heights of these ancestors.

2. After explaining the background information, discuss with your students how they can help the scientists answer the question, "How tall were these ancestors at Laetoli?" Tell your students they are going to focus on whether foot length and height are related. **Note:** Depending on the sensitivity of measuring feet, you may have your students measure shoe length instead. (However, to be realistic, the ancestors did not wear anything on their feet.) Although collecting real data is desirable, there is a sample set of class data given in Table 5.2.2.

3. Lead your students to formulate the statistical question, "What, if any, is the relationship between height and foot size of humans?"

Collect Appropriate Data

1. Discuss with your students how they are to measure the length of their right foot. The measurements should be made in centimeters and without shoes from the back of the foot to the longest forward point of their toes. **Note:** You may want to have your students press the back of their right foot against a wall to increase the accuracy of the measurement. When measuring the height of the person, remind students they should stand straight with their back against a wall.

2. After students have measured the length of both their right foot and their height, collect the class data on the recording form (available on the CD). Table 5.2.2 is a sample set of data collected from a class of 8th-graders.

Table 5.2.2 Sample Set of 8th-Grade Class Data

Student Number	Foot Length cm	Height cm	Student Number	Foot Length cm	Height cm
1	28	175	14	24	168
2	26	181	15	23	168
3	24	168	16	23	176
4	26	168	17	27	177
5	27	178	18	25	171
6	24	174	19	22	160
7	28	179	20	27	187
8	23	157	21	28	167
9	29	190	22	27	184
10	26	170	23	29	181
11	23	169	24	27	174
12	23	166	25	22	155
13	26	174	26	24	170

Analyze the Data

1. Ask your students to find the mean length of the right foot data and the mean of the height data.

2. Draw a large scatterplot on the board and plot the ordered pair (mean foot length, mean height) with a black dot. For the sample class data, the point is (25.4, 172.6).

3. Ask all students with above-average right foot length and above-average height to stand. Give them a green "sticker dot" and have them place their stickers on the graph at the appropriate coordinates.

4. Ask all students with below-average right foot length and above-average height to stand. Give them a blue "sticker dot" and have them place their stickers on the graph at the appropriate coordinates.

5. Ask all students with below-average right foot length and below-average height to stand. Give them an orange "sticker dot" and have them place their stickers on the graph at the appropriate coordinates.

6. Ask all students with above-average right foot length and below-average height to stand. Give them a red "sticker dot" and have them place their stickers on the graph at the appropriate coordinates. Figure 5.2.1 is a scatterplot of the sample set of data collected from an 8th-grade class. Note the ordered pair of means is the black dot in the scatterplot.

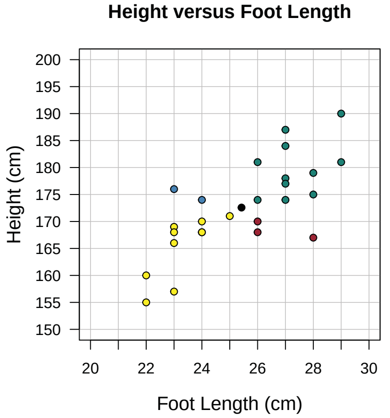


Figure 5.2.1 Class scatterplot of height versus foot length. **Note:** There is a duplicate data point at (24, 168).

7. Ask your students what trends they observe in the graph. Note that they should say there is a positive trend with longer foot lengths related to taller heights and shorter foot lengths related to shorter heights. Discuss

with your students that, in statistics, single summary numbers are calculated for a data set that tell us something about the data set. For example, when asked to characterize a central tendency for a data set, three summary statistics have been developed: the mode (most often), median (middle of the ordered data), and mean (a fair share value or balance point). Each is a single number. Similarly, when characterizing the spread of a data set, three summary statistics have been developed: the range (overall span of the data), interquartile range (span of the middle 50% of the data), and mean absolute deviation (MAD, a fair share value for how far the data are in terms of absolute distance from their mean). In this investigation of two variables, we want to develop a summary statistic (single number) that measures how related two quantitative variables are to each other. The following steps help your students develop such a summary statistic.

