Making Sense of Statistical Studies

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I am honored to write the foreword for *Making Sense of Statistical Studies*, a capstone experience for high-school students. As statistics is increasingly recognized as a necessary component of the high-school mathematics curriculum and other high-school curricula, there is an urgent need for materials such as *MSSS*.

In its *Principles and Standards for School Mathematics* (*PSSM*, 2000), the National Council of Teachers of Mathematics (NCTM) articulates a vision for mathematics education that includes data analysis and probability as one of five major content strands. To support and further elaborate on the Data Analysis and Probability standard, the American Statistical Association (ASA) produced the report *Guidelines for Assessment and Instruction in Statistics Education (GAISE): A Pre-K–12 Curriculum Framework*. The goals of the GAISE Report include the following:

Presenting the statistics curriculum for grades Pre-K–12 as a cohesive and coherent curriculum strand

Promoting and developing statistical literacy

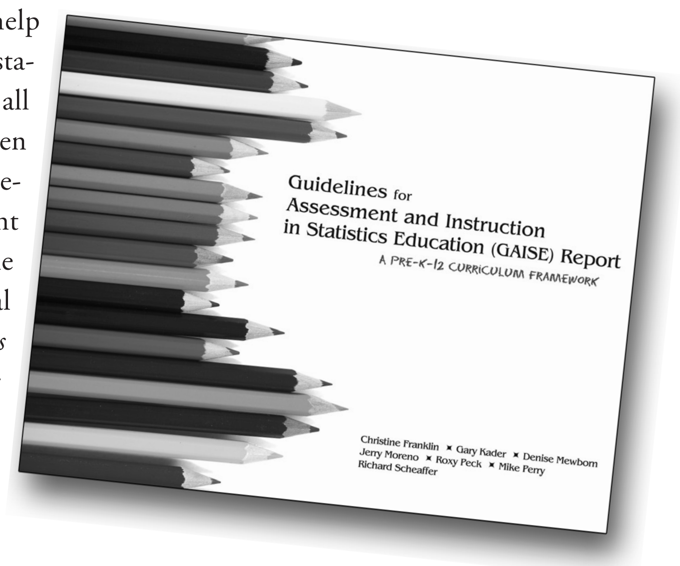
Providing links to the NCTM Standards

Articulating the differences between mathematical and statistical thinking—in particular, the importance of context and variability with statistical thinking

Clarifying the role of probability in statistics

Illustrating concepts associated with the data analysis process

The GAISE Report was endorsed by the ASA in 2005 and, with funding provided by the ASA/NCTM Joint Committee on Curriculum in Statistics and Probability, appeared in print in 2007.



The GAISE Framework was developed to help identify the essential topics and concepts in statistics and probability in grades pre-K–12 for all students as they progress from pre-kindergarden to graduation from high school. The Framework has become an instrumental document in defining the role of statistics within the school mathematics curriculum in national documents such as *College Board Standards for College Success: Mathematics and Statistics* (2006) and NCTM's forthcoming report *Focus in High School Mathematics: Reasoning and Sense-Making* (2009). The GAISE Report provides guidance for curriculum directors, faculty of teacher preparation colleges, and pre-K–12 teachers involved with statistics education. Finally, GAISE has had an effect on state standards and writers of assessment items. For example, the data analysis and probability strand is a major component of the new Georgia Mathematics Performance Standards.

The GAISE Report submits that every high-school graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy and productive life. It also holds that statistics is a must-have competency for high-school graduates to thrive in this modern world of mass information. Taken together, *PSSM* and the GAISE Report describe statistical problem solving as an investigative process that involves the following four components:

Formulate a question (or questions) that can be addressed with data

Design and employ a plan for collecting data

Analyze and summarize the data

Interpret the results from the analysis and answer the question on the basis of the data

The GAISE Report outlines a conceptual structure for statistics education in a two-dimensional framework model with one dimension defined by the statistical problem solving process, plus the nature of variability. The second dimension is comprised of three levels of statistical development (Levels A, B, and C), which students must progress through to develop statistical thinking. Grade ranges for attainment of each level are intentionally unspecified. It is paramount for students to have worthwhile experiences at Levels A and B during their elementary and middle-school years in order to prepare for future development at Level C during the secondary level. A high-school student with no prior experience with statistics will need experiences with concepts from both Levels A and B before moving to Level C.

Making Sense of Statistical Studies is an excellent set of investigations that supports the spirit of the GAISE Framework recommendations and is ideal for a capstone experience of the high-school student who has evolved through Levels A and B and is on to Level C. The investigations in *MSSS* bring the real world to the student and provide students the opportunity to understand the necessity of statistical reasoning and sense-making for everyday life and post-secondary education.

I'm most grateful to the writers of *MSSS* and the ASA/NCTM Joint Committee for developing this valuable resource in support of both the recommendations of GAISE and the importance of statistical reasoning in our high-school curriculum. These dear colleagues share the vision of promoting statistical reasoning for all high-school graduates.

Christine Franklin
Chair of the GAISE Report for Grades Pre-K–12

Preface

Making Sense of Statistical Studies (*MSSS*) is designed as a stand-alone experience with the methods of designing and analyzing statistical studies. It is written for an upper middle-school or high-school audience having some background in exploratory data analysis and basic probability.

The *MSSS* student module consists of an introduction and four distinct sections. Each section begins with an overview that contains essential background information for students. The remainder of the section is devoted to guided student investigations. These investigations start with a research question on some topic of interest. Students are then led through a series of questions that help them examine the study design, analyze data, and interpret results. Later investigations ask students to design, carry out, and analyze results from their own studies. A description of each section follows.

The **Introduction** gives students a snapshot of the statistical problem-solving process. It includes a brief discussion of the primary methods of data production—surveys, experiments, and observational studies—as well as the difference between a sample and a population. Ethical issues involved in data collection are also mentioned here.

Section I: Observational Studies shows students how much can be learned just by watching and recording data. The first two investigations in this section help students review the primary graphical and numerical tools for analyzing data. The remaining investigations incorporate random selection, which allows students to generalize the results of their data analysis to some larger population of interest.

Section II: Surveys begins with two investigations that require students to examine data from surveys and critique the design of surveys that have already been conducted. The questions include a review of some basic ideas of probability. In the final investigation of this section, students are led through the process of administering their own survey.

Section III: Experiments starts with an investigation in which students practice using the terminology of experiments as they review the details of two studies involving dieting and weight loss. In the next investigation, students are guided through the process of designing an experiment to test the effect of listening to music on memory. Once they have collected the data, students must use the data analysis and interpretation skills they developed in Section I to help answer the research question. Students get to design, execute, and analyze results from their own experiments in the final investigation of this section.

Section IV: Drawing Conclusions introduces students to the basic ideas of inference—estimating a population characteristic and testing a claim about a population characteristic. Simulation is used to quantify the sample-to-sample variability that occurs in repeated random sampling. This chance variation is reflected in the margin of error for an estimate and in the decision-making process for evaluating the validity of a claim about some population characteristic.

Acknowledgements

We would like to thank the American Statistical Association (ASA) for its steady support over the past five years as we have worked on this project. The initial funding for *MSSS* came from an ASA Member Initiative. With the publication of the GAISE Report, the ASA took a leadership role in promoting statistical literacy in the pre-K–12 mathematics curriculum. *MSSS* is designed to support the curriculum framework proposed in the GAISE Report.

Our special thanks go to the ASA/NCTM Joint Committee on Curriculum in Statistics and Probability (JC) for its unwavering commitment to making *MSSS* a reality. At a key point in the project, the JC provided additional funding that allowed the lead authors to meet for three uninterrupted days of writing. Under the leadership of Jerry Moreno, the JC also assisted in reviewing drafts of the student and teacher modules to get *MSSS* into production. We offer a heartfelt thank you to our colleagues who served as reviewers: Martha Aliaga, Brad Hartlaub, Dan Lotesto, Deborah Lurie, Jerry Moreno, Tom Short, and Jeff Witmer.

We particularly wish to acknowledge the contribution of Wanda Bussey and her students from Rufus King High School in Milwaukee, Wisconsin, as well as the students from Katedralskolan in Lund, Sweden, for the survey and data provided in Investigation #15.

Two individuals deserve our heartfelt appreciation for their tireless efforts during the final stages of the project. Nicholas Horton managed all aspects of the production process with remarkable professionalism. His close attention to detail and personal touch were essential in resolving many last-minute issues. Last, but certainly not least, Valerie Snider from the ASA deftly coordinated all elements of the design and production process. Her creative flair and passion for the project is apparent in the final publications. For those late nights and long weekends, and for her uncanny ability to meet seemingly impossible deadlines, we owe Val our sincere gratitude.

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Are hot dogs unhealthy? What percent of people wear their seat belts when driving? Which works better—a low fat diet or a low carbohydrate diet? Would most teenagers keep an extra $10 they received in incorrect change at a store, or return it? Does listening to music hurt students' concentration and ability to study? How are peoples' heights and foot lengths related? These are just a few examples of the types of questions that statistics can help us answer. Getting clear answers to such questions requires data that have been produced according to a careful plan, as the following example illustrates.

Research question: *Do most people wash their hands after using the bathroom?*

Not according to a December 2005 newspaper article titled "Many Adults Report Not Washing Their Hands When They Should, and More People Claim to Wash Their Hands than Who Actually Do."[1] But before you believe such a headline, you should always ask, "Where did the data come from?"

The article mentioned in the previous paragraph was based on two studies that were done in August 2005. In the first study, 1,013 U.S. adults were asked questions about their hand-washing habits by telephone. This is an example of a **survey**. In the second study, observers watched and recorded the actual hand-washing behaviors of 6,336 adults in public restrooms in four major U.S. cities. This is an example of an **observational study**. Both studies were carried out by Harris Interactive, a company that specializes in these kinds of statistical research.

Now that you know how the data were produced, you might be interested in some results from the two studies.

> While 91% of surveyed adults *claimed* to always wash their hands after using the bathroom, only 83% of the adults in the observational study did so.
>
> In the survey, 94% of women claimed to always wash their hands after using the bathroom, compared with 88% of men. In the observational study, 90% of the women actually washed their hands, compared with 75% of men.
>
> A similar observational study done in 2003 revealed that 78% of the adults observed actually washed their hands after using the bathroom. In that study, 83% of the women and 74% of the men were observed washing their hands.

Based on these studies, what can we conclude? Can we conclude that 83% of *all* U.S. adults always wash their hands after using the bathroom? No, because researchers only observed a **sample** of 6,336 adults, not the entire **population** of U.S. adults. If another group of 6,336 adults was observed on a different day, the percent who washed would probably not be exactly 83%. Can we at least say that the actual percent of all U.S. adults who always wash their hands after using the bathroom is "close" to 83%? That depends on what you mean by "close."

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1 Harris Interactive, December 14, 2005.

The process of carrying out a statistical study—like the survey or observational study in the previous example—begins with the clear statement of a **research question**. Basically, the research question describes what you want to know in simple terms. Most research questions relate to some population of interest—a group of people, animals, or things. Once a research question has been established, you need to collect some useful data. It's usually not practical to get data from every individual in the population (a **census**). Instead, we usually try to obtain data from a representative sample of individuals chosen from the population. So how do we get the data?

There are three preferred methods for producing data—**observational studies**, **surveys**, and **experiments**. In an experiment, we deliberately do something to one or more groups of individuals—such as giving a drug to people who are sick—and then measure their responses. Observational studies and surveys, on the other hand, attempt to gather data on individuals as they are. In an observational study, we record values of one or more variables—like gender or height—for a sample of individuals. We can obtain these values from direct observation, measurement, or existing data. In a survey, we select a sample of people and have them answer one or more questions. You have already seen examples of a survey and an observational study about people's hand washing habits. How might an experiment shed more light on this subject?

Some people might argue that having an observer present in the restroom—even if the observer isn't washing his or her hands—could influence an individual's hand-washing behaviors. To test this idea, we could design an experiment. Half of the time, we would station an observer at one of the sinks. The other half of the time, we would "hide" the observer in one of the bathroom stalls with a clear view of the sink area. Then, we could compare the percent of people who washed their hands under each of these experimental conditions, called **treatments**.

Each data production method comes with advantages and limitations that you need to understand before you can plan a study. The method used to produce the data also determines the kinds of conclusions that can be drawn. Choosing the best method for a given research question requires careful thought and a lot of practice.

Once we have our data in hand, we must try to figure out what they're telling us. We begin by making graphs and calculating numerical summaries. Then, we interpret the results of our analysis. Of course, our goal is to answer the original research question. Finally, we can communicate our findings to others who might be interested.

Here is a brief outline that summarizes the entire process.

Carrying Out a Statistical Study[1]

I. Formulate the research question

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1 *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*, The American Statistical Association, January 2007. *www.amstat.org/education/gaise*

Do some background research to understand the nature of the problem.

Think carefully about what you expect to find and why.

II. Collect data

Decide what to measure and how to obtain the measurements. Which method—survey, observational study, or experiment—would be best?

Think about how you will analyze the data.

Be sure to consider ethical issues.

Produce data according to your stated plan.

III. Analyze the data

Use graphical and numerical summaries to describe the data.

If appropriate, use inference methods to estimate population values or test claims about characteristics of the population.

IV. Interpret your results

Draw conclusions from your data analysis. Remember to answer the research question!

Address any limitations in your conclusions that result from the process of data collection and data analysis.

Communicate your findings.

In this module, you will learn how to analyze surveys, observational studies, and experiments that have been planned by others. Then, you'll get to design and carry out your own studies. As you go, keep this in mind: You can't draw sound conclusions from badly produced data.

Here's another important principle to remember: Statistical studies should be conducted in an ethical manner. Avoid the use of deception whenever possible and ensure that survey participants and experimental subjects are informed about the purpose of the study and any potential risks associated with their participation. If study subjects are people, they must provide their informed consent to participate after being made aware of any potential risks that may result from taking part in the study. For studies involving minors, parent/guardian permission is required. If the study uses animals as subjects, researchers should follow published guidelines for humane treatment of animals, such as those published by the American Psychological Association (see *www.apa.org/science/anguide.html*). Researchers should also ensure the anonymity and confidentiality of peoples' responses and behaviors unless participants have been informed that responses will not be confidential. For experiments, it is common to have a review board approve the experimental design in advance and monitor the results of the experiment as data are collected.

Investigation #1: Did you wash your hands?

1. Why should we care whether people wash their hands after using the bathroom?

2. In the Harris Interactive survey, people were contacted by telephone. One of the questions the interviewers asked was, “How often do you wash your hands after using a public restroom?”

(a) Which U.S. adults were not included in this study?

(b) The survey estimated that 91% of all U.S. adults would claim that they always wash their hands after using the bathroom. Do you think this estimate is too high, too low, or about right given your answer to (a)? Explain.

(c) Several people refused to participate in the survey. Give a reason that this might happen.

(d) In any survey, it is possible that some people will not answer a question accurately or honestly. Thinking about the hand-washing survey, do you think this is likely to happen? Explain your answer.

3. The observational study of hand washing was conducted at a baseball field in Atlanta, a museum and an aquarium in Chicago, a bus and train terminal in New York, and a farmers' market in San Francisco.

(a) Observers in the public bathrooms combed their hair or put on make-up at one of the available sinks while they were watching individuals' hand-washing behaviors. If the observation had been done by hidden camera instead (with no observer present), do you think the percent who washed their hands would have been greater than, less than, or about the same as 83%? Justify your answer.

(b) Suppose the observational study had been conducted using hidden cameras in the homes of the same 6,336 adults. Do you think the percent of these individuals who washed their hands would have been greater than, less than, or about the same as 83%? Justify your answer.

4. (a) Comment on the conclusion reached in the newspaper headline: "More People Claim to Wash Their Hands than Who Actually Do."

(b) Describe a study design involving only one group of people that might help us better evaluate the validity of the quoted claim in part (a).

5. Both studies were paid for by the American Society for Microbiology and the Soap and Detergent Association. Should you take this information into account when interpreting the results of the studies? If so, how?

6. You have been asked to help design a study to investigate how often teenagers wash their hands after using the bathroom.

(a) Define a research question for your study.

(b) Would you recommend using a survey, an observational study, or an experiment to produce the data? Explain.

(c) Do you think the percent of teenagers who always wash their hands after using the bathroom is higher than, lower than, or about the same as the percent of adults who do so? Justify your answer.

7. For each of the following research questions, decide which method of data production—a survey, an experiment, or an observational study—would be most appropriate. Justify your choice of method.

(a) What percent of teenagers leave the water running while they brush their teeth?

(b) Which of two drugs is more effective at preventing nausea following the onset of a migraine headache?

(c) Do male teenagers or female teenagers tend to have more numbers stored in their cell phones?

(d) What percent of drivers come to a complete stop at a stop sign near a local elementary school?

(e) Does printing suggested tip amounts on the bottom of a restaurant bill increase the average amount that customers leave in tips?

8. A follow-up study conducted in 2007 by Harris Interactive revealed that while 92% of adults said that they always washed their hands after using the bathroom, only 77% of the adults observed in public restrooms actually did. According to Harris Interactive's Hand Washing Fact Sheet, "The overall decline in hand washing observations is largely due to males. The percentage of males observed washing their hands fell from 75% in 2005 to 66% in 2007. Overall, the percentage of females observed washing their hands is down slightly from 90% in 2005 to 88% in 2007."

Did people's hand washing habits improve or get worse from 2005 to 2007? Justify your answer with specific evidence from the reports describing the Harris Interactive studies.

Section I: Observational Studies

You can learn a lot just by watching. That's the idea of an observational study. If you want to know how often people wash their hands after using the bathroom, don't ask them! Observe them. As we saw in the Introduction, what people say and what they actually do can be quite different. But be sure to keep in mind the old adage: "The observer influences the observed." Merely having an observer present in the restroom might affect the percent of people who wash their hands.