8. Draw a vertical line through the center point (black dot) extended to the x-axis; indicate the mean of X, 25.4 cm, on the x-axis. Similarly, draw a horizontal line through the center point (black dot) extended to the y-axis; indicate the mean of Y, 172.6 cm, on the y-axis.

9. Point to different colored dots and ask your students to explain what the dots represent in relation to the center point, given by the coordinate pair (the mean foot length, the mean height).

10. Number the four quadrants as shown in Figure 5.2.2.

    Quadrant I: Green dots

    Quadrant II: Blue dots

    Quadrant III: Orange dots

    Quadrant IV: Red dots

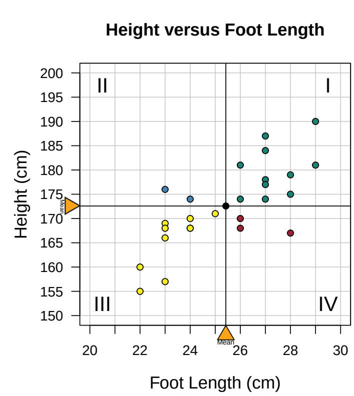


Figure 5.2.2 Scatterplot of height versus foot length showing the quadrants

The QCR

The beauty of the QCR is its simplicity and ease of calculation while providing a conceptual measure of relationship. The QCR clearly has some shortcomings. For example, the same number of points within a quadrant could occur in a very different orientation so that the data do not exhibit linearity at all and yet the QCR value would remain the same. The difficulty, of course, is that the QCR measure is based on only counts within a quadrant and does not consider, for example, how far a data point is from the horizontal and vertical axes. In high school, this topic is revisited and a more sophisticated measure (the Pearson correlation) is developed that corrects for QCR's limitations.

11. Ask your students where most of the stickers are. Determine the number of dots in each quadrant and put the number on the graph in the respective quadrants. Figure 5.2.3 shows the sample class data with the number of ordered pairs written in each quadrant. Note that there are two data points at (24,168) so there are 10 data points in quadrant III.

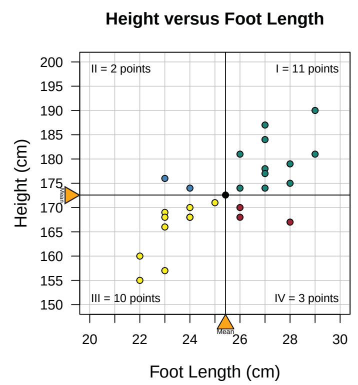


Figure 5.2.3 Scatterplot showing number of ordered pairs in each quadrant

12. Ask your students what a dot in Quadrant I represents. (People with above average foot length and above-average height). Ask what a dot in Quadrant III represents. (People with below-average foot length and below-average height)

13. Explain to your students that this graph indicates a **positive relationship** between the variables foot length and height. Generally, two numeric variables are **positively related** when above-average values of one variable tend to occur with above-average values of the other and when below-average values of one variable tend to occur with below-average values of the other. **Negative relationship** between two variables occurs when below-average values of one variable tend to occur with above-average values of the other and when above-average values of one variable tend to occur with below-average values of the other.

14. Explain to your students that we would like to have a single number that helps describe the degree of relationship seen in the graph. A **correlation coefficient** is a number that measures the direction and strength of a relationship between two variables. One such correlation coefficient is called the **Quadrant Count Ratio** (QCR). The QCR is defined as:

$$\text{QCR} = \frac{(\text{Number of Data Points in Quadrants I and III}) - (\text{Number of Data Points in Quadrants II and IV})}{\text{Total Number of Points}}$$

15. Have your students find the QCR for the class data. For the example:

$$QCR = \frac{((11+10)-(2+3))}{26} = \frac{(21-5)}{26} = \frac{16}{26} = 0.62$$

16. Ask your students to find the value of the QCR if all the ordered pairs are located in quadrants I and III. Ask your students if it is possible to get a QCR greater than 1 (such as 1.5). Ask your students to find the value of the QCR if all the ordered pairs are located in quadrants II and IV. Ask your students if it is possible to get a QCR less than -1 (such as -1.9).

17. Explain to your students that the closer the value is to +1, the stronger the positive relationship is. The closer to -1 suggests a stronger negative relationship. Close to 0 would indicate no relationship. Have them look at the scatterplot and explain why that should be.

18. Indicate on a number line where the class value of QCR is and ask students what the value indicates about the strength of the relationship between height and foot length. Figure 5.2.4 shows the location of the QCR for the sample set of data.

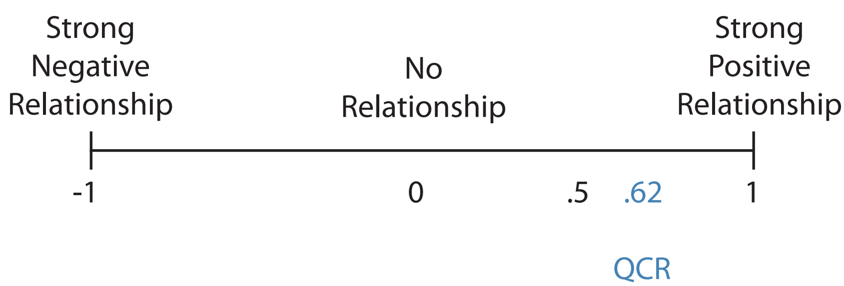


Figure 5.2.4 Strength of relationship on a number line

19. Ask your students to look at Figure 5.2.3 and, assuming the relationship of height to foot length is the same for the ancestors of Laetoli as it is in this data set, what color of sticker would the ancestors have based on a mean footprint of 21.5 cm? Note that they should say orange and that the ancestors had shorter feet than their own and were shorter in height than their own average height.

Interpret the Results in the Context of the Original Question

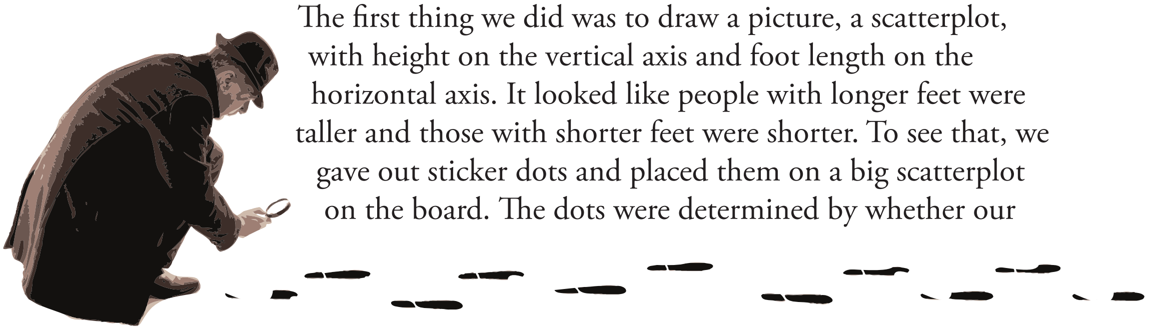
1. Have your students recall the original statistical question, "What, if any, is the relationship between height and foot size of humans?" Have your students write a brief report that answers the question and justifies their answer by using the analysis they did in class. In addition, remind your students that what prompted the statistical question involving height and foot length was an interest in trying to estimate the heights of the two ancestors from Laetoli, as we know only the length of their footprints. Have your students include in their report how they might go about coming up with an estimate of the height of the Laetoli ancestors. Indicate that you are not as interested in their actual estimate as you are in the process they are suggesting for determining the estimate. (See the extension for further development.)

2. Ask your students how they think the relationship between height and foot length would change from what they found using their class data if they collected data on height and foot length from all the teachers in the school.

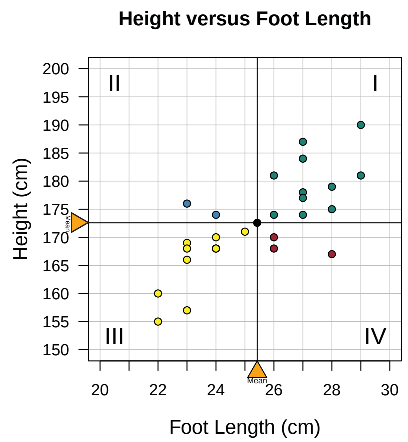
Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

For a statistics project, we got an idea from an anthropological study by Dr. Leakey, who found footprints of 3.6 million–old ancestors in Laetoli, Tanzania. The study had the ancestors footprint lengths, and we were wondering how tall they might have been. One of the set of footprints had a mean footprint of 21.5 cm. Our statistical question was, "Is there a relationship between human height and foot length?" Our data were the lengths of our right foot and our height. There were 26 paired data points in our class.