In her book, *The Female Brain*, Dr. Louann Brizendine claimed that women talk almost three times as much as men. Some researchers at the University of Arizona were skeptical, so they designed an observational study to examine this claim. About 400 male and female college students participated in the study. The students wore specially designed recording equipment that turned on automatically at pre-set intervals over several days without the students' knowledge. Researchers then counted words used by the male and female participants. Their findings? Both males and females tended to speak an average of about 16,000 words per day. Dr. Brizendine later admitted that her claim had little factual basis.

Let's consider one further example from industry. Suppose you are in charge of quality control at a factory that produces potato chips. Imagine a string of thousands of very similar looking chips moving one behind the other down a conveyor belt, hour after hour, day after day. At some point in the process, salt is added to each chip. How can you be sure that the chips your factory is producing today don't contain too much or too little salt? Do you have to measure the salt content of every potato chip made today? Of course not. It isn't practical to observe every chip. Even if it were, you wouldn't choose to do that, because measuring the amount of salt on a chip actually destroys the chip. If you examined the salt content of every chip produced that day, you'd have no potato chips left to sell! What should you do instead? Select a sample of chips from that day's production and measure the salt content of the chips in the sample.

The potato chip example reminds us of an issue that was discussed briefly in the Introduction. If we want to get information about some characteristic of a population, such as the salt content of the potato chips produced today, we often tend to measure that characteristic on a sample of individuals chosen from the population of interest. We'd like to draw conclusions about the population based on results from the sample. To generalize from sample to population in this way, we need to know that the sample is representative of the population as a whole.

Suppose you measured the salt content of the last 100 potato chips produced at the factory today and found that the chips were generally too salty. Should you conclude that the entire batch of chips produced today is too salty? Not necessarily. Something may have happened during the last hour of production that affected the saltiness of the chips made at the end of the day. The last 100 chips produced may not be a representative sample from the population of today's potato chips.

So how do we get a representative sample? If we choose the first 100 potato chips, or the last 100, or even 100 chips "willy-nilly" off the conveyor belt, we may obtain a sample in which the chips tend to be consistently saltier than or less salty than the entire batch of chips produced that day. The best way to avoid this problem is to let chance select the sample. For example, you might choose one time "at random" in each of the 10 hours of production and measure the salt content of the next 10 potato chips that pass a certain point on the conveyor belt at those times. This incorporates **random selection** into the way the sample is chosen.

Random selection involves using some sort of chance process—such as tossing a coin or rolling a die—to determine which individuals in a population are included in a sample. If the individuals are people, one simple method of random selection is to write people's names on identical slips of paper, put the slips of paper in a hat, mix them thoroughly, and then draw out one slip at a time until we have the number of individuals we want for our sample. An alternative would be to give each individual in the population a distinct number and use the "hat method" with this collection of numbers, instead of people's names. Notice that this variation would work just as well if the individuals in the population were animals or things instead of people.

The hat method works fine if the population isn't too large. If there are too many individuals in the population, however, we would need a very big hat and many small slips of paper. In such cases, it would be easier to "pretend" that we're using the hat method, but to choose the numbers in a more efficient (but equivalent) way.

Technology is the answer. Computers and many calculators have the ability to select numbers "at random" within a specified range, just like drawing the numbers out of a hat. These devices can generate many numbers at random in a short period of time.

Many statistics textbooks contain entire pages filled with rows of "random digits"—numbers from 0 to 9 generated at random using technology. Such tables of random digits were especially useful before the invention of graphing calculators. Here are four rows of random digits that might appear in such a table:

5 2 7 1 1	3 8 8 8 9	9 3 0 7 4	6 0 2 2 7
4 0 0 1 1	8 5 8 4 8	4 8 7 6 7	5 2 5 7 3
9 5 5 9 2	9 4 0 0 7	6 9 9 7 1	9 1 4 8 1
6 0 7 7 9	5 3 7 9 1	1 7 2 9 7	5 9 3 3 5

Now let's consider an example. Kayla wants to conduct an observational study investigating the average number of contacts stored in teenagers' cell phones. She decides to restrict her attention to seniors, most of whom have cell phones. There are 780 seniors in her high school. How might Kayla use random selection to choose a sample of 30 seniors to participate in the cell phone study?

It would be tedious to write 780 names on slips of paper, so Kayla decides to pretend that she's using the hat method. After getting an alphabetized list of the school's seniors from the office, Kayla numbers the students from 1 to 780 in alphabetical order. To choose 30 seniors at random, Kayla can then use either a random digits table or a random number generator.

Random digits table: To use a random digits table, Kayla could look at groups of three digits, which could range from `000` to `999`. If she lets `001` correspond to student 1 on the list, `002` correspond to student 2, and so forth, then `780` would correspond to student 780, the last senior on the list. Numbers `781, 782, …, 000` would not correspond to any of the students on the list. By starting at the left-hand side of a row in the table and reading across three digits at a time, Kayla would continue until she had chosen 30 distinct numbers between `001` and `780`. The corresponding seniors would be the chosen sample.

Using the lines of random digits on the previous page, for example,

5 2 7 **1 1**	**3** 8 8 8 **9**	**9 3** 0 7 4	**6 0 2** 2 7
4 **0 0 1** 1	8 5 **8 4 8**	4 8 7 **6 7**	**5** 2 5 7 **3**
9 5 5 9 2	**9 4 0** 0 7	6 **9 9 7** 1	9 1 **4 8 1**
6 0 7 **7 9**	**5** 3 7 9 **1**	**1 7** 2 9 7	**5 9 3** 3 5

the senior numbered `527` would be chosen first, and the senior numbered `113` would be selected second. Kayla would skip the numbers `888` and `993` because they don't correspond to any seniors, and so on. Continuing likewise, the first 10 students in the sample would be the seniors numbered `527, 113, 074, 602, 274, 001, 185, 487, 675`, and `257`. The eleventh student selected would be the senior numbered `395`. Do you see why?

Random number generator: Kayla could also use her calculator or computer to generate a "random integer" from 1 to 780. She would repeat this until she got 30 distinct numbers from 1 to 780. The seniors on the alphabetized list with the corresponding numbers would be the chosen sample.

In this example, Kayla entered the command `randInt(1,780)` on a TI-84 calculator and pressed ENTER several times to repeat the command. The first ten resulting numbers were `718, 512, 653, 416, 190, 89, 689, 519, 470`, and `44`. So the seniors with these numbers would be included in her sample.

Random Integer Generator

Here are your random numbers:

741	72	355	297	755
559	398	629	47	310
536	304	752	397	483
388	405	149	634	699
739	152	721	516	640
293	589	714	771	566

We used the "random integer generator" at *www.random.org* as an alternative and came up with the numbers here.

If random selection is accomplished by using the hat method or mimicking it with random numbers, the resulting sample is called a **random sample**. To be classified as a random sample, the *n* selected individuals must have been chosen by a method that ensures:

(1) each individual in the population has an equal chance to be included in the sample

(2) each group of *n* individuals in the population is equally likely to be chosen as the sample

In the cell phone study example, Kayla did obtain a random sample. Once she selected the students for her observational study, it might have been quite difficult for Kayla to locate the 30 seniors who were chosen in a school with so many students, however. For practical reasons, Kayla might have used a method of random selection that didn't result in a truly random sample.

If, for example, the 780 seniors were assigned to 30 homerooms of 26 seniors each based on their last names, Kayla might have decided to select one student at random from each homeroom for her cell phone study. Notice that this alternative method of random selection does give each senior in Kayla's school an equal chance to be included in the sample, but it does not give every group of 30 seniors an equal chance to actually be chosen as the sample. In fact, with this method, the chance of getting a sample with two or more students from the same homeroom is zero!

Think back to the potato chip example for a minute. Can you imagine how difficult it would be to take a random sample from all of the potato chips produced in one day? Just picture someone numbering the individual potato chips for starters! It would be much more feasible to select, say, 10 consecutive potato chips from a particular spot on the conveyor belt by choosing a time at random during each hour of production.

Some observational studies do not use random selection to select the individuals who participate. In the hand-washing study from the Introduction, for example, observers simply watched whoever happened to be in public restrooms at the time. Perhaps the kinds of people who use public restrooms at sporting events, in museums or aquariums, and in train stations have different hand-washing habits than the population of adults as a whole.

The researchers from the University of Arizona used volunteer college students from the United States and Mexico in their observational study of talking patterns by gender. Because of the way in which their sample was chosen, their conclusion about male and female talking tendencies wouldn't necessarily apply to older adults or to college students from other countries. In fact, the results might not even extend to all college students, since some—perhaps those who talk a lot—might have refused to participate in the study. Lack of random selection limits our ability to generalize from the sample to a larger population of interest.

In the investigations that follow, you will learn more about designing and analyzing results from observational studies. You will see firsthand how the presence or absence of random selection affects our ability to generalize.

Investigation #2: Get Your Hot Dogs Here!

If baseball is America's game, then hot dogs are America's food. Whether you are at a sporting event, a backyard barbecue, or even a local convenience store, you are bound to see folks wolfing down frankfurters. Why do so many people like to eat hot dogs? For the yummy taste, of course! But what makes hot dogs taste so good? Unfortunately for health-conscious eaters, it's probably the fat and sodium they contain. Not all hot dogs are created equal, however. Some are made from beef, others from poultry, and still others from a combination of meats. With so many varieties available, can hot dog lovers find a healthy option that still tastes great?

Several years ago, Consumers Union, an independent nonprofit organization, tested 54 brands of beef, meat, and poultry hot dogs. For each brand tested, they recorded calories, sodium, cost per ounce, a protein-to-fat rating, and an overall sensory rating that included taste, texture, and appearance. The table below and those on the following two pages summarize some of their findings, which were published in *Consumer Reports*.[1] Note that the hot dogs are categorized by type—meat, beef, and poultry.

Meat Hot Dogs

Brand	Protein-to-Fat	Calories per Frank	Sodium per Frank (mg)	Overall Sensory Rating
Armour Hot Dogs	Poor	146	387	Average
Ball Park	Poor	182	473	Above Avg.
Bryan Juicy Jumbos	Poor	175	507	Average
Eat Slim Veal	Average	107	144	Average
Eckrich Jumbo	Poor	179	405	Average
Eckrich Lean Supreme Jumbo	Average	136	393	Average
Farmer John Wieners	Below Avg.	139	386	Average
Hormel 8 Big	Below Avg.	173	458	Above Avg.
Hygrade's Hot Dogs	Poor	195	511	Average
John Morrell	Poor	153	372	Average
Kahn's Jumbo	Poor	191	506	Above Avg.
Kroger Jumbo Dinner	Poor	190	545	Above Avg.
Oscar Mayer Wieners	Poor	147	360	Above Avg.
Safeway Our Premium	Below Avg.	172	496	Above Avg.
Scotch Buy with Chicken & Beef	Poor	135	405	Below Avg.
Smok-A-Roma Natural Smoke	Poor	138	339	Below Avg.
Wilson	Poor	140	428	Below Avg.

1 "Hot dogs: There's not much good about them except the way they taste," *Consumer Reports*, June 1986.

Beef Hot Dogs

Brand	Protein-to-Fat	Calories per Frank	Sodium per Frank (mg)	Overall Sensory Rating
A & P Skinless Beef	Poor	157	440	Average
Armour Beef Hot Dogs	Poor	149	319	Average
Best's Kosher Beef	Below Avg.	131	317	Average
Best's Kosher Beef Lower Fat	Average	111	300	Average
Eckrich Beef	Poor	149	322	Average
Hebrew National Kosher Beef	Poor	152	330	Average
Hygrade's Beef	Poor	190	645	Average
John Morrell Jumbo Beef	Poor	184	482	Average
Kahn's Jumbo Beef	Poor	175	479	Average
Kroger Jumbo Dinner Beef	Poor	190	587	Average
Mogen David Kosher Skinless Beef	Below Avg.	139	322	Average
Nathan's Famous Skinless Beef	Below Avg.	181	477	Above Avg.
Oscar Mayer Beef	Poor	148	375	Average
Safeway Our Premium Beef	Poor	176	425	Above Avg.
Shofar Kosher Beef	Below Avg.	158	370	Average
Sinai 48 Kosher Beef	Below Avg.	132	253	Below Avg.
Smok-A-Roma Natural Smoke	Below Avg.	141	386	Average
Thorn Apple Valley Brand	Poor	186	495	Above Avg.
Vienna Beef	Below Avg.	135	298	Average
Wilson Beef	Poor	153	401	Average

Poultry Hot Dogs

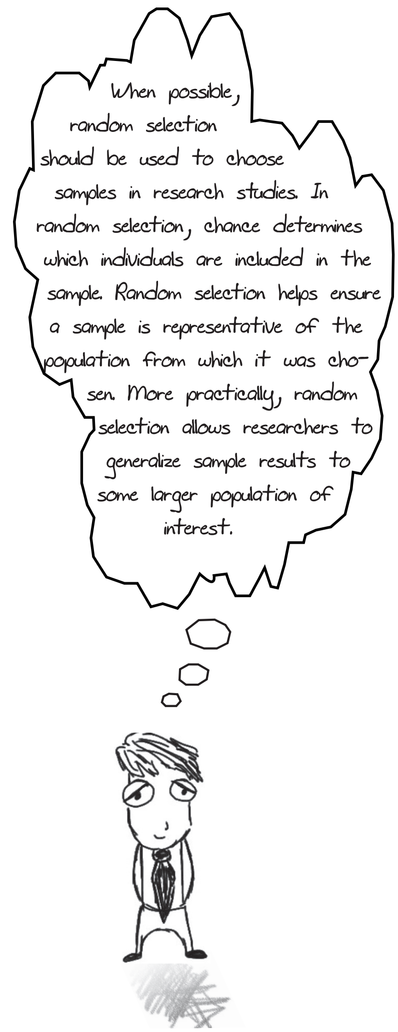
Brand	Protein-to-Fat	Calories per Frank	Sodium per Frank (mg)	Overall Sensory Rating
Foster Farms Jumbo Chicken	Below Avg.	170	528	Average
Gwaltney's Great Dogs Chicken	Below Avg.	152	588	Average
Holly Farms 8 Chicken	Below Avg.	146	522	Average
Hygrade's Grillmaster Chicken	Average	142	513	Average
Kroger Turkey	Excellent	102	542	Average
Longacre Family Chicken	Above Avg.	135	426	Average
Longacre Family Turkey	Above Avg.	94	387	Average
Louis Rich Turkey	Average	106	383	Average
Manor House Chicken (Safeway)	Average	86	358	Average
Manor House Turkey (Safeway)	Excellent	113	513	Average
Mr. Turkey	Average	102	396	Average
Perdue Chicken	Average	143	581	Average
Shenandoah Turkey Lower Fat	Above Avg.	99	357	Average
Shorgood Chicken	Below Avg.	132	375	Average
Tyson Butcher's Best Chicken	Average	144	545	Below Avg.
Weaver Chicken	Below Avg.	129	430	Above Avg.
Weight Watchers Turkey	Excellent	87	359	Average

The *Consumer Reports* article did not provide many details about how the hot dog data were produced. Our best guess is that Consumers Union first obtained one package of each of the 54 brands of hot dogs they intended to test. For each brand, they could then determine the protein-to-fat rating and the calories and sodium per frank from information provided on the package. To prepare the hot dogs for taste testing, Consumers Union cooked each frankfurter in boiling water.

1. Did Consumers Union produce these data using a survey, an experiment, or an observational study? Justify your answer.

2. According to the data table, Oscar Mayer beef hot dogs have 148 calories per frank. Does this mean that *every* Oscar Mayer beef hot dog has exactly 148 calories, or is there some variability in calorie count from frank to frank? Explain.

3. Why didn't Consumers Union cook some hot dogs in the microwave, others on a grill, and the rest in boiling water?



4. For the taste testing, would it have been better to rate one hot dog of each brand, or to get an average sensory rating for several hot dogs of each brand? Why?

5. It is possible that someone from Consumers Union went to one grocery store in a particular city and picked up one easy-to-reach packet of each brand of hot dogs. Would this **convenience sampling** method result in a representative sample of each brand of hot dogs? Why or why not?

6. Suppose Consumers Union had used random selection to choose a package of Armour beef hot dogs from a single grocery store for testing. If they obtained an average sensory rating for all the hot dogs in the selected package, to what population could they generalize their results—all Armour beef hot dogs ever produced, all Armour beef hot dogs that have ever been sent to this store, or all Armour beef hot dogs in this store at the time the sample was chosen? Justify your answer.

In this study, Consumers Union recorded several variables for each brand of hot dog, including type of hot dog, protein-to-fat rating, calories, sodium, and sensory rating. Two of these are **quantitative variables**—calories and sodium. Type of hot dog, protein-to-fat rating, and sensory rating are **categorical variables**. When we analyze data, the types of graphs and numerical summaries we should use are determined by the type of data we are analyzing. We begin by examining two of the categorical variables: type of hot dog and protein-to-fat rating.

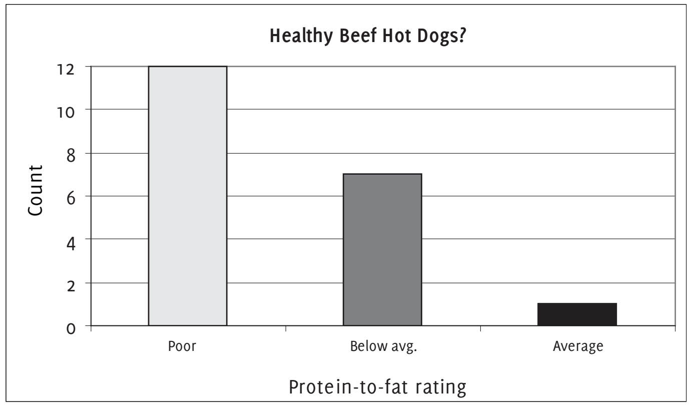
7. Here is a **two-way table** that summarizes the protein-to-fat ratings by type of hot dog.