The first thing we did was to draw a picture, a scatterplot, with height on the vertical axis and foot length on the horizontal axis. It looked like people with longer feet were taller and those with shorter feet were shorter. To see that, we gave out sticker dots and placed them on a big scatterplot on the board. The dots were determined by whether our height was above or below the mean height of 172.6 cm and how our foot length compared to the mean 25.4 cm. Green dots were for (above 25.4 foot length, above 172.6 height); blue for (below, above); orange for (below, below); and red for (above, below). We added vertical and horizontal lines through the paired mean point. The scatterplot looked like this:



We could see a definite trend from the lower left to the upper right. In statistics, single numbers called summary statistics are often calculated to indicate the degree of some characteristic. So, our teacher suggested we count the number of points in the first and third quadrants and subtract the numbers in quadrants two and four, and then take the mean and call the result the Quadrant Count Ratio (QCR). For our data, QCR = ((11+10) – (2+3))/26 = 0.62. If all the data had been in quadrants one and three, the QCR would have been 1. So, we decided that .62 was pretty good and that it reflected a positive relationship. We then decided that our Laetoli ancestors would have had orange stickers, since the mean footprint we had for them was 21.5 and, from our scatterplot, there was no way the sticker could be blue. We were thinking about doing this study on all our teachers to get a new data set and see if it differs from ours. There's a difference of opinion. Some of us think it would have more variation because the ages of the teachers are more spread out than our ages.

Assessment with Answers

A group of students measured their height and arm span in centimeters. Table 5.2.3 shows the data they collected, and the scatterplot of the data is shown in Figure 5.2.5.

Table 5.2.3 Height and Arm Span (cm)

Height	Arm Span	Height	Arm Span
155	151	173	170
162	162	175	166
162	161	176	171
163	172	176	173
164	167	178	173
164	155	178	166
165	163	181	183
165	165	183	181
166	167	183	178
166	164	183	174
168	165	183	180
171	164	185	177
171	168	188	185

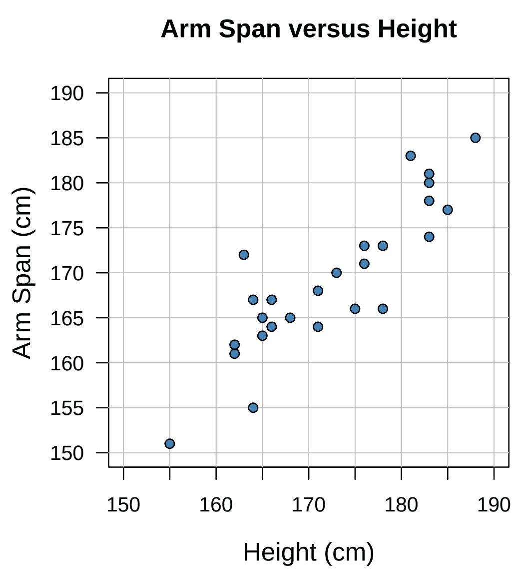


Figure 5.2.5 Scatterplot of arm span versus height

1. Describe the relationship between arm span and height. There is a positive relationship between arm span and height. Higher values of arm span tend to occur with higher values in height; lower values of arm span tend to occur with lower values in height.

2. Find the mean height and the mean arm span. Mean height = 172.5 cm and the mean arm span = 169.3.

3. Locate the point (mean height, mean arm span) on the graph and draw a horizontal line and a vertical line through the point.

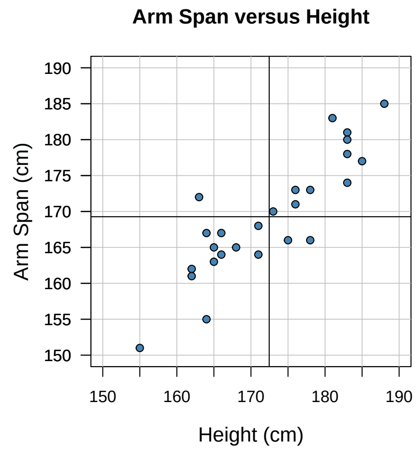


Figure 5.2.6 Scatterplot showing means and vertical and horizontal lines

4. Find the value of the QCR. QCR = ((11 + 12) - (1 + 2))/26 = (23 - 3)/26 = 20/26 = .77

5. Interpret the value of the QCR. Fairly strong positive relationship between height and arm span. This indicates that height is a pretty good predictor for arm span.

Extension

To determine an estimate for the height of the Laetoli ancestors, suggest the following:

1. Consider the scatterplot of height versus foot length as shown in Figure 5.2.7. Hand out a copy of this scatterplot to your students.

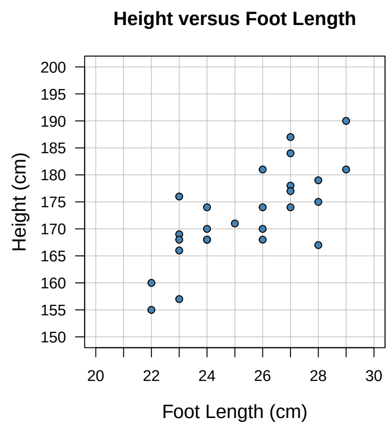


Figure 5.2.7 Scatterplot of height versus foot length. **Note:** There is a duplicate data point at (24,168).

2. Demonstrate and discuss an eyeball line on the class scatterplot using a piece of string or yarn. Ask your students what property the line should have. Lead them to suggesting that the line should "fit" the data fairly well.

3. Draw the line on the class graph and demonstrate making a prediction using the line. For a given value of X—for example, 21.5 for Laetoli ancestor 1—from 21.5 on the X axis, move vertically up to the line, then horizontally to the y-axis, as shown in Figure 5.2.8.

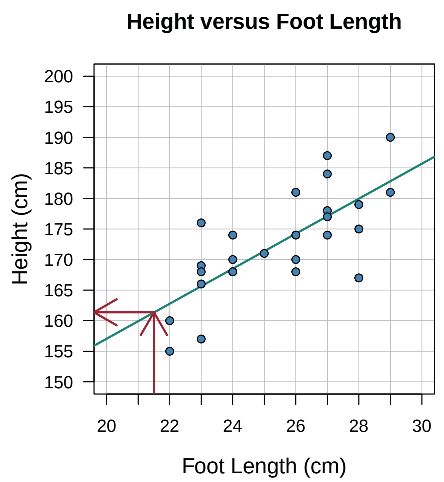


Figure 5.2.8 Scatterplot of height versus foot length with eyeball fit line. **Note:** There is a duplicate data point at (24,168).

4. Have each student draw their eyeball fit line and use the line to make a prediction for the two Laetoli ancestors.

References

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry, and R. Scheaffer. 2007. *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k–12 curriculum framework*. Alexandria, VA: American Statistical Association. *www.amstat.org/education/gaise*.

National Council of Teachers of Mathematics. 2000. *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Common Core State Standards for Mathematics, *www.corestandards.org*.

Laetoli, *http://en.wikipedia.org/wiki/Laetoli.*

Investigation 5.3
How Long Does It Take to Perform the Wave?

Overview

The focus of this investigation is looking for a **relationship** between two **quantitative** variables. Specifically, students will investigate whether there is a relationship between number of people and how long it takes them to perform the "wave." As part of this investigation, students will collect, organize, and analyze data by conducting an experiment to time how long it takes a varying number of students to perform the wave. Students will construct a **scatterplot**, use the plot to look for patterns in the data, and draw a line to summarize the data.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Construct a scatterplot
- Describe a relationship between two variables
- Draw a line and describe the rate of change

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Common Core State Standards Grade Level Content

8.SP1 Construct and interpret scatterplots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatterplots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6-8 Students should formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; select, create, and use appropriate graphical representations of data, including histograms, boxplots, and scatterplots.