		Type of Hot Dog		
Protein-to-Fat Rating		Beef	Meat	Poultry
	Poor	12	12	0
	Below Avg.	7	3	5
	Average	1	2	6
	Above Avg.	0	0	3
	Excellent	0	0	3

(a) What percent of hot dogs with a below average protein-to-fat rating were made from poultry?

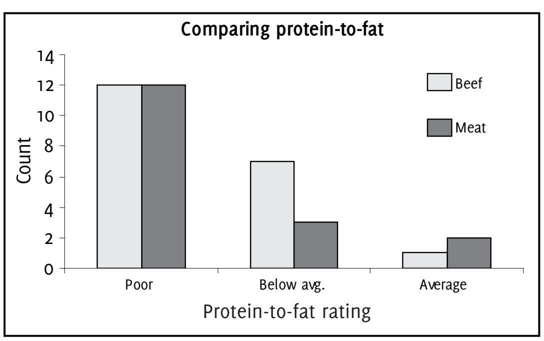
(b) What percent of poultry hot dogs had below average protein-to-fat ratings?

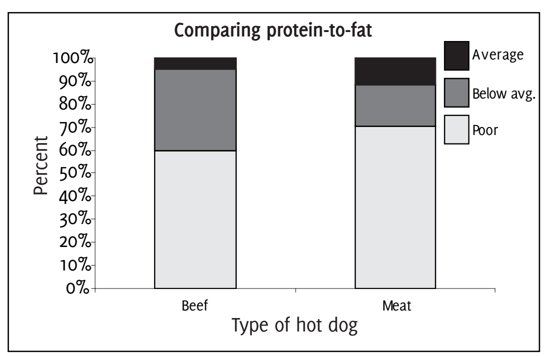
8. Here is an Excel bar graph of the protein-to-fat rating data for the beef hot dogs.



Describe what the graph tells you about protein-to-fat ratings in beef hot dogs.

9. Two Excel bar graphs that could be used for comparing the protein-to-fat ratings for beef and meat hot dogs are displayed below.

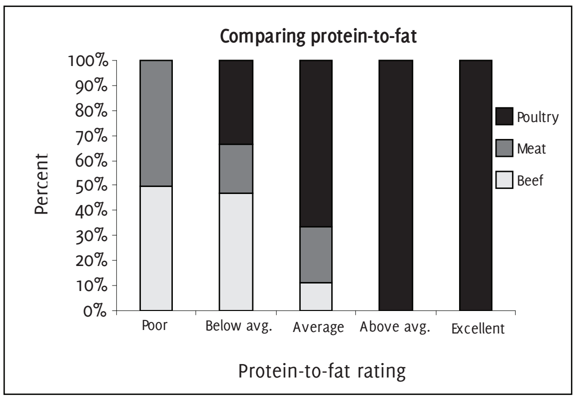


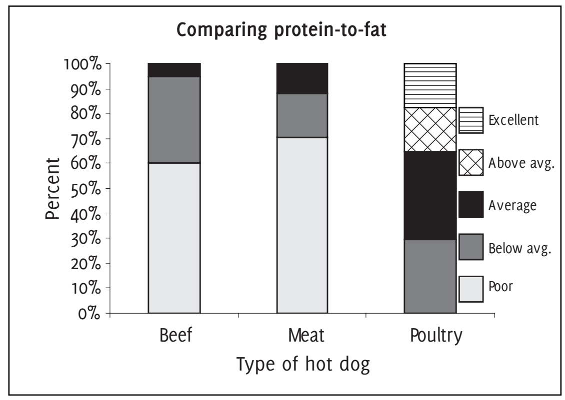


(a) Which graph is more appropriate for making this comparison? Explain.

(b) Write a few sentences comparing protein-to-fat ratings for beef and meat hot dogs.

10. Two different bar graphs that could be used for comparing the protein-to-fat ratings for all three types of hot dogs are displayed below.



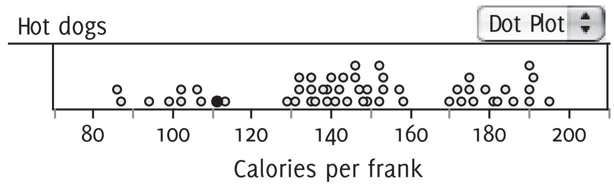


(a) Which graph is more appropriate for making this comparison? Explain.

(b) In terms of protein-to-fat ratings, which type of hot dogs is healthiest? Justify your answer with appropriate graphical and numerical evidence.

Now let's look at the calorie content for different brands of hot dogs.

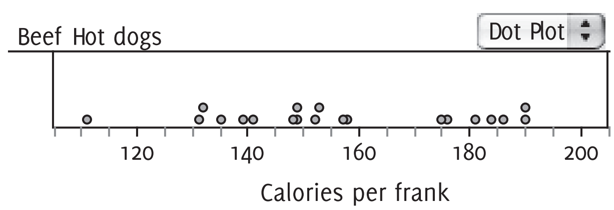
11. A dotplot of the calorie data for all 54 brands of hot dogs is shown below.



(a) Why do you think this distribution has three distinct clusters? Check whether your hunch is accurate.

(b) Identify the brand and type of hot dog for the highlighted point.

12. A dotplot of the calorie content for the 20 brands of beef hot dogs is shown below. Describe the interesting features of this distribution.



13. How does the calorie content of beef and meat hot dogs compare? A partially completed back-to-back stemplot of the calorie data for these two types of hot dogs is shown below.

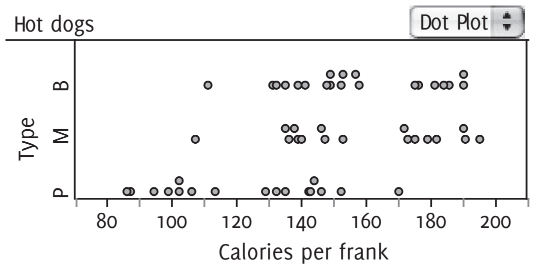
```
Meat              Beef

          |10|
          |11| 1
          |12|
          |13| 1 2 5 9
          |14| 1 8 9 9
          |15| 2 3 7 8
          |16|
          |17| 5 6
          |18| 1 4 6
          |19| 0 0
```

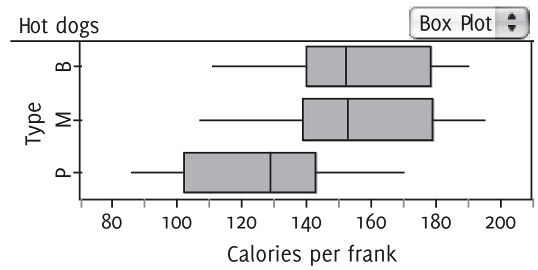
Key: 13| 1 means this brand advertises 131 calories per frank

(a) Add the calorie data for the meat hot dogs to the stemplot. Note that in a back-to-back stemplot, the "leaves" increase in value as you move away from the "stem" in the center of the graph.

(b) Comment on any similarities and differences in the distributions of calories per frank for these two types of hot dogs. Be sure to address center, shape, and spread, as well as any unusual values.

14. To compare calories per frank for all three types of hot dogs, we used computer software to construct graphs and numerical summaries.





Descriptive Statistics: Calories per Frank by Type

Variable	Type	N	Mean	Median	TrMean	StDev
Calories	B	20	156.85	152.50	157.56	22.64
	M	17	158.71	153.00	159.73	25.24
	P	17	122.47	129.00	121.73	25.48

Variable	Type	SE Mean	Minimum	Maximum	Q1	Q3
Calories	B	5.06	111.00	190.00	139.50	179.75
	M	6.12	107.00	195.00	138.50	180.50
	P	6.18	86.00	170.00	100.50	143.50

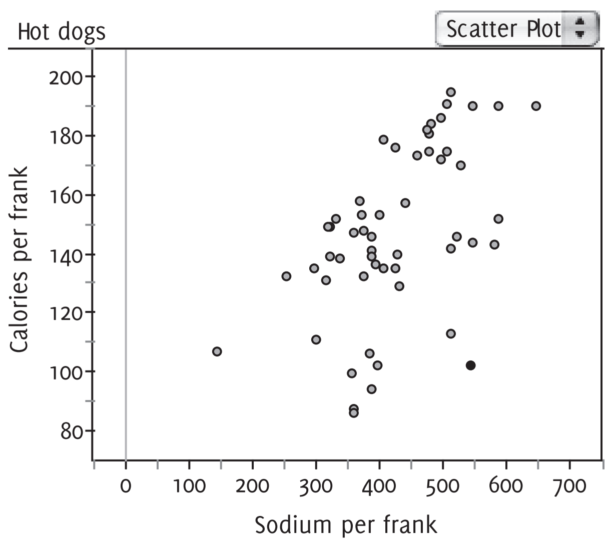
(a) Describe one advantage of using the dotplot instead of the boxplot to display these data.

(b) Describe one advantage of using the boxplot instead of the dotplot to display these data.

(c) How do beef, meat, and poultry hot dogs compare in terms of calorie content? Justify your answer using appropriate graphical and numerical information.

Research Question: Is there a relationship between the calorie content and the amount of sodium per frank in these brands of hot dogs?

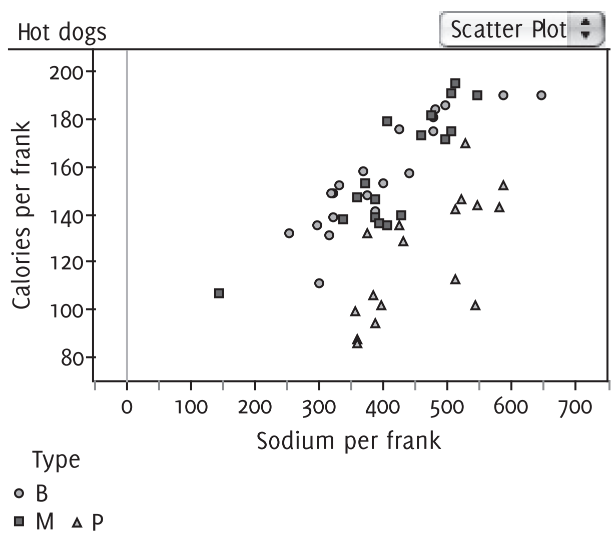
15. The scatterplot below summarizes the sodium and calorie data for the 54 brands of hot dogs in the Consumers Union study.



(a) Describe any interesting features of the scatterplot in the context of this study.

(b) What is unusual about the highlighted point in the scatterplot on the previous page?

Here is another scatterplot of the sodium and calorie data with the type of hot dog identified.



(c) What more can you say about the relationship between sodium and calories per frank when type of hot dog is considered?

16. The next two displays show some numerical summaries of the calorie and sodium data.

Hot dogs

	Calories per frank	Sodium per frank (mg)
	146.611	424.833
	29.0773	95.8564

S1 = mean ( )
S2 = stdDev ( )

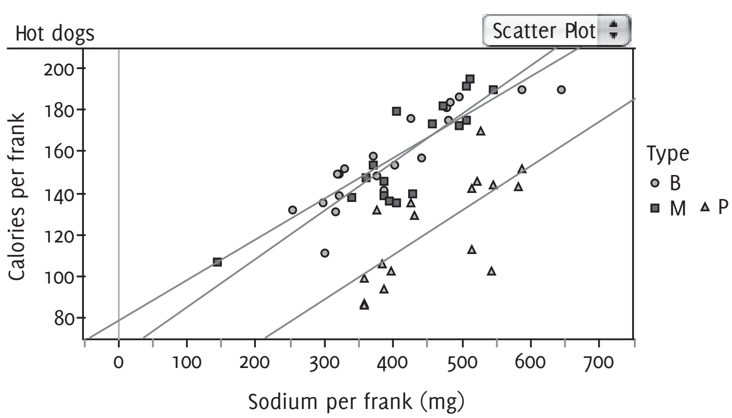
Hot dogs

	Calories per frank
Sodium per frank (mg)	0.516054

S1 = correlation ( )

(a) What additional information about the relationship between sodium and calorie content of hot dogs do these numerical summaries provide?

The graph below includes three summary lines—one describing the relationship for each type of hot dog.



— ○ Calories per frank = 78 + 0.196Sodium per frank (mg); $r^2 = 0.79$
— ■ Calories per frank = 62 + 0.232Sodium per frank (mg); $r^2 = 0.75$
— △ Calories per frank = 24 + 0.214Sodium per frank (mg); $r^2 = 0.51$

(b) Interpret the slope and the *y*-intercept of the summary line for beef hot dogs.

(c) Suppose Consumers Union had chosen another brand of meat hot dog, beef hot dog, and poultry hot dog, each having 300 milligrams of sodium per frank. What would you predict for the calories per frank in each case? Explain how you made your prediction.

(d) Based on the graph on the previous page, which of the predictions in the previous question do you think would be most accurate? Explain.

17. In the Consumers Union study, beef hot dogs had a mean calorie content of 156.85 calories per frank, compared to 158.71 calories per frank for meat hot dogs and 122.47 calories per frank for poultry hot dogs. Would you feel comfortable generalizing this result about calorie content to the *population* of all brands of beef, meat, and poultry hot dogs? Why or why not?

18. What about the taste? Consumers Union gave an overall sensory rating, which included texture, taste, and appearance. The following table summarizes the ratings by type of hot dog.

		Sensory Rating		
Type of Hot Dog		Above Avg.	Average	Below Avg.
	Beef	3	16	1
	Meat	6	8	3
	Poultry	1	15	1

Which type of hot dog had the best overall sensory ratings? Prepare a brief report that includes graphical and numerical evidence to support your answer.

19. How salty are they? Which have more sodium per frank—beef, meat, or poultry hot dogs? Carry out an analysis that includes graphs and numerical summaries to help answer this question. Write a brief report that summarizes your analysis on a separate piece of paper.

Investigation #3: What's in a Name?

According to the *Seattle Times* (Oct. 5, 2003), there will be a lot of Jacobs and Emilys in the high-school graduating class of 2020—those were the most popular baby names in the United States in 2002 according to Social Security card applications.

It's nice to be popular, and great to be "cool." The authors of the book *Cool Names for Babies* (Satran, Pamela & Rosenkrantz, Linda, Harper Collins Publishers, 2004) say that it is the unusual names that are most cool.

In this activity, you will carry out an observational study to assess the popularity and coolness of your class based on the names of the students in class.

Getting Started

To complete this activity, you will need to use the Social Security Administration's Popular Baby Name web site. It can be found at *www.ssa.gov/OACT/babynames*.

On this site, you will be able to find lists of the 10 most popular baby names for boys and girls in each year starting in 1880. These lists were compiled using a random sample consisting of 1% of all babies born in a particular year who subsequently applied for a social security card. You will also find a list of the top 1,000 names for each decade from the 1900s to the 2000s.

Spend a few minutes familiarizing yourself with the information available on this web site. Then, start answering the questions that follow.

1. Let's start with an easy question! What is your first name?

2. Are you male or female?

3. In what year were you born?

4. Is your name one of the 10 most popular names for the year in which you were born?

5. Is your name one of the 10 most popular names for the most recent year for which data are available?

6. Is your name one of the most popular 1,000 for the decade in which you were born? If so, record your name's rank. If your name is not in the top 1,000, just record that your name is "cool"!

7. After each student in your class has answered questions 1–6, enter the data from the entire class into the following table.

First Name	Gender	Year Born	In Top 10 for Year Born? (Yes or No)	In Top 10 for Most Recent Year? (Yes or No)	Rank for Decade Born (1-1,000 or cool)

8. Is there a most common name for the class? If so, what is the most common name?

9. What is the most common year of birth for the class?

10. In the year that was the most common birth year for the class, what is the most popular name for boys according to the popular baby names web site? For girls? Does anyone in the class have these most popular names?

11. What proportion of the class has "cool" names?

12. Omitting the cool names from the data set, construct a graphical display that shows the distribution of the decade ranks data. How would you describe this distribution? (Comment on shape, center, spread, and any unusual values.)

13. What proportion of the class has names that were in the top 10 names for the year in which they were born?

14. Based on your answers to questions 11 and 13, is your class more popular or more "cool?"

15. What proportion of the class has names that are listed in the top 10 for the most recent year for which data are available?

16. Is the proportion from question 15 lower than, about the same as, or higher than the proportion from question 13? How does this suggest that the popularity of the class' names has changed over time?

17. What makes this study an observational study, rather than an experiment?

18. Was there random selection in the data collection for this study? How does this affect your ability to generalize from the study?

19. How might you modify this study if your goal was to generalize to all students at your school? To all high-school students in your school district? To all high-school students in your state?

Investigation #4: If the Shoe Fits . . .

Welcome to CSI at School. Over the weekend, a student entered the school grounds without permission. Even though it appears the culprit was just looking for a quiet place to study undisturbed by friends, school administrators are anxious to identify the offender and have asked for your help. The only available evidence is a suspicious footprint outside the library door.

In this activity, you will use data on shoe print length, height, and gender to help develop a tentative description of the person who entered the school.

After the incident, school administrators arranged for the data in the table below to be obtained from a random sample of this high school's students. The table shows the shoe print length (in cm), height (in inches), and gender for each individual in the sample.

Shoe Print Length	Height	Gender	Shoe Print Length	Height	Gender
24	71	F	24.5	68.5	F
32	74	M	22.5	59	F
27	65	F	29	74	M
26	64	F	24.5	61	F
25.5	64	F	25	66	F
30	65	M	37	72	M
31	71	M	27	67	F
29.5	67	M	32.5	70	M
29	72	F	27	66	F
25	63	F	27.5	65	F
27.5	72	F	25	62	F
25.5	64	F	31	69	M
27	67	F	32	72	M
31	69	M	27.4	67	F
26	64	F	30	71	M
27	67	F	25	67	F
28	67	F	26.5	65.5	F
26.5	64	F	27.25	67	F
22.5	61	F	30	70	F
			31	66	F

Use the data provided to answer the questions that follow.

1. Construct an appropriate graph for comparing the shoe print lengths for males and females.

2. Describe the similarities and differences in the shoe print length distributions for the males and females in this sample.

3. Explain why this study was an observational study and not an experiment.

4. Why do you think the school's administrators chose to collect data on a random sample of students from the school? What benefit might a random sample offer?

5. If the length of a student's shoe print was 32 cm, would you think the print was made by a male or a female? How sure are you that you are correct? Explain your reasoning.

6. How would you answer question 5 if the suspect's shoe print length was 27 cm? 29 cm?

7. Construct a scatterplot of height versus shoe print length using different colors or different plotting symbols to represent the data for males and females. Does it look like there is a linear relationship between height and shoe print length?

8. Does it look like the same straight line could be used to summarize the relationship between shoe print length and height for both males and females? Explain.

9. Based on the scatterplot, if a student's shoe print length was 30 cm, approximately what height would you predict for the person who made the shoe print? Explain how you arrived at your prediction.

10. The shoe print found outside the library actually had a length of 31 cm. Based on the given data and the analysis of questions 1–9, write a description of the person who you think may have left the print. Explain the reasoning that led to your description and give some indication of how confident you are that your description is correct.

Investigation #5: Buckle Up

Do you wear your seat belt when driving? Do most people? Is seat belt use changing over time? To answer questions such as these (well, at least the last two questions—only you know the answer to the first question, but we sure hope the answer is yes!), the National Center for Statistics and Analysis published data on seat belt use for 48 states. No data were available for New Hampshire or Wyoming.

The data shown in the table at the top of the next page are from a large-scale study conducted annually by the National Highway Traffic Safety Administration.[1] The study involves actual observation of drivers' seat belt use at a random selection of roadway sites in each state.

The table gives the percentage of drivers observed who used seat belts in 2004 and in 2005. The table also shows the change in seat belt use percentage from 2004 to 2005 (computed as 2005 use percentage – 2004 use percentage).

Use the data in the table to answer the following questions.

1. Would comparative dotplots or comparative boxplots be better for comparing the seat belt use rates for 2004 and 2005? Make the graph that you pick. Then write a sentence or two describing the similarities and differences in the seat belt use rate distributions in 2004 and 2005.

2. Construct an appropriate graph that shows the change in seat belt use by state from 2004 to 2005. Comment on any interesting features of the distribution.

1 "Seat Belt Use in 2006—Use Rates in the States and Territories," Traffic Safety Facts, National Highway Traffic Safety Administration, January 2007.

State	2004 Use	2005 Use	Difference	State	2004 Use	2005 Use	Difference
Alabama	80	82	2	Missouri	76	77	1
Alaska	78	83	5	Montana	81	80	-1
Arizona	95	94	-1	Nebraska	79	79	0
Arkansas	64	68	4	Nevada	87	95	8
California	90	93	3	New Jersey	82	86	4
Colorado	79	79	0	New Mexico	90	90	0
Connecticut	83	82	-1	New York	85	85	0
Delaware	82	84	2	No. Carolina	86	87	1
Florida	76	74	-2	North Dakota	67	76	9
Georgia	87	90	3	Ohio	74	79	5
Hawaii	95	95	0	Oklahoma	80	83	3
Idaho	74	76	2	Oregon	93	93	0
Illinois	83	86	3	Pennsylvania	82	83	1
Indiana	83	81	-2	Rhode Island	76	75	-1
Iowa	86	87	1	So. Carolina	66	70	4
Kansas	68	69	1	South Dakota	69	69	0
Kentucky	66	67	1	Tennessee	72	74	2
Louisiana	75	78	3	Texas	83	90	7
Maine	72	76	4	Utah	86	87	1
Maryland	89	91	2	Vermont	80	85	5
Massachusetts	63	65	2	Virginia	80	80	0
Michigan	91	93	2	Washington	94	95	1
Minnesota	82	84	2	West Virginia	76	85	9
Mississippi	63	61	-2	Wisconsin	72	73	1

3. In what way is the graph in question 2 more informative than the graph in question 1?

4. Did most states increase seat belt use from 2004 to 2005? What aspect of the graph you made in question 2 could be used to justify your answer?

5. Compute the mean and median change in seat belt use.

6. What aspect of the graph you made in question 2 explains the large difference between the mean and the median?

7. Would you recommend using the mean or the median to describe the seat belt use change data? Why?

8. Are there any states that stand out as unusual in this data set? If so, which states and what makes them unusual?

9. How did seat belt use in your state change from 2004 to 2005? Would you describe your state as typical with respect to seat belt use change? Explain. (If your state is one of the two states for which no data are given, choose a neighboring state and answer this question for that state.)

10. What makes this seat belt use study observational, rather than an experiment?

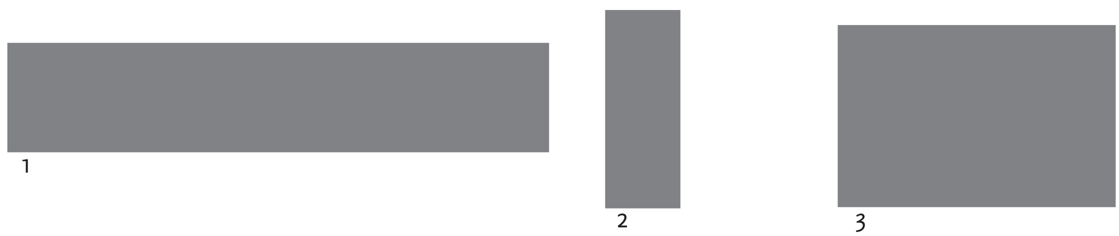
11. Why do you think the study was based on actual observation of drivers, rather than a survey of drivers asking if they use a seat belt when driving?

12. Based on the sampling method used in this study, do you think it would be reasonable to generalize the seat belt use results to drivers at all locations in a given state? Explain.

13. Write a brief summary report describing how seat belt use changed from 2004 to 2005. Include graphs and numerical summaries as appropriate.

Investigation #6: It's Golden (and It's Not Silence)

Which of the three rectangles shown here do you find the most pleasing?



If you picked the third one, you selected the "golden" rectangle. Because they are generally thought to be the most pleasing, golden rectangles are common in art, architecture, and even in the boxes designed for packaging products that are sold in grocery stores.

A rectangle is "golden" if the ratio of its longest side to its shortest side is approximately 1.618.

In this activity, you will design and carry out an observational study to determine if students at your school do, in fact, find golden rectangles more pleasing than other, less-golden ones.

Since the goal is to be able to generalize the study findings to all students at your school, the first thing to think about is how you will select the students who will participate in your study.

1. Describe a way to select study participants that would result in a random sample of students from your school. Don't worry at this point if your plan cannot be easily implemented—instead, focus on what it would take to get a true random sample of students at your school.

2. Do you think it would be possible to actually implement the plan you described in the previous question? Explain.

3. If it would not be possible to carry out the selection plan described in question 1, describe another sampling method that you think would result in a "representative" sample, but not a truly random sample, from your school. Explain why you think a sample selected in the way you propose here could be considered representative of the students at your school.

Now let's think about how you will collect data from the selected students in a way that will enable you to determine if students really do find golden rectangles more pleasing than nongolden rectangles.

4. In the space below, draw a few rectangles that are golden and several nongolden rectangles.

5. In this study, you will be showing the selected students some rectangles and asking which of the rectangles is most pleasing. How many rectangles will you have the selected students choose between? Why did you select this number?

6. Prepare a separate page containing the rectangles to be shown to your study participants.

After your teacher has approved the data collection plan and your page of rectangles, you can proceed to collect the data for your study.

7. Summarize your data in table form and construct an appropriate graphical display of the data.

8. Write a brief report on separate paper that addresses the question "Do students at your school find golden rectangles to be the most pleasing?" Use tables and graphs to support your conclusions.

Section II: Surveys

Three methods for producing data—surveys, observational studies, and experiments—were discussed in the Introduction. In this section, we examine surveys in more detail. A survey is a type of study in which individuals are asked one or more questions. The survey questions are worded so that the resulting responses will provide data that help answer questions about some population of interest.

If every individual in the population provides responses to the survey questions, the study is called a **census**. A census is the usual method of collecting data only if the population of interest is very small—the students in your math class, for example. However, if the population is large, it is more common for only a subset of the population to provide responses to a survey. In this case, the group of individuals who respond to the survey is referred to as a **sample**.

When only a sample participates in a survey, the way in which the individuals in the sample are selected is critical. As with observational studies, if we want to generalize the results of a survey to the entire population, we need to select the sample in a way that is likely to result in a representative sample.

A popular classic movie called "Magic Town" (1936) featured an actor named Jimmy Stewart playing a very successful pollster. He was able to accurately determine the opinions of the entire United States simply by surveying all the residents of a small town called Magic Town. Because this town was a flawless mirror of the entire country, its residents constituted the perfect sample. Unfortunately for those planning surveys, Magic Town is fictional and much more care needs to go into sample selection!

Just as with observational studies, sample selection can be random or nonrandom. To be reasonably confident that the selected sample will be representative of the population, some type of random selection is required. It is sometimes tempting to select the sample in a nonrandom way just because it is convenient to do so. For example, it might be easy to use the students in your math class as a sample of the students at your high school, but there are many reasons why this sample may not be representative of the entire school—the class may consist of mostly seniors, for example. Because there is no way to tell by just looking at a sample if it is representative of the population, our only assurance comes from the method that was used to choose the sample and from the role that random selection played in the choice.

In addition to being thoughtful about how the sample will be selected, it is also important to think carefully about how the actual survey questions will be worded. Each question should be evaluated to determine if it uses appropriate vocabulary and simple sentence structure and to make sure that the question is clear. This will help to ensure that the survey responses, in addition to being representative of the population, are unambiguous and can be generalized in a straightforward manner.

There is one last thing to think about when planning a survey—how large should your sample be? You want the sample to be large enough so that it can reasonably represent the population of interest. On the other hand, it can be both costly and time-consuming to carry out a survey with a large sample size. Because larger samples tend to provide more information than smaller samples, you will need to consider both the desire for a large sample and the available resources for carrying out the survey to arrive at a reasonable sample size.

Planning and carrying out a good survey is a complex task. This overview and the following investigations just provide the basics. You can learn more about surveys in a course in statistics and data analysis. In the investigations that follow, you will explore aspects of planning surveys and analyzing the data that result from them.

In Investigation #9, you will have the opportunity to design and carry out a survey. Collecting survey data involves asking people to share personal opinions or ideas. Not everyone feels comfortable doing that. Any individual has the right to refuse to participate in a survey. When you are in the role of researcher, you must respect that right. It is also your responsibility to preserve the anonymity and confidentiality of responses.

Investigation #7: Welcome to Oostburg!

Oostburg is a small town in Wisconsin. The 306 residents of this town are very data-driven! They are willing and anxious to respond to surveys and give their opinions about various issues. A recent survey was conducted in Oostburg and every person who lives there responded. (Although baby Edna, the youngest citizen of Oostburg at only 8 months old, was not able to answer any of these questions, her parents were willing to respond for her.) This particular survey included questions about age, sex, voting behavior, and participation in various activities during the last month. Data from the survey are summarized in the following two tables.

Age	**17 and Younger**		**18 to 40 Years Old**		**41 to 60 Years Old**		**61 and Older**	
Sex	Male	Female	Male	Female	Male	Female	Male	Female
Number of Responses	36	41	32	35	38	46	32	46

Age	**17 and Younger**		**18 to 40 Years Old**		**41 to 60 Years Old**		**61 and Older**	
Sex	Male	Female	Male	Female	Male	Female	Male	Female
Voted in last town election	0	0	10	12	28	40	29	43
Attended a movie during the last month	4	5	16	22	12	22	4	7
Ate fast food at least once during the last month	6	7	28	33	8	5	3	4
Shopped for clothes online during the last month	5	6	26	28	4	7	0	1
Watched "The Simpsons" during the last month	23	27	29	31	12	8	1	2

What do the data tell us about Oostburg residents? Given that the entire population of Oostburg was surveyed, the above data is a census of the town. Use the given data to answer the following questions.

1. If a resident of Oostburg is to be selected at random, what is the probability that the person selected:

(a) attended a movie during the last month?

(b) attended a movie and is 18 to 40 years old?

(c) is male?

(d) is between 18 and 60 years old?

2. Estimate the probability that a person selected at random is between 10 and 30 years old. Why is this probability more difficult to compute than those of question 1?

3. What is the probability that a person selected at random is male and did not watch "The Simpsons" in the past month?

4. Pose two other probability questions that could be answered using the survey data and then answer those questions by computing the relevant probabilities.

5. Below are seven headlines from the *Oostburg Herald*, a local newspaper. Evaluate the accuracy of each headline based on the survey data. Write a sentence or two giving your assessment of the headline, using the survey data to support your evaluation.

(a) "70% of Eligible Voters Turned Out for Election" (Assume the eligible age of voting in Oostburg is 18.)

(b) "Over 60 Crowd Not Responding to Online Shopping"

(c) "Movies Are Reaching Across ALL the Generations"

(d) "Fast Food Eating a Big Thing with the Younger Crowd"

(e) "'The Simpsons' Not Popular with Older TV Viewers"

(f) "40% of People Over 60 Voted in the Election!"

(g) "Oostburgians Eating Preferences Dependent on Age!"

6. The section overview describes "Magic Town," a town that is a flawless mirror of the entire country. Do you think Oostburg could be such a magic town? Explain your reasoning.

Hugo VanHorn, a senior at Oostburg High School, did not have access to the data from the survey described here. For a school project, Hugo decided to investigate the popularity of online shopping in Oostburg. After band practice, he quickly asked 40 band members if they had shopped for clothes online in the past month. The results from his survey are summarized below:

Have you shopped for clothes online during the past month?	
Yes	No
28	12

Hugo was quite impressed with his results so he wrote a report about the popularity of online shopping in Oostburg. His report indicated that 70% of the residents of Oostburg had shopped for clothing online in the past month.

7. Is the statement that 70% shopped for clothing online in the past month an accurate summary of Hugo's sample? Explain your answer.

8. Is Hugo's statement that 70% of all Oostburg residents shopped for clothing online during the past month an accurate statement? Justify your answer.

9. Hugo's sample was a convenience sample; he did not randomly select his survey participants from the residents of Oostburg. As a consequence, Hugo's sample was not representative of the Oostburg population. In fact, residents in one of the age groups were over-represented in his sample. Based on the census survey data, which age group do you think was over-represented in Hugo's sample? Explain your reasoning.

10. Why would it have been better for Hugo to have used random selection in choosing the 40 people who would participate in his survey?

11. Assuming that Hugo would like to be able to use survey data to generalize to the Oostburg population, write a brief set of instructions that Hugo could use to select 40 participants for a new survey.

Investigation #8: Student Participation in Sports

The short article below is from the student newspaper at Rufus King High School. Use the information from the article to answer the following questions.

Student Survey Finds Females More Involved in Sports

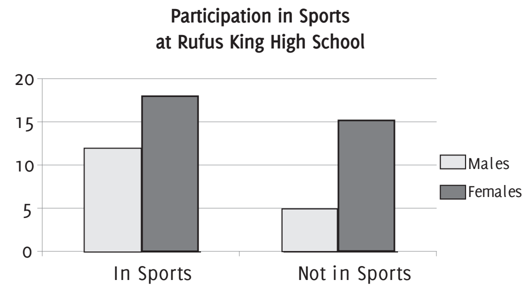
By Kayla Johnson

A Rufus King mathematics class conducted a survey to investigate student participation in extra-curricular activities. Fifty randomly selected students participated in the survey.

One question on the survey asked about participation in school sports programs. The accompanying graph shows participation by sex.

Shauna Rafferty, a junior at Rufus King and captain of the girls' soccer team, noted that there were more females in the sample that participated in sports. She said, "I think the girls in our school are more active in sports than the males. I am sure the success of our soccer team played a large role in this increased interest."

Bryon Jones, a junior on last year's state championship boys' basketball team did not agree. He responded, "All of the guys in my classes belong to one of the sports programs in the school."



Mr. Samuelson, the athletic director of the school, indicated that the number of sports programs and the number of students participating in the programs has posed a real problem in scheduling practice sessions. "Hopefully we will not have to eliminate some of the programs available to our students because students are not able to get adequate practice session time," Mr. Samuelson indicated.

Mr. Samuelson further stated that it was difficult to balance the demand for time in the weight room, the gym, and the outdoor fields.

1. How many of the 50 students surveyed were females involved in sports?

2. How many of the 50 students surveyed were females not involved in sports?

3. How many of the 50 students surveyed were males involved sports?

4. How many of the 50 students surveyed were males not involved in sports?

5. The article says that the 50 students surveyed were randomly selected. Describe one way in which this random selection might have been accomplished.

6. For each of the sample selection methods listed below, give two reasons why random selection of survey participants would be preferable.

(a) Give the survey to the first 50 students who arrive on campus on a Friday morning.

(b) Give the survey to all of the students enrolled in the school's two sections of pre-calculus.

(c) Give the survey to all students who use the weight room on a particular day.

7. What percent of the 50 students in the survey sample were female?

8. What percent of the 50 students in the survey sample were male?

9. What are two reasons that the percent of females in the survey sample might not be 50%?

10. Based on the results of the survey, do you think that the number of girls attending the school is about the same as the number of boys attending the school? Explain your reasoning.