Materials

- Stopwatch
- Graph paper for each student
- Ruler or straightedge for each student
- Data collection sheet (available on the CD)

Estimated Time

One day

Instructional Plan

✏ Formulate a Statistical Question

1. Begin this investigation by asking your students if they have ever been at a sporting event where the crowd performed the wave. Note that you may wish to show a YouTube video of spectators at a sporting event performing the wave. Share with your students that some people claim the wave was first performed at Fenway Park in Boston. Others claim it originated

at Pacific Lutheran University in the early 1960s. But no matter where it started, it occurs at many sporting events. Ask your students how they might predict how long it takes to perform the wave in a large football stadium. Ask your students what they would need to know to answer this question. Students usually suggest the number of people, number of sections in the stadium, and how fast the wave was performed.

2. After discussing your students' ideas, lead them to the statistical question, "Is there a relationship between number of people and length of time it takes them to perform the wave?"

Collect Appropriate Data

1. Before collecting data, discuss with your students how they are to perform the wave. Suggest they remain seated, but push their chair away from their desk. To perform the wave, they are to stand while raising their arms straight up in the air over their head, and sit back down. Have one student demonstrate the wave. Also, have your students agree on how fast they are to perform the wave. Should each student perform the wave as fast as possible or be deliberate in the motion? It is recommended to have your students be deliberate in the wave motion. It also is recommended that each student practice performing the wave to keep the procedure the same.

2. Appoint a timekeeper. The same person should do all the timing.

3. Start with three students as your first group to perform the wave. When the timekeeper says "go," the first student stands, moves his/her arms, then sits down. As soon as the first person sits down, the second student starts the wave. When the second student sits down, the third student performs the wave. Record the number of students who performed the wave and the time elapsed in Table 5.3.1.

Table 5.3.1 Data Collection Sheet

Number of Students	Time (sec) to Complete the Wave
3	
6	
9	
…	

4. Add three more students and have all six students perform the wave. Again, record the results. Continue to add three students until the entire

class has been included in performing the wave. Table 5.3.2 shows the results of 24 8th-graders having performed the wave experiment.

Table 5.3.2 Results of the Wave Experiment for a Group of 8th-Graders

Number of Students	Time (sec)
3	4
6	8
9	13
12	17
15	20
18	24
21	27
24	30

Analyze the Data

1. Ask your students if they see any patterns in the table. Students should recognize that the number of students increased by three and the time increase varied by 3, 4, or 5 seconds. To help the students focus on the change in time for the wave, add a column to the data collection sheet labeled Change in Time. Table 5.3.3 shows how the time increased as the number of students increased. Ask your students why they think the change in time varied.

Table 5.3.3 Change in Time

Number of Students	Time (sec)	Change in Time
3	4	
6	8	4
9	13	5
12	17	4
15	20	3
18	24	4
21	27	3
24	30	3

2. Ask your students to find the median number of seconds by which the change in time increased. In the example, the median is 4 seconds.

3. Ask your students about how much longer it takes to perform the wave for every additional three people. See if they realize you are asking them for a rate of change, or slope if the data turn out to be linear. For this

example, the median increase is 4 seconds for three people, or 4/3 second increase per person.

4. Ask your students how long it would take all the students in grades 6, 7, and 8 to perform the wave based on the 4/3 second per person estimate.

5. Explain to your students that a scatterplot is also useful in finding patterns or relationships between the number of students and the time to perform the wave. Have your students make a scatterplot on their graph paper. Put Number of Students on the horizontal axis (x-axis) and Length of Time on the vertical (or y) axis. Plot the ordered pairs (number of students, time). Figure 5.3.1 is a scatterplot for the example 8th-grade class data.

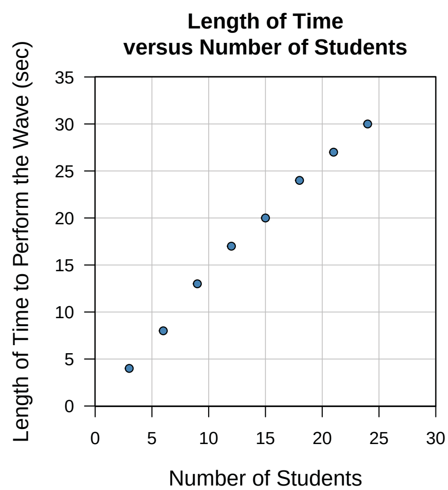


Figure 5.3.1 Scatterplot of length of time versus number of students

6. Have your students examine the scatterplot. Ask your students to describe what they observe from the scatterplot. Ask what type of relationship there is between the number of people and the length of time. And how strong is this relationship? Note that you may wish to have your students find the QCR (Quadrant Count Ratio) described in Investigation 5.2.

7. Explain to your students that you would like for them to draw a straight line through the data matching the pattern in the data as closely as they can. This line will be used to help look for patterns and make predictions about how long the wave takes. Ask them for criteria to use for determining their line. Ask them to justify why they want their line to go through or not go through the origin (0,0). Have them use a straightedge or ruler to draw the line. Figure 5.3.2 shows an example of a line drawn through (0,0) on the scatterplot from the example 8th-grade class data.

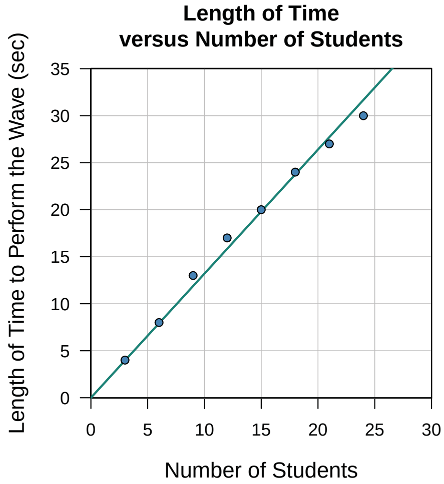


Figure 5.3.2 Scatterplot with line drawn through the (0,0)

8. Have your students locate a point on the line they drew. For this eyeball line example, a point on the line is (20,26). Ask your students what the coordinates of the ordered pair they listed represent.

9. Using the line that was drawn in Figure 5.3.2, ask your students to describe how much longer it takes to perform the wave for each additional person added. Remind them of the question you asked them in Step 3 of the Analyze the Data section. Tell your students that this value is called a **rate of change** or the **slope** of their line. For this example, the rate of change is 26/20, or about 1.3, which means that for every additional person, the time to perform the wave will increase by about 1.3 seconds.

Interpret the Results in the Context of the Original Question

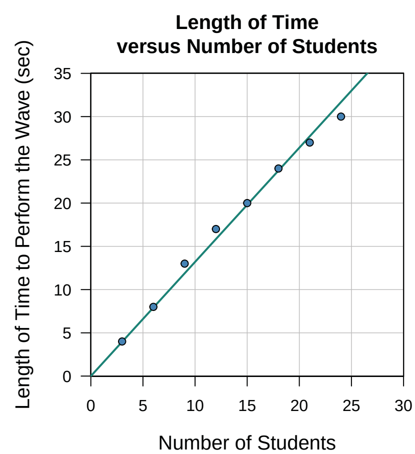
Have your students recall the original statistical question: "Is there a relationship between the number of people and the length of time to perform the wave?" Have your students answer this question in a paragraph in which they support their answers in depth using the analysis they performed. In this answer, they should refer to the relationship they observed between the number of people performing the wave and the length of time to complete the wave.

Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

This activity was really fun because we got to perform the wave in class. The statistical question we came up with was "Is there a relationship between the number of people and the length of time to perform the wave?" We actually collected data in our classroom, starting with timing how long it took three of us to perform the wave.

First, we all had to practice so we were doing the procedure the same. Otherwise, we would bias our data. We also had one timekeeper maintain all the times so no bias would enter there, either. We made a data chart by increasing the number of us performing the wave by three each time and the time it took us. We calculated that it took a median increased time of 4 seconds for every three students we added, so the rate of change is an increase of 4/3 seconds for every additional person. We also figured out that if our whole grade level of 243 students lined up to perform the wave and our rate of change was accurate, it would take 243*(4/3) = 324 seconds or about 5.4 minutes to perform the wave. Wow. We showed our data in another way by graphing the points in a scatterplot. Here it is.