11. Is Shauna correct in her statement that there were more girls in the sample who participated in sports than boys who participated in sports?

12. Explain why Shauna may not be correct in her statement that the survey results imply that girls at the school are more interested in sports than boys.

13. The headline in the school newspaper states more females participate in sports. Explain how this statement could be considered accurate and explain how this statement is at the same time misleading.

14. Write a replacement headline that is not misleading, and then write a few sentences that you think accurately summarize the survey results.

Investigation #9: Planning and Conducting a Survey

In this investigation, you will develop a sampling plan and carry out a survey to investigate tooth-brushing behavior of students at your school. Consider the following two recommendations.

From *www.animated-teeth.com*:

> As you might guess, many humans simply aren't self-disciplined enough to brush properly when they use a manual toothbrush. As a general rule, most people should brush their teeth at least twice a day with each **brushing period encompassing at least two to three minutes**. The fact of the matter is that most of us fail to routinely meet these guidelines.

From a *Los Angeles Daily News* (December 15, 2007) article titled "Water District Asks Users to Cut Back by 10 Percent. Drought Depleted Supplies Spur Voluntary, Mandatory Measures to Limit Consumption"

> The Las Virgenes Water District is asking residents to reduce water use by 10 percent and is ordering farmers to cut back by a third. In seeking voluntary and mandatory cutbacks, the district follows the lead of Long Beach and other cities responding to an ongoing drought. "With no relief to the drought in sight, we must take steps now to ensure we have adequate supplies for the coming year," said John Mundy, the district's general manager. "We are dealing with water cutbacks throughout the state." … Since nearly 70 percent of water is used outdoors, the district is asking residents to reduce use, water every other day and to sweep, rather than hose off, driveways. They also called upon residents to fix leaks, take shorter showers, and **shut off faucets while shaving or brushing teeth**.

1. Write a set of survey questions that would allow you to get responses regarding the following three characteristics of selected students at your school:

Sex of the survey participant

Whether or not the survey participant leaves the water on or turns the water off while brushing his or her teeth

How long, in seconds, the survey respondent thinks that he or she spends when brushing his or her teeth

You can also include other questions you think might be of interest.

2. As a class, discuss the proposed survey questions and come to an agreement on the wording of the questions to be included in the survey. Record the final version of the survey questions below.

3. As a class, discuss whether you think it would be easy or difficult to obtain a random sample of 50 students at your school and to obtain the desired survey information from all the students selected for the sample. Write a few sentences summarizing the class discussion in the space below.

4. As a class, decide how you will go about selecting a sample of 50 students that reasonably could be considered representative of the population of students from your school. Write a brief description of the sampling plan, and point out the aspects of the plan that make it reasonable to argue that it will be representative.

5. Carry out the survey and record the responses in the table below. If you included additional questions in your survey, you can modify the data sheet as needed. As a reminder: *Collecting survey data involves asking people to share personal opinions or ideas. Not everyone feels comfortable doing that. Any individual has the right to refuse to participate in a survey. When you are in the role of researcher, you must respect that right. It is also your responsibility to preserve the anonymity and confidentiality of students' responses.*

Survey Data

Respondent	Sex (M or F)	Water Off (Y or N)	Time Spent Brush-ing	Respondent	Sex (M or F)	Water Off (Y or N)	Time Spent Brushing
1				26			
2				27			
3				28			
4				29			
5				30			
6				31			
7				32			
8				33			
9				34			
10				35			
11				36			
12				37			
13				38			
14				39			
15				40			
16				41			
17				42			
18				43			
19				44			
20				45			
21				46			
22				47			
23				48			
24				49			
25				50			

Now use the survey data to answer the following questions.

6. Construct a bar chart of the "water off" data. What proportion of survey respondents reported that they turn the water off while brushing their teeth?

7. Think for a minute about how the students in the sample were chosen. Do you think the proportion of students at your school who report that they turn the water off while brushing is likely to be much smaller than, much larger than, or somewhere near the value of the proportion computed in question 6? What aspect of the survey design supports your answer?

8. Sometimes there is a difference between what people say they do and what they *actually* do. Do you think this might be the case for the "water off" question? Explain your reasoning.

9. What proportion of the girls in the survey sample report that they turn the water off while brushing? How does this compare to the proportion of boys that say they turn off the water?

10. Use the reported brushing time data to construct a dotplot. Write a few sentences describing what the dotplot tells you about the distribution of brushing times.

11. Now construct a dotplot that uses color to distinguish between the reported brushing times of females and the reported brushing times of males (use one color for dots that correspond to responses that came from females and a different color for the dots that represent responses from males). Does this plot suggest that females tend to report longer brushing times? Explain.

12. Find the median of the data set consisting of the 50 reported brushing times. Divide the survey responses into two groups—those whose reported brushing times were less than the median brushing time and those whose reported brushing times were equal to or greater than the median brushing time. Use the table below to organize the information needed to compute the proportion that report turning off the water while brushing for each of these two groups. Do these proportions suggest that people who brush longer may be more likely to turn off the water while brushing? Explain.

	Below the Median Brushing Time	Equal to or Greater than the Median Brushing Time
Number in the Sample		
Number Who Report They Turn Off Water		
Proportion Who Report They Turn Off Water		

13. The web site referenced earlier (*www.animated-teeth.com*) also included the following:

> Actually, the statement that most people aren't self-disciplined enough to brush properly when they use a manual toothbrush is probably a little bit harsh. Research has found that there can be a major discrepancy between the amount of time that a person actually does brush, as compared to the amount of time that they perceive they have brushed.
>
> One study (*Journal of Clinical Dentistry*, 1998, 9(2):49-51) found that their test subjects, on average, brushed their teeth for 78 seconds (a little longer than a minute) when they actually thought they were brushing for 141 seconds (over two minutes, an adequate amount of time). So, the intention of these people was appropriate but in reality their actions (actual brushing time) were lacking.

Compute the mean of the 50 reported brushing times in the survey data set. How does your sample mean compare to the value of 78 seconds in the quote above?

14. As a class, discuss how you might design a study that would help you determine if there is a discrepancy between reported brushing times and actual brushing times for students at your school. Write a few sentences summarizing the class discussion.

Section III: Experiments

In an observational study, researchers make observations and record data. As much as possible, the observer tries not to influence what is being observed. In an experiment, researchers deliberately do something and then measure a response. The "participants" in an experiment are called **experimental units**. Experimental units can be people, animals, or objects. When the experimental units are people, they are often referred to as **subjects**. The specific conditions researchers impose on the experimental units are called **treatments**. As experimental units may differ from one another in many important ways, the method of assigning treatments to experimental units is an important concern in the experimental design process.

Let's look at an example. A biologist would like to determine which of two leading brands of weed killer is less likely to harm the broad-leafed plants in a garden at the university. Before spraying near the plants in the garden, the biologist decides to conduct an experiment that will allow her to compare the effects of these two brands of weed killer on broad-leafed pansy plants (one of the varieties in the garden). The biologist obtains 24 individual pansy plants to use in the experiment. In this simple experiment, the *experimental units* are the individual pansy plants and the *treatments* are the two brands of weed killer.

Consider the following two plans for assigning treatments to the pansy plants:

Plan A: Choose the 12 healthiest looking pansy plants. Apply brand X weed killer to all 12 of those plants. Apply brand Y weed killer to the remaining 12 pansy plants.

Plan B: Choose 12 of the 24 individual pansy plants at random. Apply brand X weed killer to those 12 plants and brand Y weed killer to the remaining 12 plants.

Which plan seems preferable? Let's evaluate what might happen with each of these plans.

Under Plan A, suppose the pansy plants treated with brand Y weed killer have many more dead or dying leaves than the pansy plants treated with brand X. Can the biologist feel confident recommending brand X to the campus gardener as the safer weed killer? Not at all. Since the healthier plants received the brand X treatment and the less healthy plants received the brand Y treatment, it could be that more leaves were dead or dying on the pansy plants treated with brand Y because those plants were less healthy to begin with. We really can't separate the effects of the two brands of weed killer from the effect of the original healthiness of the plants in the two groups. The inability to separate the effects of the treatments from the effects of another variable in a study is known as **confounding**.

With Plan B, individual pansy plants are assigned at random to one of the two weed killer treatments. This **random assignment** helps to ensure that the group of plants treated with brand X and the group of plants treated with brand Y are fairly similar to begin with in terms of all characteristics that might affect the plants' responses to the treatments. If the biologist then observes that the pansy plants treated with brand Y weed killer have many more dead or dying leaves than the pansy plants treated with brand X, there are two plausible explanations for the observed difference.

First, it is possible that there is no difference in the effects of the two brands of weed killer on pansy plants. Some pansies are heartier than others, and, just by chance, the random assignment placed more of those healthy plants in the group that was treated with brand X. In other words, the observed difference could be simply due to chance.

The second possible explanation is that brand X weed killer actually results in greater harm to pansy plants than brand Y. In that case, we could say the difference in the number of dead or dying leaves between the two groups of pansy plants is a direct result of the brand of weed killer used. Put another way, the difference in brand of weed killer *caused* the difference in the number of dead or dying leaves.

Random assignment of treatments to subjects is an essential component of well-designed experiments. One of the big advantages of such experiments is their ability to help the researcher establish that changes in one variable (like brand of weed killer) cause changes in another variable (like number of dead or dying leaves). Since establishing causation is often a goal of experiments, we find it useful to give names to the two variables mentioned in the previous sentence. We call the variables that the experimenters directly manipulate the **explanatory variables** or **factors** and the variables that measure the subjects' responses to the treatments the **response variables**. The treatments in an experiment correspond to the different possible values of the explanatory variables. For the weed killer experiment above, there is one factor—brand of weed killer—and one response variable—number of dead or dying leaves.

In addition to randomly assigning treatments to experimental units, there are two other important considerations in designing experiments. The first is to **control** for the effects of variables that are not factors in the experiment but that might affect experimental units' responses to the treatments. Some variables can be controlled by trying to keep them at a constant value. For example, the biologist would want to ensure that the plants all receive the same amount of water and are exposed to the same amount of light. If everything is roughly equivalent for the two groups of plants except for the treatments, and we observe a difference in the response variable, then that difference is either a result of the random assignment or is caused by the difference in treatments.

Some variables can't be easily controlled by keeping them at a constant value. One such variable in the weed killer example was the current state of health of the plant. In this case, the random assignment of plants to treatments should help spread the healthy and less healthy plants out in a fairly balanced way between the two groups of pansy plants. Then, any differences in the number of dead or dying leaves that appear should not be a result of differences in initial plant health.

The other important experimental design principle is **replication**. In a nutshell, replication means giving each treatment to enough experimental units so that any difference in the effects of the treatments is likely to be detected. Imagine the biologist treating one pansy plant with brand X weed killer and one pansy plant with brand Y weed killer. If the plant treated with brand Y has more dead or dying leaves, can the biologist conclude that brand X is safer to use on the university's pansy plants? Of course not. Individual pansy plants vary widely in terms of general health and other characteristics that might affect their response to a particular brand of weed killer. With only one experimental unit available for each treatment, the random assignment can't be counted on to produce roughly "equivalent" groups prior to administering the treatments. Any difference we observe in the number of dead or dying leaves on the two pansy plants could simply be due to the difference in the initial health of the plants.

Now imagine the biologist conducting the same weed killer experiment, but with 50 pansy plants receiving each treatment. If the pansies treated with brand Y have a much higher number of dead or dying leaves than the pansies treated with brand X, the biologist should feel much more confident concluding that the difference in treatments caused the observed difference in the response variable.

Let's look at one more example. In the fall of 1982, researchers launched a now famous experiment investigating the effects of aspirin and beta carotene on heart disease and cancer. Over 22,000 healthy male physicians between the ages of 40 and 84 agreed to serve as *subjects* in the experiment. The two *factors* being manipulated by the researchers were whether a person took aspirin regularly and whether a person took beta carotene regularly. Researchers decided to use four treatments: (1) aspirin every other day and beta carotene every other day, (2) aspirin every other day and "fake" beta carotene every other day, (3) "fake" aspirin every other day and beta carotene every other day, and (4) "fake" aspirin every other day and "fake" beta carotene every other day.

The "fake" pills looked, tasted, and smelled like the pills with the active ingredient, but had no active ingredient themselves. (We call such "fake" treatments **placebos**.) Subjects were randomly assigned in roughly equal numbers to the four groups. Several *response variables* were measured in the study, including whether the individual had a heart attack and whether the individual developed cancer. Neither the subjects nor the people measuring the response variable knew who was receiving which treatment. We say this experiment was carried out in a **double-blind** manner. If either the subjects or the people measuring the response variable knows who is receiving which treatment, but the other doesn't, then the experiment is **single-blind**.

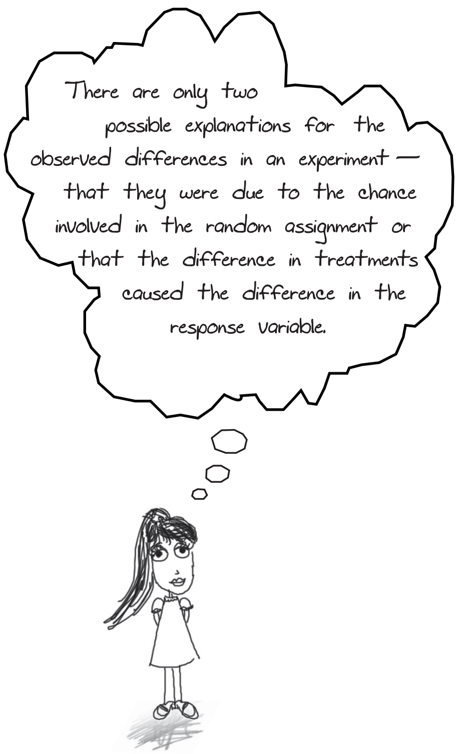
An outside group of statisticians that was monitoring the Physicians' Health Study reviewed data from the experiment on a regular basis. To everyone's surprise, the data monitoring board stopped the aspirin part of the experiment several years ahead of schedule. Why? Because there was compelling evidence that the subjects taking aspirin were having far fewer heart attacks than those who were taking placebo aspirin. It would have been unethical to continue allowing some physicians to take a placebo with clear evidence that aspirin reduced the risk of heart attack.

Even though the Physicians' Health Study was an exceptionally well-designed experiment, it does have some limitations. Researchers decided to use male physicians as subjects because they felt doctors would be more likely to understand the importance of taking the pills every other day for the duration of the study. That may be true, but because only male physicians were used in the study, we cannot generalize the findings of this study to women, or even to all male adults. We can feel pretty confident concluding that taking aspirin regularly *caused* a reduction in heart attack risk. However, the benefits of taking aspirin regularly might be offset by other effects of the drug, such as an increased risk of stroke. In spite of its limitations, the Physicians' Health Study provided a template for other researchers who wanted to design experiments to help answer important questions.

In many published reports of experimental studies, we see conclusions such as "the observed difference in heart attack rates was **statistically significant**." This tells us that the differences in the response variable between those in different treatment groups cannot reasonably be explained by the chance involved in the random assignment of treatments to subjects. Recall what we said earlier: There are only two possible explanations for the observed differences in an experiment—that they were due to the chance involved in the random assignment or that the difference in treatments caused the difference in the response variable. Saying that the results of a particular experiment are *not* statistically significant means that we can't rule out the possibility that there is no difference in the effects of the treatments, and that the differences in response are simply due to the random assignment.

You may have noticed that in both the examples presented here, the subjects were *not* randomly selected from a larger population. This is usually the case with experiments. It often isn't practical to choose subjects at random from the population of interest. Consider how you would go about randomly selecting 24 pansy plants from the population of *all* pansy plants, for example. Or how researchers might randomly select 22,000 male physicians. As you learned earlier, the lack of random selection limits our ability to generalize results to the population of interest.

However, even if experimental units are not randomly selected, well-designed experiments can give convincing evidence that changes in one variable cause changes in another variable. Establishing causation is much more difficult with observational studies, because researchers cannot hold other variables constant and cannot assign individuals at random to treatment groups. As an example, consider early observational studies that suggested people who smoked were much more likely to get lung cancer than people who didn't smoke. Cigarette company executives argued that *confounding* was at work. They claimed that the kinds of people who smoked were also much more likely to engage in other unhealthy activities—such as drinking, overeating, and failing to exercise—than people who didn't smoke. It was these other unhealthy behaviors, they said, that led to increased risk of cancer, not smoking cigarettes. After many other observational studies showed the strong connection between smoking and lung cancer, and experiments on animal subjects demonstrated that smoking caused cancerous growths, cigarette company executives finally conceded.



Investigation #10: Do Diets Work?

The Atkins Diet is one of many popular weight loss diets. It is based on reducing the consumption of carbohydrates. For years, such "low-carb" diets have been touted as being effective for weight loss and other health benefits. But before 2001, no one had attempted to demonstrate the effectiveness of a low-carb diet in a well-designed comparative experiment. Then, two separate groups of researchers attempted to do just that.

At Duke University Medical Center, Dr. William Yancy and his colleagues recruited 120 people between the ages of 18 and 65. All of the participants were obese and had high cholesterol, but were otherwise in generally good health. Researchers randomly assigned half of the participants to a low-carbohydrate, high-protein diet (similar to an Atkins Diet) and the other half to a low-fat, low-cholesterol diet. At the end of six months, researchers measured the change in each participant's weight and cholesterol levels.[1]

In the second study, Dr. Linda Stern and her colleagues recruited 132 obese adults at the Philadelphia Veterans Affairs Medical Center in Pennsylvania. Half of the participants were randomly assigned to a low-carbohydrate diet and the other half were assigned to a low-fat diet. Researchers measured each participant's change in weight and cholesterol level after six months and again after one year.[2]

1. Complete the following table using the details provided above about the two studies.

	Duke University Study	**Philadelphia Study**
Subjects		
Factor(s)/Explanatory Variable(s)		
Treatments		
Response Variable(s)		

1 "A Low-Carbohydrate, Ketogenic Diet versus a Low-Fat Diet To Treat Obesity and Hyperlipidemia," by Yancy, William S. et al, *Annals of Internal Medicine*, May 2004, 140(10) 769-777.

2 "The Effects of Low-Carbohydrate versus Conventional Weight Loss Diets in Severely Obese Adults: One-Year Follow-up of a Randomized Trial," by Stern, Linda et al, *Annals of Internal Medicine*, May 2004, 140(10) 778-785.

2. Explain why both of these studies are experiments, and not observational studies or surveys.

3. How did the researchers in both studies determine which subjects received which treatments? Why did they use the method they did?

4. Could these experiments have been carried out in a single-blind or double-blind manner? Justify your answer.

5. Each of the following quotations describes the subjects in the Duke University experiment. Explain how each is an example of control and why it is important in terms of the design of the study.

(a) "None had dieted or used weight loss medications in the previous six months."

(b) "All subjects were encouraged to exercise 30 minutes at least three times per week and had regular group meetings at an outpatient research clinic for six months."

Let's look at some results from the two studies.

> In the Duke University experiment, over the six-month duration of the study, weight loss was 12.9% of original body weight in the low-carbohydrate diet group and 6.7% of original body weight in the low-fat diet group. The low-carb diet group showed a greater increase in HDL (good) cholesterol than the low-fat diet group.
>
> In the Philadelphia experiment, subjects in the low-carbohydrate diet group lost significantly more weight than subjects in the low-fat diet group during the first six months of the study. At the end of a year, however, the average weight loss for subjects in the two groups was not significantly different. The low-carbohydrate diet group did show greater increase in HDL (good) cholesterol level after a year than the low-fat diet group.

6. Briefly summarize what the results of these two experiments seem to suggest about the relative effectiveness of low-carbohydrate diets and low-fat diets on weight and cholesterol.

7. In the Philadelphia experiment, the subjects in the low-carbohydrate diet group lost an average of 5.1 kg in a year. The subjects in the low-fat diet group lost an average of 3.1 kg. Explain how this information could be consistent with the statement above about the average weight loss in the two groups not being significantly different.

8. Here is an excerpt from a report about the Duke University experiment: "Participants in the low-carbohydrate diet group had more minor adverse effects, such as constipation and headaches, than did patients in the low-fat diet group." How would you modify your summary in question 6 based on this additional information?

When you look at experimental results, it's important to consider possible limitations of the study. The next few questions will help you look critically at the two experiments described earlier.

9. Explain how the following excerpts from a report about the two experiments might affect your conclusions about the effectiveness of low-carb versus low-fat diets:

Duke University study: "The study was completed by 76% of participants in the low-carbohydrate diet group and by 57% of participants in the low-fat diet group."

Philadelphia study: "Study limitations include high dropout rate of 34% …"

10. In both experiments, participants were assigned at random to a low-fat or low-carbohydrate diet group. What exactly does that mean? The subjects in the low-fat diet group attended counseling sessions about how to restrict their caloric intake from fat. The subjects in the low-carbohydrate group attended counseling sessions about how to restrict their carbohydrate intake. These counseling sessions continued on a weekly or monthly basis throughout the experiment. It is possible that some people in each group did not restrict their diets as instructed. How might this affect conclusions based on the experiment?

11. In the Duke University study, subjects in the low-carbohydrate group all received daily nutritional supplements. Subjects in the low-fat group did not. How might this affect conclusions based on the experiment?

12. Give an example of a potential confounding variable in one of the two experiments. Explain carefully how the factor you choose could result in confounding.

13. Is it reasonable to generalize the results of these two experiments to the population of all overweight adults? Justify your answer.

14. Now that you have considered possible limitations of these two experiments, summarize what the results of these two experiments seem to suggest about the relative effectiveness of low-carbohydrate diets and low-fat diets on weight and cholesterol. You may want to refer to what you wrote earlier in response to question 6.

Investigation #11: Distracted Learning

While you study, do you watch TV, listen to music, check your MySpace page, surf the Internet, chat on e-mail, talk or text on your cell phone? Do your parents insist that you can't possibly concentrate on studying while you're distracted by one of these activities? Maybe the conversation goes something like this:

> **Parent:** "Take off your headphones and do your homework!"
>
> **Student:** "I am doing my homework, and I work better with my music on."
>
> **Parent:** "Turn it off! You can't study with that distraction!"
>
> **Student:** "Yes I can. It helps me relax."
>
> **Parent:** "Turn off that racket and concentrate on your school work!"
>
> **Student:** "I study better with it on!"

Who is right? Some say that any distraction might interfere with your focus on the work you're doing, which may in turn affect the quality of the finished product. But others argue that listening to music actually helps them concentrate because the music "drowns out" other potential distractions. What do you think? Can previous research help us sort this out?[1]

In 1993, Frances Raucher and his colleagues designed an experiment to test whether listening to Mozart would help students improve their performance on a spatial reasoning task. They recruited 36 college students to participate in the experiment. The subjects were randomly assigned to three groups, with 12 students per group. Subjects in Group 1 listened to a 10-minute selection from a Mozart piece. Group 2 listened to a relaxation tape for 10 minutes. Subjects in Group 3 sat in silence for 10 minutes. Each subject took a pretest on spatial reasoning two days before the experiment and a post-test on spatial reasoning immediately after the 10-minute treatment. The results of the experiment seemed surprising: Students who listened to Mozart showed significantly higher gains in their scores on spatial-reasoning tasks than students in the other two groups.

After hearing the results of Rauscher's experiment, some eager parents started playing Mozart tapes for their children in hopes of increasing their spatial reasoning skills. One state even passed legislation requiring preschools to play 30 minutes of classical music a day. Other researchers tried to confirm this so-called "Mozart effect" in experiments of their own, but with little success.

So the question remains: Does listening to music help or hinder students' learning? The answer may depend on what type of "learning" we mean. In this investigation, your class will design and carry out an experiment to test whether listening to music

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1 *www.madsci.org/posts/archives/mar98/889467626.Ns.r.html* served as inspiration for part of this investigation.

helps or hinders students as they perform a memorization task. Then, you will analyze data from the experiment and draw some preliminary conclusions from your research.

1. For simplicity, the members of your class will serve as the subjects in your experiment. How might this affect your ability to generalize the results of your study?

2. One possible design for the experiment would be to randomly assign about half of the students in your class to perform the memorization task while listening to Mozart, and the other half to perform the task in a silent room nearby. Then, you could compare the scores of students who listened to Mozart while memorizing with the scores of students who didn't. What flaw(s) do you see in using this design to conduct the experiment?

3. Some people are better at memorizing things than others. Here's another possible design for your experiment that takes this fact into account. Begin by having each student perform a memory task. Based on students' performance on this task, split the class into two roughly equal-sized groups containing the "good memorizers" and the "not-so-good memorizers." Randomly assign about half of the good memorizers to perform a second memory task while listening to Mozart, and the other half to perform the task in a silent room nearby. Use the same random assignment strategy for the not-so-good memorizers. To analyze the data from the experiment, you would compare the change in scores from the first memory task to the second for the good memorizers who listened to Mozart and those who didn't, and separately for the not-so-good memorizers who did and didn't listen to Mozart while memorizing.

(a) In what ways does this design improve on the design from question 2?

(b) How might you further improve the design of this experiment using the idea that some people are better memorizers than others? Explain.

4. Perhaps the best way to take individual differences in memorization skills into account in this experiment is to have each person perform two memory tasks—one while listening to Mozart and one in silence. Then, you can analyze data on the difference in performance for all students in your class and determine whether listening to Mozart seems to help or hurt memorization.

To carry out the experiment in this way, you will need two different but similar memory tasks. Let's call them task A and task B.

(a) Explain why you should not have all students perform task A while listening to Mozart and task B while in a silent room.

(b) Explain why you should not have all students perform their first memory task while sitting in a silent room and their second memory task while listening to Mozart, or vice versa.

(c) Discuss with your classmates how you could use random assignment to most effectively address the issues raised in parts (a) and (b). Once you have settled on a plan, propose it to your teacher.

(d) Describe carefully how you will perform the random assignment required by your approved plan from part (c).

5. Now that we have settled on a design for the experiment, let's confirm some of the details.

(a) Who are the subjects in this experiment?

(b) What factor(s)/explanatory variable(s) is this experiment investigating?

(c) What treatments are being administered? Explain why task A and task B are not treatments.

(d) Let's take a look at the tasks. Each subject will be presented with a list of 20 randomly generated two-digit numbers, such as the list shown below. The student will then have one minute to memorize as many of the numbers in the list as possible. At the end of the minute, each student will have two minutes to write down as many of the numbers as he or she can remember.

26	86	64	65	75	11	49	47	85	19
23	57	97	00	62	43	66	94	79	50

A wily student might just write down a bunch of two-digit numbers during the two minute period, hoping to match as many as possible. How might you score performance on this task to reward students for actual memorization and not for guessing?

(e) Based on your answer to (d), describe the response variable(s) this experiment will measure.

Now it's time to do the experiment! Your teacher will assist with logistics so that all students can participate.

6. Carry out the random assignment required for your experiment from question 4(d). Indicate clearly what each student will be doing first and second. You may find it helpful to make a chart like the one below that summarizes how the experiment will be carried out.

Subject	First Task	First Treatment	Second Task	Second Treatment
1	A	Music	B	Silence
2	A	Silence	B	Music
3	B	Music	A	Silence
4	B	Silence	A	Music

Subject	Which Task First? (A or B)	Music First? (Yes or No)	Score With Music	Score Without Music	Difference

7. Have students perform the two memorization tasks as specified in question 6. Record data from the experiment in the table on the previous page.

8. Construct comparative dotplots or boxplots of the scores with music and the scores without music. Describe any similarities and differences you see in a few sentences.

9. Calculate the difference in scores for each student when listening to Mozart versus sitting in a silent room. As a class, decide on which order you will subtract the values. Record these values in the right-most column of the table on the previous page.

10. Construct an appropriate graph of the difference in memorization scores. Describe what the graph tells you in a couple of sentences.

11. In what way is the graph you constructed for question 10 more informative than the comparative graph from question 8?

12. Calculate a measure of center (mean or median) and a measure of spread that you think summarize the differences well. Explain why you chose the measures you did.

13. Was this experiment single-blind, double-blind, or neither? Justify your answer.

14. Based on the results of your experiment, does it appear that listening to Mozart helps or hinders students' performance on memorization tasks? Give appropriate graphical and numerical evidence to support your answer.

15. Can we generalize the results of this experiment to any kind of task that requires memorization? Justify your answer.

16. Why did we have all students listen to the same piece of Mozart music, rather than letting each student choose music he or she liked? Explain.

Investigation #12: Would You Drink Blue Soda?

Does what you see affect your perception of how it tastes? If color can influence how people think a food tastes, what implications does this have for companies that make and market food and beverages?[1]

PepsiCo might be interested in your answer to these questions, as they have had two marketing failures based on introducing nontraditional colored beverages. In the early 1990s, PepsiCo introduced Pepsi Clear, a cola-flavored drink that was clear instead of brown in color. Pepsi Clear was later discontinued because sales were low. In 2002, PepsiCo tried again with Pepsi Blue.[2] Pepsi Blue was a berry-flavored cola drink that was blue in color. The Pepsi web site (*www.pepsi.com*) says that Pepsi Blue was "created by and for teens. Through nine months of research and development, Pepsi asked young consumers what they want most in a new cola. Their response: Make it berry and make it blue."

Unfortunately for PepsiCo, Pepsi Blue, like Pepsi Clear, was not a successful product, and it was discontinued a few years later. So what happened? Was the mistake adding a berry flavoring to cola, making the cola blue, or a combination of both?

In this investigation, you'll investigate whether teens have a preference for or a dislike for blue-colored soda.

Getting Started

To decide whether coloring a soda blue is a good or bad strategy if the drink is going to be marketed to teenagers, you will design and conduct an experiment, collect and analyze the data, and then make a recommendation.

For this experiment, you can start with a clear-colored soda, such as 7-Up or Sprite. Experiment with adding blue food coloring to the soda to create a "recipe" for a blue version of the soda. Food coloring is tasteless, so the addition of food coloring will not change the actual taste of the soda.

Once you have developed your new product, think carefully about how you would design an experiment to determine if teens have a preference for the clear soda or the blue soda.

Note: Be sure to discuss the ethical considerations involved in performing an experiment with human subjects. Your teacher will require you to obtain informed consent from all students (and possibly their parents) before they can participate in your experiment.

Once you have a plan in mind, answer the following questions. Be as specific as possible in your answers. It is OK to modify the design of your experiment if any of these

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1 The page titled "Does the Color of Foods and Drinks Affect the Sense of Taste?" on the Neuroscience for Kids web site, *http://faculty.washington.edu/chudler/coltaste.html*, has a list of references to studies that have examined how color affects perceived taste.

2 You can find an announcement describing the launch of Pepsi Blue at *http://money.cnn.com/2002/05/07/news/companies/pepsi*.

questions reveal a weakness in your original plan. Now is the time to revise, before you actually carry out the experiment and collect the data!

1. In taste test experiments like the one you are designing, it is usual to randomize the order in which subjects taste the two drinks. That is, some subjects should taste the clear drink first and then the blue drink, while others should taste the blue drink first and then the clear. A random mechanism would be used to determine the order for each subject. Why do you think it is important to randomize the order in which the drinks are presented in an experiment of this type?

2. What would be a good way to determine the order (clear then blue or blue then clear) for each subject?

3. What are the two treatments for this experiment? *Hint:* In an experiment, subjects are assigned at random to one of the treatments.

4. Explain why it is not possible in this experiment to "blind" the subjects with respect to which experimental group they are in.

5. How will you select the subjects for your experiment, and how many subjects will participate? Be specific!

6. To what group, if any, will you be able to generalize the results of your experiment? Explain why you think it is reasonable to generalize to this particular group.

7. What question will you ask each subject after he or she has tasted the two sodas? Make sure that you will be able to determine from the response which of the two drinks was preferred.

8. After considering your answers to questions 1 through 7 and modifying your plan as needed, write a summary of your plan for conducting the experiment on *separate paper*. Include enough detail that someone who has not been part of your design team could read the summary and be able to carry out the experiment as you intended. Be sure to address ethical issues of using human subjects.

After your teacher has approved your experimental plan, carry out the experiment and collect data. Be sure to record the order in which the two drinks were tasted and the response for each subject.

Once you have collected the data, use it to fill in the four cells of the table below.

		Order	
		Clear then Blue	Blue then Clear
Preference	Clear		
	Blue		

9. Construct a graphical display that allows you to compare the preferences for the two experimental groups (clear then blue and blue then clear).

10. Based on your display, do you think there is a difference in preference for the two experimental groups? That is, do you think the order in which the drinks were tasted makes a difference? Explain.

11. Based on the data from this experiment, do you think there is a preference for one of the drinks (clear or blue) over the other? Explain, justifying your answer using the data from the experiment.

12. Write a report that makes recommendations to a soft drink company that is considering introducing a blue soft drink that will be marketed to teens. Include appropriate data and graphs to support your recommendations.

The objective of many statistical studies is to use sample data to tell us something about the population from which the sample was selected. As we have seen in the previous sections, if the sample is selected in an appropriate way and involves some type of random selection, it may be reasonable to regard the sample as representative of the population of interest. Suppose that 15 in a sample of 50 randomly selected students favor banning soda machines from campus—a proportion of 15/50 = .30 or, equivalently, 30%. Is it also reasonable to say that exactly 30% of the students at your school favor banning soda machines from campus? The value of the sample proportion depends on the outcome of the random selection—which 50 students are selected to participate. As a consequence, the value of the sample proportion will vary from one random sample to another, and we can't expect that the sample proportion will be exactly equal to the actual value of the population proportion.

Does this mean the sample proportion doesn't tell us anything about the population proportion? Fortunately, the answer to that question is no! If the sample has been chosen appropriately and can be regarded as representative of the population, we can expect that the sample value will, on average, be "close" to the true population value. But, to really tell us something useful about the population, we need to be a bit more specific about what we mean by "close."

In this section, we will begin to investigate how we can use data from a well-designed study to draw conclusions about a population. This requires an understanding of the nature of **sampling variability**—the chance differences that occur from one random sample to another as a consequence of random selection.

Statistical inference is the process of drawing conclusions about a population using data from a sample. In most statistical studies, sample data is collected so that the investigator can either estimate some population characteristic of interest (such as the proportion of students at your school who favor the ban of soda machines on campus) or evaluate the plausibility of some claim that has been made about the population (for example, a claim that more than 50% of the students at the school favor a ban on soda machines).

In an **estimation** situation, we need to understand sampling variability to be able to assess how close an estimate from a sample is likely to be to the actual value of the corresponding population characteristic. In published reports, you will often see statements that include a **margin of error**. For example, the analysis of data from a survey on public support for the president might include a statement such as "The proportion of U.S. adults who believe the president is doing a good job is .45 (45%) with a margin of error of .03 (3%)." The reported margin of error acknowledges that the true population proportion is not likely to be *exactly* .45 and indicates that, based on what was seen in the sample, plausible values for the population proportion might be anything between .42 and .48. How is the margin of error computed? It is based on an assessment of sampling variability. Investigation 13 illustrates the reasoning that leads to a margin of error calculation.