We eyeballed a line through the data. We decided the line should go through the origin because it made sense that if there are no people, then the time to perform the wave is 0. We calculated a rate of change by finding a point that was on our line. The point (20,26) looked like it was on our line. So, the rate of change or slope is 26/20 = 1.3, which is about what we got before for the rate of change, 4/3. This rate means that for every additional person added, the time to perform the wave goes up about 1.3 seconds.

Assessment with Answers

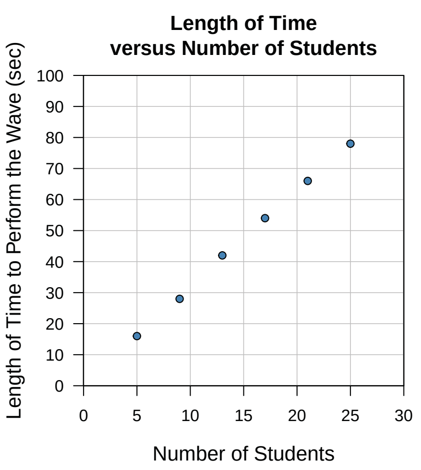
A group of 8th-grade students wanted to investigate the relationship between how long it takes to perform the wave and the number of people participating. The table below shows the results of an experiment that students conducted.

The experiment started with a group of five students. The timer said "Go" and the five students made a wave. The first student stood up, threw his/her hands in the air, turned around, and sat down. The second student did the same, and so on. The last student said "Stop" when he/she sat down. The timer recorded the elapsed time in seconds. The experiment was repeated with 9, 13, 17, 21, and 25 students.

Table 5.3.4 Number of Students and Length of Time to Perform the Wave

Number of Students	Time (sec)
5	16
9	28
13	42
17	54
21	66
25	78

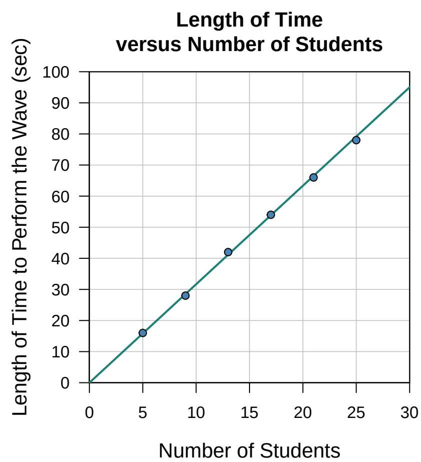
1. Draw a scatterplot of the length of time (sec) versus the number of students.



2. Is there a relationship between the number of students and the length of time to perform the wave? Describe the relationship. There is a strong positive relationship.

3. Describe any patterns you observe in the collected data for both the number of students and the length of time. As the number of students increases by four, the time to perform the wave increases by 12.4 seconds (the average of the increase changes in time).

4. Draw a line that matches the pattern in the data as closely as you can. List an ordered pair that lies on the line. Describe what the coordinates of the ordered pair represent.



The x-coordinate is the number of people and the y-coordinate is the predicted length of time to perform the wave for that number of people.

5. For each additional student added, how much longer does it take to perform the wave? Use words, numbers, and/or graphs to explain your answer. For each additional student, the wave would take a little more than 3 seconds. In the chart, the change of time for each addition of four students was about 12 seconds, which would give about 3 seconds for each additional person. The rate of change of the line on the graph is about 3.2 seconds per person.

Extension

1. Ask your students to write the equation of the line they drew through the scatterplot of Time versus Number of Students. Ask them to interpret the slope of the line in terms of the scenario.

2. Have your students predict how long it would take all the students in your school to perform the wave. Ask the principal if this could be done during an all-school program.

3. Investigate the size of a stadium near or in your community. Write down how many sections there are and how many seats are in a row. Have your students calculate how long they would predict it would take the spectators to perform the wave at the stadium.

References

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