The second way data from a sample is typically used to draw a conclusion about a population is in the evaluation of the plausibility of a claim about the population. For example, the *Youth Monitor* report titled "Coming of Age in America: Part IV – The MySpace Generation" (*www.greenbergresearch.com*) includes the following statement:

> Half (52%) of 18–24 year olds report they have a page on MySpace. A third report membership on Facebook, though that number rises to over half (54%) among students.

What about students at your school? Does a majority (more than 50%) have a page on MySpace? Suppose the proportion in a random sample of students from your school who report that they have a MySpace page was .52. Can you conclude that a majority of *all* students at the school have a MySpace page? This requires some careful thought. There are two reasons why the sample proportion might have been larger than .50. One reason is sampling variability—we don't expect the sample proportion to be exactly equal to the population proportion. So, maybe the population proportion is .50 (or maybe even something smaller) and a sample proportion of .52 is "explainable" just due to sampling variability. If this is the case, we can't interpret the sample proportion of .52 as convincing evidence that a majority of *all* students at your school have a MySpace page. However, another reason the sample proportion might have been greater than .50 is that the true population proportion is, in fact, greater than .50. Is .52 enough larger than .50 that the difference can't be explained as being due to sampling variability alone? If so, we would say that, based on the sample data, there is convincing evidence that more than half of the students at the school have a MySpace page. How do we make this determination? Again, it is based on an assessment of sampling variability. Investigations 14 and 15 illustrate the reasoning that enables us to use data from a random sample to evaluate the plausibility of a claim about a population.

The investigations in this section are designed to introduce you to reasoning about sampling variability in each of these two types of settings—estimation and evaluating the plausibility of a claim. In this brief introduction, we can illustrate the reasoning, but for a more complete treatment of the inferential process, we encourage you to consider taking a course devoted to statistics and data analysis!

Investigation #13: The Internet—Information or Social Highway?

The report "Coming of Age in America: Part IV—The MySpace Generation" referenced in the Overview to this section also included the following:

> For many young people, the Internet represents not just an information superhighway but, indeed, a social highway. In fact, nearly two thirds (64%) agree that "I don't know how I would keep up with my friends or family if I didn't have the Internet."

What proportion of the students at your school agree with the statement above? In this investigation, you will carry out a simple survey and then use the resulting data to estimate the proportion who agree. You will also see how we can obtain a conservative margin of error for your estimate.

Let's begin by taking a look at margin of error. A **conservative estimate of the margin of error** when a proportion from a random sample is used as an estimate of the corresponding population proportion is $\frac{1}{\sqrt{n}}$, where n denotes the sample size. For a sample size of 40, it is reasonable to use $\frac{1}{\sqrt{n}}$ as a conservative estimate of margin of error for sample proportions between .15 and .85. For larger sample sizes, this way of computing a conservative margin of error is reasonable for an even larger range of values of the sample proportion. When we say that this is a conservative estimate of the margin of error, we mean that the actual margin of error will be equal to or smaller than this conservative value. That is, when we use this conservative estimate, we may be overstating our potential error. This is generally considered to be a better alternative than understating potential error. It is possible to obtain more precise estimates of the margin of error, but that is beyond the scope of this introductory investigation.

To see why it is reasonable to use $\frac{1}{\sqrt{n}}$ as a conservative estimate of the margin of error, we need to consider the role of sampling variability and how the sample proportion varies from one random sample to another. To do this, we will carry out a simulation.

How much will sample proportions vary from one random sample to another and how much will these sample proportions tend to differ from the actual value of the corresponding population proportion? Suppose that the proportion of students at your school who have some characteristic of interest is actually .60. Think of this as a population where 60% have the characteristic we are interested in (called them "successes"). What kind of sample proportions would you expect to see for random samples of size 40 from this population? We can find out by creating a hypothetical population with 60% "successes." One way to create such a population is to use random digits. Because each of the digits 0, 1, 2, …, 9 is equally likely to occur in a list of random digits, if we think of the digits 1, 2, 3, 4, 5, and 6 as representing individuals who are successes and the digits 0, 7, 8, and 9 as representing individuals who are not successes, we can view a collection of random digits as a population with 60% successes.

We can now take a random sample of size 40 from this hypothetical population by selecting 40 random digits. Each member of the sample (each digit) can then be classified according to whether it is a success, and then the sample proportion of success can be computed. For example, consider the list of 40 random digits below.

0 5 2 3 5 7 2 3 2 3 0 7 1 1 0 0 3 2 5 8 1 2 4 6 9 4 5 3 1 7 0 2 1 5 9 3 9 4 7 6

Identifying the 1s, 2s, 3s, 4s, 5s, and 6s as successes (the underlined digits above), we could then compute the proportion of successes for this random sample:

$$\text{sample proportion} = \frac{\text{number of successes in the sample}}{n} = \frac{27}{40} = .675$$

Repeating this process a large number of times and noting the values of the sample proportions obtained allows us to see what values of the sample proportion are expected as a result of sampling variability alone.

1. Use a sequence of random digits to represent a random sample from a population that has 60% successes (a population proportion of .60). Identifying successes as described in the paragraph above, determine the number of successes in the sample and the sample proportion.

Number of successes in the sample =

Proportion of successes for this sample =

2. Record the sample proportion obtained in step 1 in the table below. Then, repeat the process four more times. Enter each of the four new sample proportions into the table below.

Sample Number	1	2	3	4	5
Sample Proportion					

3. Enter a tally mark for each of your observed sample proportions in the table on the following page. Then enter a tally mark for each of the five sample proportions observed by other students in your class. You can do this by making tally marks as each student reads his or her sample proportions. When finished, count the tally marks for each row in the table and enter the total in the frequency column for the appropriate row.

4. Use the information in the table from step 3 to complete the following sentences.

(a) About 2.5% of the observed sample proportions are less than ______.

(b) About 2.5% of the observed sample proportions are greater than ______.

(c) About 95% of the observed sample proportions are between ______ and ______.

Sample Proportion	Tally	Frequency	Sample Proportion	Tally	Frequency
0.000			0.525		
0.025			0.550		
0.050			0.575		
0.075			0.600		
0.100			0.625		
0.125			0.650		
0.150			0.675		
0.175			0.700		
0.200			0.725		
0.225			0.750		
0.250			0.775		
0.275			0.800		
0.300			0.825		
0.325			0.850		
0.350			0.875		
0.375			0.900		
0.400			0.925		
0.425			0.950		
0.450			0.975		
0.475			1.000		
0.500					

5. Now try to complete the following statement:

> About 95% of the time, the sample proportion was within ______ of the actual population proportion (.60).

Hint: Look at your answer to part (c) of question 4. How far are the endpoints of the interval you give there from .60?

6. Compute the conservative margin of error for a sample size of 40.

Conservative margin of error = $\frac{1}{\sqrt{n}}$ =

7. How does the conservative margin of error from step 6 compare to your number from step 5?

What if the proportion of successes in the population was .2 instead of .6? What kind of sample proportions should we expect to observe? We can investigate this by carrying out a simulation that is similar to the one just carried out. Only a few modifications are needed.

8. If a sequence of random digits is to be used to represent a random sample from a population that has 20% successes, what digits will you use to represent successes in the sample?

9. Use a sequence of random digits to represent a random sample of size 40 from a population that has 20% successes (a population proportion of .20). Determine the number of successes in the sample and the sample proportion.

Number of successes in the sample =

Proportion of successes for this sample =

10. Record the sample proportion obtained in step 9 in the table below. Then repeat the process four more times. Enter each of the four new sample proportions into the table.

Sample Number	1	2	3	4	5
Sample Proportion					

11. Enter a tally mark for each of your observed sample proportions in the table on the next page. Then enter a tally mark for each of the five sample proportions observed by other students in your class. You can do this by making tally marks as each student reads his or her sample proportions. When finished, count the tally marks for each row in the table and enter the total in the frequency column for the appropriate row.

12. Write a few sentences describing how the observed sample proportions for random samples of size 40 differ for a population with 60% successes and a population with 20% successes.

Sample Proportion	Tally	Frequency	Sample Proportion	Tally	Frequency
0.000			0.525		
0.025			0.550		
0.050			0.575		
0.075			0.600		
0.100			0.625		
0.125			0.650		
0.150			0.675		
0.175			0.700		
0.200			0.725		
0.225			0.750		
0.250			0.775		
0.275			0.800		
0.300			0.825		
0.325			0.850		
0.350			0.875		
0.375			0.900		
0.400			0.925		
0.425			0.950		
0.450			0.975		
0.475			1.000		
0.500					

13. Use the information in the table to complete the following sentences.

(a) About 2.5% of the observed sample proportions are smaller than ______.

(b) About 2.5% of the observed sample proportions are greater than ______.

(c) About 95% of the observed sample proportions are between ______ and ______.

14. Now try to complete the following statement:

About 95% of the time, the sample proportion was within ______ of the actual population proportion (.20).

Hint: Look at your answer to part (c) of question 13. How far are the endpoints of the interval you give there from .20?

15. How does the conservative margin of error for a sample of size 40 (see step 6) compare to your number from step 14?

16. Consider the following statement from the introduction to this investigation:

> When we use this conservative estimate, we may be overstating our potential error. This is generally considered to be a better alternative than understating potential error. It is also worth noting that the conservative estimate overstates to a larger degree the more that the sample proportion differs from .5.

Is this statement supported by the two simulations that you have carried out? Explain.

Now that you understand what is meant by conservative margin of error and know how to compute it, you are ready to see how this is used when reporting survey data. Recall the question posed at the beginning of this investigation:

> What proportion of the students at your school agree with the statement "I don't know how I would keep up with my friends or family if I didn't have the Internet"?

17. As a class, discuss how you will go about selecting a sample of students at your school to participate in a survey. Remember that the sample should be selected in such a way that it will be reasonable to generalize from the sample to the population of all students at the school. Be sure to consider the ethical issues involved in conducting a survey—informed consent, right of refusal, and preserving anonymity and confidentiality. Write a brief summary of your plan for carrying out the survey.

18. Once your teacher has approved your plan, carry out the sampling that is required. Then, ask each individual selected for inclusion in the sample whether he or she agrees with the statement "I don't know how I would keep up with my friends or family if I didn't have the Internet." Use the resulting data to compute the sample proportion and a conservative margin of error.

Sample size =

Number in sample who agree with statement =

Sample proportion =

Conservative margin of error =

19. Write a paragraph describing the results of your survey. Be sure to reference the estimate of the proportion of students at your school who agree with the statement and the margin of error. Also, indicate whether you think the statement from the report referenced at the beginning of this investigation is accurate for your school and why or why not. (The report said nearly two-thirds of young people agree with the statement.)

Investigation #14: Evaluating the MySpace Claim

In the Section IV Overview, the following question was posed: Does a majority (more than 50%) of students at your school have a page on MySpace? In this investigation, you will carry out a survey of 60 students at your school. Then, you will use the resulting data to determine if there is convincing evidence that a majority of students at your school have a MySpace page.

Before we collect data, let's consider what it would take to convince us that a majority of students at the school have a MySpace page. Suppose we select a random sample and find that the proportion in the sample who report they have a MySpace page is less than .50 (less than 50%). That certainly wouldn't be convincing evidence that more than 50% have such a page. But what if our sample proportion is greater than .50? Should we be "convinced" that a majority of *all* students (not just those in the sample) have a MySpace page? This decision isn't so easy!

What we need to see to be *convinced* is not just a sample proportion greater than .50—it has to be *enough* greater that it is not likely to have occurred just by chance due to sampling variability. That is, to be convinced, we would need to rule out sampling variability—those chance differences that occur from one sample to another as a consequence of random selection—as a plausible explanation for what we see in the sample. For our MySpace example, if we decide that the sample proportion is enough larger than .50 that we don't think it is just due to sampling variability, we would conclude that there is convincing evidence that the actual proportion in the population is larger than .50.

So, how do we decide just how much greater than .50 our sample proportion must be in order to convince us that what we are seeing is not just sampling variability? To figure this out, we need to know what kind of sample proportions are likely to be observed just by chance when the population proportion is .50 or less (not a majority). If we see one of these proportions, we would not want to say we were convinced that a majority of students have a MySpace page.

The first steps in this investigation use simulation to explore what values of the sample proportion would *not* be convincing evidence of a majority. Suppose that in fact 50% or fewer of the students at your school have a MySpace page. We will focus on the most extreme of these possible population values—50%. We use 50% (a proportion of .50) in the simulation that follows because to be convinced that a majority of students have a MySpace page, we need to be convinced that the population proportion is greater than .50.

To determine what sample proportions are likely to be observed when the population proportion is .50, we will use a hypothetical population of random digits. We create a population with 50% "successes" by designating the digits 1, 2, 3, 4, and 5 to represent individuals who are successes and the digits 6, 7, 8, 9, and 0 to represent individuals who are not successes.

We can now take a random sample of size 60 from this hypothetical population by selecting 60 random digits. Each member of the sample (each digit) can then be classified according to whether it is a success. Then, the sample proportion of successes can be computed. For example, consider the list of 60 random digits below.

8 4 6 6 4 3 7 8 6 7 3 0 1 3 5 4 6 7 6 2 6 3 8 2 9 9 1 9 0 0
3 0 6 3 7 7 0 5 9 1 8 1 3 4 3 6 5 4 0 3 0 9 2 2 5 9 7 1 5 9

Identifying the 1s, 2s, 3s, 4s, and 5s as successes (the underlined digits above), we could then compute the proportion of successes for this random sample:

$$\text{sample proportion} = \frac{\text{number of successes in the sample}}{n} = \frac{28}{60} = .467$$

Repeating this process a large number of times and noting the values of the sample proportions obtained allows us to see what values of the sample proportion are expected as a result of sampling variability alone.

1. Use a sequence of 60 random digits to represent a random sample of size 60 from a population that has 50% successes (a population proportion of .50). Identifying successes as described in the paragraph above, determine the number of successes in the sample and the sample proportion.

Number of successes in the sample =

Proportion of successes for this sample =

2. Record the sample proportion obtained in step 1 in the table below. Then, repeat the process four more times. Enter each of the four new sample proportions into the table.

Sample Number	1	2	3	4	5
Sample Proportion					

3. Enter a tally mark for each of your observed sample proportions in the table on the next page. Then, enter a tally mark for each of the five sample proportions observed by other students in your class. You can do this by making tally marks as each student reads his or her sample proportions. When finished, count the tally marks for each row in the table and enter the total in the frequency column for the appropriate row.

4. Use the information in the table from step 3 to complete the following sentence.

About 5% of the observed sample proportions are greater than ______.

5. Based on the statement in step 4, it is reasonable to say that when the true population proportion is .50, a value as large as or larger than the number in step 4 would

Sample Proportion	Tally	Frequency	Sample Proportion	Tally	Frequency
0.017			0.517		
0.033			0.533		
0.050			0.550		
0.067			0.567		
0.083			0.583		
0.100			0.600		
0.117			0.617		
0.133			0.633		
0.150			0.650		
0.167			0.667		
0.183			0.683		
0.200			0.700		
0.217			0.717		
0.233			0.733		
0.250			0.750		
0.267			0.767		
0.283			0.783		
0.300			0.800		
0.317			0.817		
0.333			0.833		
0.350			0.850		
0.367			0.867		
0.383			0.883		
0.400			0.900		
0.417			0.917		
0.433			0.933		
0.450			0.950		
0.467			0.967		
0.483			0.983		
0.500			1.000		

occur only about 5% of the time just by chance. That means that it would be fairly unusual to see a value this large if the population proportion is .50 (and even less likely if the population proportion were something smaller than .50). Use the number from step 4 to complete the following decision statement:

> If we take a random sample of 60 students from our school and we find that the proportion who have a MySpace page is equal to or greater than ______, we will conclude that there is convincing evidence that a majority of students at the school have a MySpace page.

6. Write a few sentences explaining why the decision statement in step 5 makes sense.

7. We are almost ready to collect some data. As a class, discuss how you will go about conducting a survey involving 60 students at your school. Remember that the sample should be selected in such a way that it will be reasonable to generalize from the sample to the population of all students at the school. Be sure to consider the ethical issues involved in conducting a survey—informed consent, right of refusal, and preserving anonymity and confidentiality. Write a brief summary of your plan for carrying out the survey.

8. Implement the sampling plan described in the previous step, asking each chosen individual whether he or she has a MySpace page. Use the resulting data to compute the sample proportion.

Sample size =

Number in sample who agree with statement =

Sample proportion =

9. Based on the observed sample proportion and the decision statement from step 5, which of the following possible conclusions is appropriate?

Conclusion 1: There is convincing evidence that a majority of students at our school have a MySpace page.

Conclusion 2: There is not convincing evidence that a majority of students at our school have a MySpace page.

10. Write a paragraph describing your conclusions. Include an assessment of whether it is plausible that 50% or fewer of the students at your school have a MySpace page and that what was observed in the sample can be explained by sampling variability alone.

Investigation #15: Are Teens the Same Everywhere?

Students from a high school in Milwaukee, Wisconsin, and from a high school in Sweden participated in a survey that included the following seven questions.

(1) Indicate your gender: ____ Male ____ Female

(2) You have lots of homework to do tonight. What choice best describes what you will probably do?

(A) You do what you want and forget to complete the homework.

(B) Start it, but quit when it gets hard.

(C) Wait until the last minute, but do it.

(D) Do it right away.

(3) You are scheduled to work, but there is a party you would like to go to. What choice best describes what you will probably do?

(A) You go to the party and don't go to work.

(B) You call in sick and go to the party.

(C) You make an excuse to leave work an hour or two early so you can go to the party.

(D) You work your entire shift and only go to the party if it is still on after work.

(4) Your parents ask you to do the dishes. What choice best describes what you will probably do?

(A) You ignore them and don't do the dishes.

(B) You say you'll get to them, but never "find the time."

(C) You procrastinate, but finally get them done.

(D) You do them quickly and without your parents nagging you.

(5) You have a curfew of midnight. What choice best describes what you will probably do?

(A) You don't call and come home when it suits you.

(B) You finally call after you are already late and make excuses.

(C) You call to tell your parents you will be late.

(D) You get home on time.

(6) The cashier mistakenly gives you an extra $10 in change. What choice best describes what you will probably do?

(A) You keep the extra money and spend it.

(B) You keep it, but feel guilty about it.

(C) You start to keep it, but then decide to return it.

(D) You give it back as soon as you realize you have it.

(7) You have a major paper due in one month. What choice best describes what you will probably do?

(A) You never do it.

(B) You finish it under pressure after the due date.

(C) You stay up late to finish it the night before it is due.

(D) You plan ahead and get it done on time without rushing.

A total of 60 students at the Milwaukee school responded to the survey and a total of 56 students from the school in Sweden responded. The sample at each school was selected in a way that was designed to produce a representative sample of the students at that school. The survey data are given in the tables that appear at the end of this investigation.

In this investigation, we will see how the data from these two samples can be used to decide if there is a "significant difference" in the proportion of students at the Milwaukee school and the proportion of students at the school in Sweden who respond in a particular way to one of these questions. By "significant difference" we mean that the difference in the response proportions for the two samples is larger than we would have expected to see just as a result of sampling variability.

The investigation will first guide you through the steps necessary to answer the following question:

> Is there a significant difference in the proportion of students who report that they would probably get home on time when given a curfew of midnight for students at the two schools?

After seeing how this question is answered, you will then be asked to answer an additional question using a similar process.

1. Let's consider the question posed: Is there a significant difference in the proportion of students who report that they would probably get home on time when given a curfew of midnight for students at the two schools?

To answer this question, we will need to look at the responses to question number 5 on the survey. In particular, we are interested in the proportions of students who answer D to this question. Answers A, B, and C can be combined into a single category here because they all represent answers where the student says he or she will probably not get home on time. For each of the two samples, we then need to know how many students answered D to question 5 and how many gave an answer other than D. We can use the data sets that appear at the end of this investigation to fill in the cells in the table below.

	Home on Time	Not Home on Time	**Total**
Milwaukee Sample			**60**
Sweden Sample			**56**
Total			**116**

Looking at the Milwaukee data set, we can count the number of D answers in the question 5 column. Verify that this number is 19 and enter it into the table above. Use the data sets provided to fill in the remaining cells and compare your numbers to those in the following table.

	Home on Time	Not Home on Time	**Total**
Milwaukee Sample	19	40	**60**
Sweden Sample	14	42	**56**
Total	**33**	**83**	**116**

2. We can now compute the relevant sample proportions. The proportion who report that they would probably be home on time for the Milwaukee sample is 19/60 = .32. Verify that the corresponding proportion for the Sweden sample is .25.

3. Notice that the two sample proportions are different: .32 for the Milwaukee sample and .25 for the Sweden sample. The difference between the two sample proportions is .07. How can we tell if this difference is significant? We know that even if there was no difference in the true proportions for the two schools, we would still not expect the sample proportions to be exactly equal because of sampling variability (those chance differences that occur from one sample to another as a result of the random selection process). To determine if a difference of .07 is significant, we need to know something about what kinds of differences are consistent with sampling variability alone.

As you might have guessed based on the previous two investigations, we will carry out a simulation. In this simulation, we create a population consisting of 116 individuals (representing the 116 survey respondents). Do this by cutting out 116 squares of paper that are the same size. You can use the templates at the end of this investigation if you don't want to make your own. Mark 33 of the 116 squares with an "H" to represent the 33 survey participants who reported they would probably be home on time. The other 83 slips of paper, which are unmarked, will represent the 83 survey participants who reported that they would probably not be home on time.

Place all the slips of paper in a paper bag and mix them well. Next, select 60 slips of paper from the bag to represent the 60 Milwaukee students. Count the number of the 60 selected squares that have an H and enter the count into the highlighted cell in the table below.

	Home on Time	Not Home on Time	**Total**
Milwaukee Sample			**60**
Sweden Sample			**56**
Total	**33**	**83**	**116**

Use the row and column totals to fill in the remaining cells in the table. Then, compute the two sample proportions and the difference between the two sample proportions:

Home on Time proportion of Milwaukee sample =

Home on Time proportion of Sweden sample =

Difference in sample proportions (Milwaukee – Sweden) =

For example, if the number in the highlighted cell was 16, the resulting table, proportions, and difference would be the following:

	Home on Time	Not Home on Time	**Total**
Milwaukee Sample	16	44	**60**
Sweden Sample	17	39	**56**
Total	**33**	**83**	**116**

Home on Time proportion of Milwaukee sample = 16/60 = .27

Home on Time proportion of Sweden sample = 17/56 = .30

Difference in sample proportions (Milwaukee – Sweden) = .27 – .30 = – .03

4. Record the difference you obtained in step 3 in the Trial 1 row of the table below.

Trial Number	**Difference in Sample Proportions**
1	
2	
3	
4	
5	

5. Put all of the slips of paper back in the bag, mix well, and then repeat step 3 four more times. Use the templates below to keep track of your results and to compute the relevant proportions. Finally, enter the resulting differences in sample proportions into the trials 2 through 5 rows of the table in step 4.

Trial 2			
	Home on Time	Not Home on Time	**Total**
Milwaukee Sample			**60**
Sweden Sample			**56**
Total	**33**	**83**	**116**

Home on Time proportion of Milwaukee sample =

Home on Time proportion of Sweden sample =

Difference in sample proportions (Milwaukee – Sweden) =

Trial 3			
	Home on Time	Not Home on Time	**Total**
Milwaukee Sample			**60**
Sweden Sample			**56**
Total	**33**	**83**	**116**

Home on Time proportion of Milwaukee sample =

Home on Time proportion of Sweden sample =

Difference in sample proportions (Milwaukee – Sweden) =

Trial 4			
	Home on Time	Not Home on Time	**Total**
Milwaukee Sample			**60**
Sweden Sample			**56**
Total	**33**	**83**	**116**

Home on Time proportion of Milwaukee sample =

Home on Time proportion of Sweden sample =

Difference in sample proportions (Milwaukee – Sweden) =

Trial 5			
	Home on Time	Not Home on Time	**Total**
Milwaukee Sample			**60**
Sweden Sample			**56**
Total	**33**	**83**	**116**

Home on Time proportion of Milwaukee sample =

Home on Time proportion of Sweden sample =

Difference in sample proportions (Milwaukee – Sweden) =

6. OK, we are almost there! Each student in your class should have carried out five trials. Now combine all the simulated differences into one large data set by recording the values obtained by each student in your class in the table on the next page.

7. Use the data from step 6 to construct a dotplot of the simulated differences.

8. The dotplot of the simulated differences in step 7 provides information about how large the difference in sample proportions can be expected to be just as a result of sampling variability. That is, these simulated sample differences are consistent with the situation where there is no difference in the population proportions.

Now, let's go back and look at the actual difference in sample proportions for the Milwaukee and Sweden samples. The observed difference was .07. Based on what you see in the dotplot of simulated differences, is sampling variability a plausible explanation for why we might see a difference in sample proportions as large as .07? Explain why or why not.

9. Write a few sentences that address the original question posed: Is there a significant difference in the proportion of students who report that they would probably get home on time when given a curfew of midnight for students at the two schools?

10. In the last part of this investigation, your teacher will assign you to a team. Each team will either choose one of the questions below or formulate a question of its own that can be answered using the Milwaukee and Sweden data. Once your teacher has approved the team's question, use the data provided to answer it and write a brief report summarizing your results. (You should use a process similar to the one used to answer the "home on time" question, but note that because only the members of your team will be carrying out the simulation, you will each need to carry out more than five trials.)

Some possible questions to investigate:

> Is there a significant difference in the proportion of students who report that they would give back an extra $10 in change for students at the two schools?
>
> Is there a significant difference in the proportion of students who report that they would plan ahead and get a paper done without rushing and on time for students at the two schools?
>
> Is there a significant difference in the proportion of students who report that they do homework right away for students at the two schools?

Data Set: Milwaukee							
Student #	**Question (1)**	**Question (2)**	**Question (3)**	**Question (4)**	**Question (5)**	**Question (6)**	**Question (7)**
1	Female	A	B	C	C	A	B
2	Female	D	D	D	D	B	D
3	Female	C	D	D	C	A	C
4	Female	A	D	D	D	C	C
5	Female	C	D	D	C	A	D
6	Female	C	D	D	D	D	C
7	Female	C	D	C	C	B	C
8	Female	D	D	C	C	C	C
9	Female	B	C	C	C	A	C
10	Female	D	C	C	B	A	C
11	Female	C	D	C	C	C	D
12	Female	C	B	C	C	D	C
13	Female	D	D	D	D	D	D
14	Male	A	D	C	C	C	B
15	Female	D	D	D	D	D	C
16	Male	D	B	C	D	D	C
17	Male	C	D	C	C	D	C
18	Male	D	D	D	D	D	D
19	Male	C	D	C	B	C	C
20	Male	C	B	D	C	A	C
21	Female	A	C	B	C	A	C
22	Male	C	D	D	B	D	C
23	Male	C	D	D	B	D	C
24	Male	C	D	A	A	B	C
25	Male	C	D	C	D	A	D
26	Female	C	D	D	C	D	C
27	Female	C	D	C	C	C	C
28	Male	C	D	A	C	D	C
29	Male	A	D	B	C	B	C
30	Female	C	C	C	B	C	C

Data Set: Milwaukee							
Student #	**Question (1)**	**Question (2)**	**Question (3)**	**Question (4)**	**Question (5)**	**Question (6)**	**Question (7)**
31	Male	D	D	D	C	C	D
32	Male	B	B	C	B	D	D
33	Female	C	C	B	A	A	D
34	Female	C	D	C	C	A	C
35	Male	C	D	D	C	D	C
36	Male	B	C	C	D	A	C
37	Male	C	D	D	A	D	C
38	Female	D	D	D	D	D	D
39	Female	C	C	D	C	A	C
40	Male	C	D	B	C	D	C
41	Male	C	D	A	C	A	D
42	Male	C	B	C	C	D	C
43	Female	C	D	D	D	D	C
44	Male	D	D	C	C	B	D
45	Male	A	A	A	A	A	C
46	Female	C	D	C	D	D	C
47	Female	C	D	C	D	C	C
48	Male	D	C	A	D	D	C
49	Male	A	B	B	B	A	C
50	Female	C	D	B	B	A	C
51	Female	D	D	B	D	B	C
52	Male	B	C	B	A	A	C
53	Male	A	D	B	D	A	C
54	Female	D	D	C	C	C	D
55	Male	C	D	C	D	A	D
56	Female	A	D	D	D	C	C
57	Male	C	D	C	C	A	C
58	Female	C	C	B	C	C	C
59	Male	C	D	D	C	B	D
60	Female	C	D	D	D	D	C

Data Set: Sweden							
Student #	**Question (1)**	**Question (2)**	**Question (3)**	**Question (4)**	**Question (5)**	**Question (6)**	**Question (7)**
1	Female	C	D	C	B	D	C
2	Female	C	D	C	B	D	C
3	Female	B	D	B	B	D	C
4	Male	C	C	B	B	A	C
5	Male	C	B	D	D	A	C
6	Female	C	C	B	C	A	C
7	Female	C	B	B	B	B	C
8	Male	A	B	D	C		C
9	Female	D	D	D	C	C	D
10	Female	C	C	D	C	B	C
11	Female	C	D	D	D	D	C
12	Male	B	D	D	D	B	C
13	Female	C	D	B	D	D	D
14	Female	D	C	B	C	D	D
15	Female	D	C	C	C	A	D
16	Male	D	C	D	C	C	C
17	Male	C	D	D	D	A	D
18	Male	C	C	C	C	A	C
19	Female	C	D	C	C	D	C
20	Female	D	C	D	C	A	D
21	Female	D	D	C	D	A	C
22	Male	C	B	C	C	A	C
23	Female	C	D	D	C	D	C
24	Female	B	C	C	B	A	C
25	Male	D	C	C	D	A	C
26	Female	C	D	D	C	D	D
27	Female	C	D	D	C	A	C
28	Female	D	D	C	D	D	C

Data Set: Sweden							
Student #	**Question (1)**	**Question (2)**	**Question (3)**	**Question (4)**	**Question (5)**	**Question (6)**	**Question (7)**
29	Female	D	D	D	D	C	C
30	Female	D	D	C	C	C	C
31	Male	B	D	C	C	D	D
32	Female	C	D	B	D	A	C
33	Female	C	D	B	C		C
34	Male	C	D	D	C	A	B
35	Female	C	D	D	C	D	C
36	Female	B	D	D	C	B	D
37	Female	C	D	D	C	B	C
38	Male	C	C	C	B	D	C
39	Female	C	C	B	C	A	C
40	Female	C	C		C	D	C
41	Female	C	C	D	C	B	D
42	Female	B	C	D	C	A	B
43	Male	D	C	C	C	A	D
44	Female	C	D	C	D	B	D
45	Female	C	C	C	C	B	C
46	Male	C	D	D	C	A	C
47	Male	C	D	D	C	A	C
48	Female	C	C	D	D	A	D
49	Male	C	C	C	B	A	C
50	Female	D	D	C	C	D	D
51	Male	C	D	D	D	D	D
52	Male	C	D	D	D	D	C
53	Male	B	C	D	C	A	C
54	Male	C	D	D	C	C	C
55	Female	D	D	D	B	B	D
56	Female	C	C	D	C	D	C

Random Number Table

7	8	7	2	1	7	2	6	4	7	0	4	8	9	9	6	5	9	4	8
1	6	5	4	2	0	8	6	3	3	7	5	6	2	6	8	4	0	0	6
6	9	7	0	9	6	8	2	1	2	6	5	7	7	8	8	0	2	8	1
5	6	9	7	7	0	8	7	2	7	6	4	5	3	9	5	8	9	9	7
7	8	7	5	1	9	0	2	4	6	4	9	2	5	5	1	9	4	0	8
4	3	3	1	0	4	4	8	9	4	4	3	4	3	5	5	5	7	9	3
1	6	7	8	0	4	5	8	1	4	8	9	6	0	3	0	8	3	2	8
9	8	5	9	9	8	1	2	8	3	2	3	3	9	9	0	5	2	7	7
1	0	1	8	2	0	0	2	3	1	9	3	4	6	7	8	0	1	4	0
6	8	8	2	2	5	0	7	7	4	1	6	7	8	1	6	7	9	2	3
6	3	1	6	3	8	2	5	1	1	5	7	3	2	6	0	7	2	0	6
4	7	5	2	1	2	4	4	7	3	1	1	1	7	0	8	0	0	0	8
9	8	0	3	0	9	2	7	0	6	3	0	5	2	2	4	3	4	0	6
3	8	3	2	5	5	4	1	6	1	0	9	1	7	6	3	3	7	9	0
3	6	4	0	6	8	0	5	0	2	0	1	4	1	0	2	1	1	5	1
5	7	0	7	1	7	5	5	3	0	0	9	2	3	8	8	9	1	1	8
3	2	8	4	4	2	6	7	5	6	8	6	4	8	6	7	6	1	0	7
0	8	9	9	7	2	4	9	7	6	8	5	4	3	8	8	3	9	4	1
1	9	4	2	0	4	3	6	5	0	5	4	2	8	0	7	3	1	0	7
3	6	7	7	7	8	9	4	2	8	6	3	5	3	2	9	9	0	7	9
3	0	9	9	0	7	8	9	8	3	3	0	9	7	0	9	3	3	0	5
7	3	8	9	5	5	9	4	9	3	3	3	3	1	3	7	1	8	6	0
5	3	5	6	1	7	5	6	4	9	1	2	4	4	8	4	9	9	2	2
7	5	2	6	4	1	4	4	3	2	3	9	2	8	8	7	3	7	8	9
9	8	1	6	7	0	2	9	6	0	4	2	4	3	2	9	5	1	7	1
0	8	0	3	8	6	4	5	3	1	1	4	5	0	1	5	6	0	1	5
7	8	1	6	0	3	3	0	5	3	6	8	2	1	8	8	5	4	2	7
5	5	9	9	7	1	0	4	6	9	3	4	5	9	5	9	3	3	3	4
1	6	5	8	1	5	6	4	8	6	7	4	4	4	4	8	1	7	7	5
2	7	9	4	5	6	2	7	9	5	8	9	4	4	8	9	9	5	5	7
8	0	6	0	1	7	6	0	2	8	9	9	0	1	8	3	1	6	6	6
5	7	6	0	4	6	9	4	5	5	7	0	5	3	7	9	6	8	4	2
4	5	3	2	5	2	0	8	5	5	0	5	1	0	6	4	2	5	6	2
3	7	9	3	2	0	6	6	6	6	6	5	7	1	5	7	8	7	1	4
0	3	1	3	5	7	8	0	6	8	7	2	4	8	3	1	1	0	9	9
7	4	9	7	2	4	7	1	7	5	2	3	3	1	4	4	6	7	5	2
2	8	3	7	3	9	0	0	1	1	7	8	1	9	5	0	0	1	3	9
5	2	8	3	1	3	0	2	5	9	0	6	6	2	1	2	7	7	2	5
9	5	1	3	8	0	0	3	3	8	7	1	1	7	0	6	8	6	0	4
5	4	3	5	0	3	8	2	2	7	0	6	8	0	5	1	9	7	6	4
0	1	2	9	7	5	1	4	3	9	5	8	8	6	7	7	4	3	7	6
2	6	1	2	0	0	8	1	2	1	7	7	5	4	0	4	8	0	1	6
4	0	6	8	9	9	6	9	0	3	6	6	5	2	6	2	2	4	2	0

Squares for Simulation
